Financial Integration and Monetary Policy Coordination*

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Abstract

Financial integration generates macroeconomic spillovers that may require international monetary policy coordination. We show that individual central banks may set nominal interest rates too low or too high relative to the cooperative outcome. We identify three sufficient statistics that determine whether the Nash equilibrium exhibits under-tightening or over-tightening: the output gap, sectoral differences in labor intensity, and the trade balance response to changes in nominal rates. We find that, independently of the shocks hitting the economy, under-tightening is possible during economic expansions or contractions. For large shocks, the gains from coordination can be substantial.

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1 Introduction

After a prolonged period of expansionary monetary policy, central banks around the world shifted to a tightening cycle in early 2022 to tame rising inflation. However, the rapid pace and synchronous nature of the increase in interest rates raised concerns that the unprecedented monetary tightening could lead to a severe economic downturn. In this context, there has been a renewed discussion on the necessity of cooperation to avert a global recession and achieve a soft landing (Obstfeld, 2022).[1,2]

At the heart of these policy discussions are the following questions: Does cooperative monetary policy necessarily prescribe lower interest rates? Or is it possible that countries may insufficiently tighten monetary policy relative to the social optimum? In a broader sense, what are the benefits from international coordination of monetary policy, and how do they depend on the degree of financial integration?

The study of international monetary policy cooperation has a long history in the international macro literature, dating back to Hamada (1976), and Canzoneri and Henderson (1991). In the context of the traditional Mundell-Flemming model, early studies argued that countries have incentives to weaken their currencies to gain a trade advantage, which results in competitive devaluations and widespread inflation. By contrast, modern international macro-models with explicit microfoundations, as exemplified by Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2001), predict that countries have incentives to appreciate their currencies to improve their terms of trade and extract more rents from foreign countries. From this perspective, dealing with the strategic manipulation of terms of trade calls for cooperation towards more expansionary monetary policies.[3] Moreover, the gains from cooperation in this literature emerge purely from trade flows and are present even in the absence of financial flows.

In this paper, we approach the questions on international monetary coordination from a different, intertemporal perspective. Central to the theory is the insight that monetary policy has effects on an intertemporal price—namely, the world real rate—and through this channel, central banks affect the ability of other central banks to stabilize demand and output. Specifically, each country's monetary policy affects the supply

[1] Maurice Obstfeld argues, "Central banks nearly everywhere feel accused of being on the back foot. The present danger, however, is that they collectively go too far and drive the world economy into an unnecessarily harsh contraction ...by simultaneously all going in the same direction, they risk reinforcing each other's policy impacts without taking that feedback loop into account."

[2] See Figure 1 for the evolution of inflation and policy rates in advanced economies.

[3] From a quantitative standpoint, however, the consensus in the literature following Obstfeld and Rogoff (1995) is that the gains from cooperation due to this trade channel are negligible.

of savings and generates international spillovers by affecting the world interest rate. Given the intertemporal nature of this mechanism, distinct from the static terms of trade manipulation, we refer to it as the *financial channel of international spillovers*. Our goal in this paper is to provide a general characterization of these financial spillovers to understand whether cooperative monetary policy requires lower or higher interest rates.[4]

Our model features a continuum of identical countries populated by identical households that can trade in a perfectly integrated capital market. The model has two types of goods, tradables and non-tradables, and labor that can reallocate across sectors. The economy is subject to nominal wage rigidities as well as sticky prices. In particular, we assume that intermediate good producers use tradable and non-tradable inputs and face Rotemberg cost of changing prices. In addition, wages are rigid in domestic currency, and labor in each sector is determined by firms' labor demand. We consider a temporary shock in this environment and evaluate the macroeconomic adjustment under optimal cooperative and non-cooperative monetary policy.

We begin by examining the optimal monetary policy for a single country. The combination of price and wage rigidities gives rise to a deviation from divine coincidence (Blanchard and Galí, 2007). Our first set of results concerns the optimal targeting rule. We show that a central bank should strike a balance between the output gap, CPI inflation, and the trade balance. The fact that the central bank targets CPI inflation, as observed in practice, contrasts with much of the literature where the central bank targets PPI inflation (see, e.g., Gali and Monacelli, 2005; Itskhoki and Mukhin, 2023).[5]

Moreover, the central bank targets the *trade balance* because a change in the trade balance induces a reallocation of demand over time and across sectors, which may help to stabilize the labor wedge and the level of inflation.[6]

We then examine the Nash equilibrium and cooperative monetary policy. Our main result is that the Nash equilibrium may feature nominal rates that are too high (*over-tightening*) or too low (*under-tightening*) relative to the cooperative outcome. Whether

[4]In an earlier working paper (Bianchi and Coulibaly, 2021), we examined how these financial spillovers can emerge in the context of liquidity traps and argued they can give rise to currency wars. The present paper provides more general foundations of the financial channel and formally characterizes the gap between the optimal cooperative and non-cooperative monetary policy. In a recent contribution, Fornaro and Romei (2023) also study optimal monetary policy coordination in an environment where monetary policy affects the world real interest rate. We discuss in detail below how our framework and conclusions differ from theirs.

[5]A notable exception is Engel (2011), who, in a setup with local currency pricing, derives a global loss function and an associated targeting rule that depends on CPI inflation under cooperation. However, in his framework, the targeting rule does not depend on CPI inflation under non-cooperative policy.

[6]Following Gali and Monacelli (2005), targeting rules in open economies often feature terms-of-trade in addition to domestic targets (see e.g., De Paoli, 2009; Corsetti, Dedola and Leduc, 2010; Egorov and Mukhin, 2023). However, the trade balance does not affect these tradeoffs.

the lack of coordination leads to over-tightening or under-tightening depends on three sufficient statistics: the output gap, the difference in labor intensity across sectors, and the response of the trade balance to movements in the exchange rate. For example, when the economy faces a recession, the Nash equilibrium displays over-tightening if non-tradables are more labor intensive than tradables and the trade balance increases in response to a devaluation. However, there are equally plausible constellations with under-tightening. For example, if non-tradables are more labor intensive, the Nash equilibrium displays too low interest rates when the economy faces *overheating* and the trade balance *increases* in response to a devaluation, or when the economy faces a *recession* and the trade balance *decreases* in response to a devaluation.

The general logic behind these results is that countries do not internalize how using monetary policy to steer capital flows affects the world real rate and how this, in turn, affects welfare abroad. As we show in Section 4.2, the effect of a change in the nominal rate set by all central banks, $\boldsymbol{R}_0$, on welfare can be expressed as follows:

$$\frac{\partial \mathcal{U}_k(R_{k,0}, \mathcal{R}_0^*)}{\partial R_{k,0}} + \frac{d\mathcal{R}_0^*}{d\boldsymbol{R}_0} \frac{\partial \mathcal{U}_k}{\partial R_0^*}, \tag{1}$$

where $\mathcal{U}_k(R_{k,0}, \mathcal{R}_0^*)$ is the indirect utility flow of a country k, which depends on its nominal rate $R_{k,0}$ and the world real rate $\mathcal{R}_0^*$. Individual countries take $\mathcal{R}_0^*$ as given and therefore equate the first term in (1) to zero. In general equilibrium, changes in nominal rates in all countries affect the world real rate, and this potentially affects other countries' welfare. In particular, we argue that to the extent that countries are unable to stabilize output and inflation, a change in the world real rate will affect their welfare. The second term in (1) captures that the global planner internalizes this externality.

Whether the Nash equilibrium features over- or under-tightening therefore is determined by the answer to two questions. The first question is whether countries benefit from an increase or decrease in the world real rate. The second question is whether the world real rate is increasing or decreasing in the nominal rate. Our analysis shows that depending on the output gap and labor intensities, countries may benefit from an increase or a decrease in the world real rate. Additionally, depending on the response of the trade balance to a change in the nominal rate, the real rate may increase or decrease with changes in the nominal rate.

To focus on a concrete example, consider an economy facing a negative output gap where non-tradables are more labor intensive. To the extent that wages are rigid and inflation is costly, the central bank finds it optimal to expand monetary policy to help reduce the output gap, at the expense of higher inflation. In this scenario, we argue that a *reallocation of employment from a low labor-intensity sector to a high labor-intensity sector helps mitigate inflation because the high labor-intensive sector has a lower elasticity of marginal cost with respect to output* (or equivalently, a flatter Phillips curve). Consequently, to the extent that the non-tradable sector is more labor intensive than the tradable sector, a shift in employment towards non-tradables would lead to an overall reduction in inflation.

In turn, the allocation of employment across sectors depends crucially on financial flows and the world real rate. If the world real rate is lower, households borrow more from abroad, which results in higher demand for both tradable and non-tradable consumption goods. In equilibrium, the higher demand for non-tradable goods leads to an increase in employment in the non-tradable sector (while employment in the tradable sector is independent of domestic demand conditions). Therefore, higher capital inflows result in relatively more employment in the non-tradable sector and help reduce inflation, as argued above. That is, in this case, we have $\partial \mathcal{U}_k / \partial R_0^* < 0$.

The other key element is how monetary policy affects financial flows and the world real rate, a point that relates back to the classic Marshall-Lerner condition.[7] Consider again the example of the country in a recession where non-tradables are more labor intensive, and suppose that an exchange rate appreciation increases capital inflows (i.e., the Marshall-Lerner condition holds). It thus follows that a central bank has incentives to try to appreciate its exchange rate and generate a reallocation of employment towards non-tradables that helps to lower inflation. However, an attempt by all countries to appreciate the exchange rate and increase capital inflows is self-defeating. While each country perceives it can do so individually, the result in general equilibrium is that the world real rate is higher (i.e., $d\mathcal{R}_0^* / d\boldsymbol{R}_0 > 0$), and central banks end up with a nominal interest rate that is too high relative to the cooperative outcome.

Putting the two elements together, we see that from (1) that in this example, the planner perceives a higher marginal utility cost from higher rates. That is, the Nash equilibrium displays over-tightening. In the case where an appreciation leads instead to capital outflows (i.e., the Marshall-Lerner condition fails), the above conclusion reverses. That is, central banks set an interest rate that is too low relative to the cooperative outcome in an attempt to generate a higher trade surplus and reduce inflation. In this case, the Nash

[7]The well-known Marshall-Lerner condition relates the change in the nominal exchange rate to the trade balance as a function of the elasticities of exports and imports. Following the convention, the Marshall-Lerner condition is said to hold when an appreciation of the exchange rate (or equivalently, an increase in the domestic nominal interest rate) generates an increase in the trade deficit.

equilibrium displays under-tightening.[8]

In sum, whether cooperation calls for lower or higher rates can be framed entirely in terms of the sign of the output gap, the sign of the product of the differences in labor intensity between the tradable sector and the non-tradable sector, and the response of the trade balance to a monetary expansion. The overall principle is that when central banks use monetary policy to steer capital flows, the world real interest rate in general equilibrium is altered, and there are adverse welfare effects.

Our quantitative analysis shows that the differences between the cooperative and the non-cooperative equilibrium can be quite substantial. Although the welfare gains are modest for small shocks, they can quickly become quite large for moderately large shocks. For example, for shocks leading to an inflation of 3% in the Nash equilibrium, the difference in the level of output between the cooperative and the Nash equilibrium is close to 2%.

In one extension, we allow for the anticipation of future shocks. In this case, we show that while the cooperative solution maintains zero inflation and zero output gap in response to the news shock, the Nash equilibrium exhibits one of two outcomes, either overheating and inflation or recession and deflation. The sign of the output gap, the differences in labor intensity, and the response of the trade balance to a monetary expansion remain the three key sufficient statistics, as in our baseline analysis. These results also hold when we extend the model to allow for costly labor reallocation or oil price shocks.

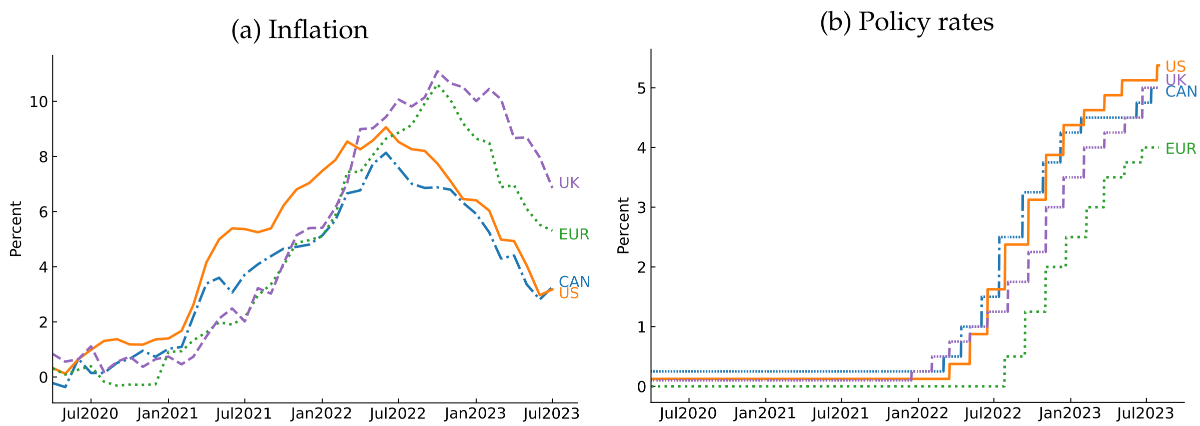

Figure 1: Synchronous Monetary Policy Tightening

[8] In an economy that is overheated, a central bank seeks to run a trade surplus to reallocate labor *away* from non-tradables insofar as non-tradables are more labor intensive. If the Marshall-Lerner condition holds (fails), central banks then under-tighten (over-tighten).

Related literature. Our paper belongs to a vast literature on international monetary policy coordination.[9] As mentioned above, a key theme in much of this literature is a terms of trade channel by which individual countries have incentives to manipulate their terms of trade in their favor at the expense of other countries. According to the optimal tariff argument, central banks generally over-tighten monetary policy relative to the socially optimal level, independently of the degree of financial integration. By contrast, we highlight a financial channel involving an intertemporal price (i.e., the world real interest rate) and show that this generates the possibility of under-tightening.

Fornaro and Romei (2023) is a notable exception that studies the gains from coordination when monetary policy affects the world real interest rate. In their model, a global increase in the preference for tradable goods leads to inflation and a negative output gap in equilibrium. They find that cooperative monetary policy prescribes higher output levels relative to the Nash equilibrium. Our model differs from theirs by considering a more general structure with elastic labor supply, diminishing returns in labor, non-unitary elasticities of substitution, and a welfare function that depends endogenously on inflation.[10] Our analysis shows that the Nash equilibrium may exhibit over-tightening or under-tightening and elucidates how this outcome depends on a set of sufficient statistics. Namely, we establish analytically that, independently of the shocks, whether cooperation calls for lower or higher rates depends on the degree of slack in the economy, the differences in labor intensities across sectors, and the response of the trade balance to a monetary expansion.[11]

Our paper is also related to the literature that examines the potential for international coordination in the context of various government policies. Chang (1990) and Kehoe (1987) study the coordination of fiscal policies when fiscal deficits in some countries make

[9]For early contributions in the context of static Mundell-Flemming models, see Hamada (1976), Oudiz and Sachs (1984), Canzoneri and Gray (1985), and Canzoneri and Henderson (1991). For modern models with microfoundations, see Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001, 2005), Tille, 2001; Obstfeld and Rogoff (2002) Clarida, Galı and Gertler (2002); Canzoneri, Cumby and Diba (2005); Devereux and Engel (2003); Benigno (2009); Egorov and Mukhin (2023); and Bodenstein, Corsetti and Guerrieri (2020).

[10]In their setup, a fixed endowment of hours implies that overheating cannot occur, linear production for non-tradables rules out inflation in non-tradables, unitary elasticities of substitution imply that the trade balance always increases in response to a depreciation, and the utility function is assumed to depend exogenously on inflation.

[11]Two other recent papers are Caldara, Ferrante, Iacoviello, Prestipino and Queralto (2023), which studies non-linear effects from monetary spillovers in a model with global banks and Acharya and Pesenti (2024) on spillovers in a model with heterogeneity. Previous work by Acharya and Bengui (2018), Eggertsson, Mehrotra, Singh and Summers (2016), Caballero, Farhi and Gourinchas (2021), and Fornaro and Romei (2019) studies the propagation of liquidity traps across countries but does not consider the scope for monetary policy cooperation. For the empirical literature on international monetary policy spillovers, see, for example, Rey (2013) and Kalemli-Ozcan (2019).

it more costly for others to finance their deficit (see also Azzimonti, de Francisco and Quadrini, 2014). In Halac and Yared (2018), governments exhibit present bias, and fiscal rules are more slack under coordination. Obstfeld and Rogoff (1996) study a two-period model where a borrower country that has market power over the world interest rate has incentives to tax foreign borrowing. In an infinite-horizon setup with a large country, Costinot, Lorenzoni and Werning (2014) show that the desire to use capital controls emerges from a dynamic terms of trade manipulation motive.[12] In our paper, countries are infinitesimal, and the case for coordination is due to a pecuniary externality, where the world real interest rate influences monetary policy tradeoffs.

The key mechanism at play in our model is also related to the literature on aggregate demand externalities. In Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2016), nominal rigidities and constraints on monetary policy create a rationale for capital controls. In our model, monetary policy faces no constraints, but divine coincidence fails, generating aggregate demand externalities. Crucially, the scope for monetary policy cooperation emerges because of the interaction between this aggregate demand externality and a pecuniary externality operating through the world real rate.

Finally, there has been an active recent literature on the rise of inflation following the COVID-19 pandemic and the connection with sectoral reallocation.[13] Besides our open economy focus, we also contribute to this literature by highlighting for the first time, to the best of our knowledge, the importance of differences in labor intensity across sectors for the determination of inflation, output, and the optimal monetary policy.

Outline. Section 2 presents the model. Section 3 presents the Nash equilibrium, and Section 4 presents the optimal monetary policy under cooperation. Section 5 presents extensions of the basic framework. Section 6 concludes.

[12] Other recent examples are Clayton and Schaab (2022) on macroprudential policy with multinational banks, Bengui and Coulibaly (2022) on capital flow management policies in a two-country model, and Chari, Nicolini and Teles (2023) on fiscal and trade policies in a multi-country business cycle model.

[13] See, for example, Guerrieri, Lorenzoni, Straub and Werning (2021, 2022), La'O and Tahbaz-Salehi (2022), Rubbo (2023), di Giovanni, Kalemli-Özcan, Silva and Yildirim (2022, 2023), Baqaee and Farhi (2022), Baqaee, Farhi and Sangani (2024), and Afrouzi and Bhattarai (2023). Baqaee and Rubbo (2023) provides a review of this literature.

2 Model

Time is discrete and infinite. We model the world economy as a continuum of identical small open economies indexed by $k \in [0,1]$. Each economy is composed of a tradable sector and a non-tradable sector, denoted with superscripts T and N, respectively. To avoid clutter in the notation, we do not index variables in each country by k. We will use $\{x_t\}$ to refer to the sequence $\{x_{k,t}\}_{t=0}^{\infty}$ for some variable x and country k.

We next describe the problem faced by households and firms in each economy k and then describe the competitive equilibrium.

2.1 Households

Preferences. Each economy is populated by a continuum of households with preferences described by

$$\sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \kappa_t n_t \right], \tag{2}$$

where $\beta \in (0,1)$ is the discount rate, c_t is a composite consumption good, and U is a strictly increasing and concave utility function with inverse intertemporal elasticity of substitution σ_t. Households face a disutility from working that is linear in total hours $n_t = n_t^T + n_t^N$.

Budget constraint. The budget constraint is given by

$$P_t c_t + \frac{b_{t+1}}{R_t} + \frac{P_t^T b_{t+1}^*}{R_t^*} = W_t n_t + \varphi_t + b_t + P_t^T b_t^*, \tag{3}$$

where P_t denotes the price of the composite consumption good, P_t^T denotes the price of tradables, W_t denotes the wage and φ_t denotes profits from domestic firms, all expressed in units of domestic currency. Notice that we have assumed implicitly that the wage is equal in both sectors, a result that follows in equilibrium because the utility function is linear in total hours. Households have two assets available, a real international bond that pays R_t^* units of tradables and a nominal domestic bond that pays R_t in units of the domestic currency. These assets are referred to as b_t^* and b_t, respectively.

Optimality conditions. The problem of the household consists of choosing a sequence of hours, asset positions, and consumption to maximize the expected present discounted value of utility (2), subject to the budget constraint (3) and a no-Ponzi-game condition.

For $t = 0$, we assume that wages are rigid and households are off their labor supply. For $t > 0$, we assume that wages are flexible and thus satisfy[14]

$$\frac{W_t}{P_t} = \frac{\kappa_t}{U'(c_t)}. \tag{4}$$

The optimality conditions with respect to assets holdings yield

$$\frac{U'(c_t)}{P_t} = \beta R_t^* \frac{P_{t+1}^T}{P_t^T} \left[\frac{U'(c_{t+1})}{P_{t+1}} \right], \tag{5}$$

$$R_t^* = R_t \frac{P_t^T}{P_{t+1}^T}. \tag{6}$$

Condition (5) is the Euler equation for the real bond. Condition (6) is a no-arbitrage condition that equates the return on the real international bond and the domestic currency bond expressed in units of tradables.

2.2 Firms and Production

Final good. The final consumption good is produced by perfectly competitive firms. They combine differentiated intermediate goods q_{jt} according to the following CES production function:

$$q_t = \left(\int_0^1 (q_{jt})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ denotes the elasticity of substitution. Cost minimization implies the following demand for each input: $q_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon} q_t$, where p_{jt} is the price of intermediate input q_{jt} and the price of the final consumption good is $P_t = \left(\int_0^1 (p_{jt})^{1-\varepsilon} ds \right)^{\frac{1}{1-\varepsilon}}$.

Intermediate good. The production of intermediate goods is conducted by retailers in a monopolistically competitive market. To produce the intermediate consumption good, a retailer $j \in [0, 1]$ has access to a technology that combines tradable consumption goods c_{jt}^T and non-tradable consumption goods c_{jt}^N according to

$$q_{jt} = \left(c_{jt}^T\right)^{\phi^T} \left(c_{jt}^N\right)^{\phi^N}, \tag{7}$$

[14] One way to rationalize an initial wage off the equilibrium value at $t = 0$ is to assume there was uncertainty at $t = -1$ when wages were set.

with $\phi^T \in (0,1)$ and $\phi^N = 1-\phi^T$. Denote by P_t^T and P_t^N the price of the tradable and non-tradable consumption good, respectively. Cost minimization implies

$$\phi^T P_t^N c_{jt}^N = \phi^N P_t^T c_{jt}^T. \tag{8}$$

That is, retailers have a constant share of expenditure in tradable and non-tradable goods. The marginal cost of producing the intermediate good is given by

$$\mathcal{M}_t = \left(\frac{P_t^T}{\phi^T}\right)^{\phi^T} \left(\frac{P_t^N}{\phi^N}\right)^{\phi^N}. \tag{9}$$

When setting prices, retailers incur a quadratic adjustment cost à la Rotemberg (1982). In particular, firms face the cost $\frac{\chi}{2}\left(\frac{p_{jt}}{p_{j,t-1}}-1\right)^2$ in units of the final consumption good. The retailer j then chooses p_{jt} to solve

$$\max_{p_{jt}} \sum_{t=0}^{\infty} \Lambda_{t,0}\frac{P_0}{P_t}\left[\left[(1+\varrho)p_{jt}-\mathcal{M}_t\right]\left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon} q_t - \frac{\chi}{2}\left(\frac{p_{jt}}{p_{j,t-1}}-1\right)^2 P_t q_t\right],$$

where $\Lambda_{t+j,t} \equiv \beta^{t+j}\frac{U'(c_{t+j})}{U'(c_t)}$ is the discount factor of households between dates t and $t+j$ and $\varrho = \frac{1}{\varepsilon-1}$ is the standard subsidy to offset the markup distortion. The optimality condition for p_{jt} evaluated at the symmetric equilibrium with $p_{jt} = P_t$ yields

$$(1+\pi_t)\,\pi_t = \frac{\varepsilon}{\chi}\left[\frac{\mathcal{M}_t}{P_t}-1\right] + \Lambda_{t+1,t}\frac{q_{t+1}}{q_t}\,(1+\pi_{t+1})\,\pi_{t+1}, \tag{10}$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$ denotes the consumer price index (CPI) inflation. Condition (10) is the dynamic Phillips curve which positively relates current CPI inflation to the marginal cost and future inflation.

Tradables and non-tradable inputs. The production of tradable goods and non-tradable goods is conducted by firms in a perfectly competitive market. Output of the two goods $i = \{T, N\}$ is produced using labor with a production function F^i so that

$$y_t^i = F^i(h_t^i, A_t^i).$$

We assume an isoelastic production function such that $F^i(h_t^i, A_t^i) = A_t^i (h_t^i)^{\alpha^i}$. We refer to α^i as the *labor intensity* parameter.

Profits are given by $P_t^i F^T(h_t^i, A_t^i) - W_t h_t^i$. At the optimum, firms equate the marginal product of labor to the nominal wage in the two sectors:

$$P_t^T F_h^T(h_t^T, A_t^T) = W_t, \tag{11}$$

$$P_t^N F_h^N(h_t^N, A_t^N) = W_t. \tag{12}$$

Given competitive markets, the labor intensity equals the labor share for each sector in equilibrium. As we will see, differences in labor intensity across sectors, $\alpha^N - \alpha^T$, will play an important role in the analysis.

We note that the fact that labor is the only factor of production or that the production function exhibits decreasing returns to scale is not restrictive. In Section 5, we incorporate oil as an additional factor of production, and we show that what matters for the results is the labor intensity and not the overall scale of the production function.

2.3 Monetary Policy

In each small open economy, there is a central bank, which chooses nominal interest rates $\{R_t\}$. Because of the assumption that wages are flexible for $t > 0$, the only source of inefficiency is the costly price adjustment. Therefore, optimal monetary policy implements a strict inflation targeting regime such that $\pi_t = 0$ for $t > 0$. For $t = 0$, we will evaluate the optimal monetary policy, comparing the cooperative and non-cooperative outcomes.

2.4 Competitive Equilibrium

We assume that the law of one price holds for the tradable good. If we denote by P_{jt}^T the price of the tradable good in terms of the country j currency, it follows that $P_{kt}^T = P_{jt}^T e_{kt}^j$ for any pair of countries k and j, where e_{kt}^j is the nominal exchange rate defined as the price of the country j currency in terms of the country k currency.

In each country, the market for non-tradable goods must clear. That is,

$$c_t^N = F^N(h_t^N, A_t^N). \tag{13}$$

At $t = 0$, households in each country supply hours in the tradable and non-tradable sectors to meet the demand by firms. For $t > 0$, the labor clears the labor market. That is, $n_t^T = h_t^T$ and $n_t^N = h_t^N$.

Market clearing for the final good consumption requires

$$c_t = \left[1 - \frac{\chi}{2}(\pi_t)^2\right] q_t, \tag{14}$$

We assume without loss of generality that the bond denominated in domestic currency is only domestically traded in each country. [15] Market clearing therefore implies

$$b_{t+1} = 0. \tag{15}$$

Finally, at the world level, real bonds are in zero net supply. To account for market clearing at the world level, we now explicitly index the policies of each country by k. We have that

$$\int b^*_{k,t+1} dk = 0. \tag{16}$$

We now define a competitive equilibrium in the global economy.

Definition 1 (Competitive Equilibrium)**.** Given initial positions $b^*_{k,0}$, a sticky wage W, and a sequence of central bank policies $\{R_t\}$ in each country k, an equilibrium is a sequence of world real rates $\{R^*_t\}$, prices $\{P^T_t, P^N_t, W_t, e^j_{k,t}\}$ and allocations $\{c^T_t, c^N_t, h^T_t, h^N_t, b_{t+1}, b^*_{t+1}, \pi_t\}$ in each country k such that

(i) households optimize, and hence conditions (8), (5), (6) hold for all $t \geq 0$, and (4) holds for all $t \geq 1$;

(ii) firms optimize, which implies (11) and (12) hold for all $t \geq 0$;

(iii) the law of one price holds for tradables: $P^T_{k,t} = P^T_{j,t} e^j_{k,t}$ for any country-pair k and j;

(iv) the market for non-tradables (13) and domestic bonds (15) clears; moreover, the labor market clears for $t \geq 1$;

(v) globally, the market for the real bond clears; that is, (16) holds.

If we combine the budget constraints of households and firms as well as market clearing conditions, we arrive at the country budget constraint for tradables, or the balance of payment condition:

$$c^T_t - F^T(h^T_t, A^T_t) = b^*_t - \frac{b^*_{t+1}}{R^*_t}, \tag{17}$$

[15] With trade of nominal bonds, monetary policy would also be guided by the incentives to alter the real value of its portfolio. We abstract from this channel to focus on the output-inflation tradeoff and the implications for international coordination. Moreover, notice that because there is no uncertainty, there is not a meaningful portfolio choice for households.

which says that if a country runs a trade deficit, it accumulates net debt, and if it runs a trade surplus, it accumulates net external assets.

We assume that all countries start at $t = 0$ with zero net foreign asset position. To the extent that all countries follow the same policies, we can therefore restrict the analysis to symmetric competitive equilibrium.

Reformulating preferences. Using that in equilibrium $q_{jt} = q_t, c^N_{jt} = c^N_t, c^T_{jt} = c^T_t$, together with (7) and (14), we can write the utility for the representative agent as a function of consumption of the two goods and inflation.[16] We denote this as $u(c^T_t, c^N_t, \pi_t)$. A feature of our environment is that CPI inflation effectively reduces the resources available for consumption. Together with wage rigidity, these two ingredients will generate tradeoffs for monetary policy.

2.5 Efficient Allocation, Output Gaps, and the Natural Wage

We conclude the description of the model by presenting the first-best allocation. We consider a benevolent social planner of the world economy who chooses allocations to maximize welfare, subject to resource constraints. The planner's problem can be written as

$$\max_{\{h^N_t, h^T_t\}} \sum_{t=0}^{\infty} \beta^t \left[u\left(F^T(h^T_t, A^T_t), F^N(h^N_t, A^N_t), 0 \right) - \kappa_t \left(h^T_t + h^N_t \right) \right].$$

Notice that because all countries are identical, we have replaced that the output of tradables must equal consumption of tradables in each country.

The necessary and sufficient conditions for optimality are

$$F^T_h(h^T_t, A^T_t)\, u_T\left(F^T(h^T_t, A^T_t), F^N(h^N_t, A^N_t), 0 \right) = \kappa_t, \tag{18}$$

$$F^N_h(h^T_t, A^N_t)\, u_N\left(F^T(h^T_t, A^T_t), F^N(h^N_t, A^N_t), 0 \right) = \kappa_t. \tag{19}$$

Let us denote by $\bar{h}^T_t$ and $\bar{h}^N_t$ the employment levels in the two sectors in the first-best allocation. The following lemma shows that the ratio of employment levels can be expressed as the product of the relative weights in preferences and the relative labor intensities.

[16] Using these expressions, we have that $U(c_t) = U\left(\left[1 - \frac{\chi}{2} (\pi_t)^2 \right] (c^T_t)^{\phi^T} (c^N_t)^{\phi^N} \right)$.

Lemma 1 (First-Best)**.** *The optimal ratio of hours in the first-best allocation is given by*

$$\frac{\bar{h}_t^N}{\bar{h}_t^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T}. \tag{20}$$

Proof. In Appendix A.1 □

We highlight that the first-best allocations coincide with those in a competitive equilibrium in a flexible wage version of our model. This can be seen by noting that if the nominal wage were flexible, we would arrive at (18) and (19) by combining firms' demand for labor (11) and (12) with households' labor supply decisions (4). This result will provide a clear benchmark for the normative analysis.

The labor wedge and the natural wage. The assumption that prices for tradables and non-tradables are flexible and the fact that wages are equalized across sectors implies that in any competitive equilibrium,

$$F_h^T(h_t^T, A_t^T) u_T(c_t^T, c_t^N, \pi_t) = F_h^N(h_t^N, A_t^N) u_N(c_t^T, c_t^N, \pi_t). \tag{21}$$

We thus have a *single labor wedge* that is common in the tradable and non-tradable sector, which we denote by τ_t and is given by

$$\tau_t \equiv 1 - \frac{\kappa_t}{F_h^T(h_t^T, A_t^T) u_T(c_t^T, c_t^N, \pi_t)}. \tag{22}$$

We define the *natural wage* as the wage, in units of the final good, that would prevail in equilibrium if prices and wages were flexible. Using (10), (11), (12) and $\chi = 0$, we obtain that the natural wage at date t is given by

$$w_t^n = \prod_{i=T,N} \left(\alpha^i \phi^i A_t^i\right)^{\phi^i} \left(\bar{h}_t^i\right)^{-(1-\alpha^i)\phi^i}. \tag{23}$$

Equation (23) implies the natural wage falls when there is a decline in productivity for tradables or non-tradables or when there is a positive labor supply shock. As we will see next, the presence of sticky wages and prices will generate tradeoffs for monetary policy.

3 Optimal Monetary Policy in a Nash Equilibrium

This section studies non-cooperative monetary policy. We model the non-cooperative game as a Nash equilibrium where central banks choose their monetary policy to maximize their own welfare, taking as given monetary policy abroad.

3.1 Monetary Policy for a Single Country

We first study the individual problem of a central bank that takes as given $\{R_t^*\}$ and policies conducted in other countries. We distinguish between the problem for $t \geq 1$ when prices are flexible and $t = 0$ when wages are sticky.

3.1.1 Time t $\geq$ 1 Problem

Given that wages are flexible for $t \geq 1$, we can focus on a situation where the central bank sets monetary policy to implement $\pi_t = 0$ for all $t \geq 1$, as this achieves the efficient allocation. The lifetime utility for a central bank with net foreign asset b_1^* in period 1 is given by

$$V_1(b_1^*) = \sum_{t=1}^{\infty} \beta^{t-1} \left[u\left(c_t^T, F^N(h_t^N, A_t^N), 0\right) - \kappa_t (h_t^T + h_t^N) \right], \tag{24}$$

where $\{c_t^T, h_t^T, h_t^N, b_{t+1}^*\}_{t=0}^{\infty}$ are the unique allocations satisfying (17),(21), $\tau_t = 0$ and

$$u_T\left(c_t^T, F^N(h_t^N, A_t^N), 0\right) = \beta R_t^* u_T\left(c_{t+1}^T, F^N(h_{t+1}^N, A_{t+1}^N), 0\right).$$

and the no-Ponzi-game condition.

3.1.2 Time t = 0 Problem

The central bank's policy choice in period 0 is the nominal interest rate. The central bank's objective is to choose an R_0 that maximizes the welfare of the domestic household subject to domestic allocations and prices consistent with a competitive equilibrium (given policies $\{R_{k,t}\}$ conducted in other countries).

Implementability constraints. Following a primal approach, we proceed to combine equilibrium conditions to express the implementability constraints in terms of allocations.

First, combining (8),(11) and (12), we arrive at an equation that determines the relative demand for hours in the two sectors as a function of the trade balance:

$$\frac{h_0^N}{h_0^T} = \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \left[1 - \frac{b_1^*}{R_0^* F^T(h_0^T, A_0^T)}\right]. \tag{25}$$

When a country accumulates net foreign assets (or equivalently, runs a larger trade balance surplus), it will display in equilibrium lower employment in non-tradables relative to tradables. The logic is as follows: the accumulation of net foreign assets implies lower available resources for consumption. Because preferences are homothetic, this means lower consumption for both tradables and non-tradables. As non-tradable goods are produced domestically, the decline in non-tradable consumption must be associated with lower hours worked in the non-tradable sector.

Second, using (9), (11), (12) and (23), we can express the Phillips curve (10) as

$$\left[1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2\right]\pi_0 = \frac{W}{w_t^n P_{t-1}} \left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T} \left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N} - 1, \tag{26}$$

which relates current inflation to employment in both sectors and the natural wage. Recall from (11) and (12) that for a given wage, higher employment in tradables or non-tradables requires higher prices in the respective sectors and thus higher inflation.

An important implication from (26) is that the labor intensities of the sectors play a crucial role in determining the extent to which higher employment in each sector raises inflation. To see this more clearly, we can totally differentiate firms' first-order conditions (11) and (12), and using that the nominal wage is constant, we obtain for $i = T, N$

$$d \log P_t^i = \frac{1-\alpha^i}{\alpha^i} d \log y_t^i.$$

That is, the higher is the labor intensity in each sector, the lower is the rise in prices needed to achieve a certain increase in output. Crucial for this result is that wages are sticky. Thus, if a good is more labor intensive, this means that firms can scale up production without significant raises in prices. As the curvature in the production function becomes lower, an increase in employment leads to a faster decline in the marginal product, thus necessitating a larger increase in prices to induce higher employment to be optimal for firms. Put differently, a higher labor intensity implies a lower elasticity of marginal cost (or equivalently, a flatter Phillips curve). To our knowledge, this role of labor intensity in shaping the response of inflation to a monetary expansion is a channel that has not received attention in the literature.

Finally, in addition to (25) and (26), the central bank is also subject to the household intertemporal Euler equation (6).

We can then write the Lagrangian for the central bank problem as

$$\begin{aligned}\mathcal{L} = u &\left(F^T(h_0^T, A_0^T) - \frac{b_1^*}{R_0^*}, F^N(h_0^N, A_0^N), \pi_0 \right) - \kappa_0 (h_0^T + h_0^N) + \beta V_1\left(b_1^*\right) \\ &+ \vartheta_0 \left[\left(1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0 - \frac{W}{w_0^n P_{t-1}} \prod_{i=T,N} \left(\frac{h_0^i}{\bar{h}_0^i}\right)^{(1-\alpha^i)\phi^i} + 1 \right] \\ &+ \eta_0 \left[\frac{\alpha^N \phi^N}{\alpha^T \phi^T} \left(1 - \frac{b_1^*}{R_0^* F^T(h_0^T, A_0^T)}\right) \frac{h_0^T}{h_0^N} - 1 \right] \\ &+ \mu_0 \left[u_T \left(F^T(h_0^T, A_0^T) - \frac{b_1^*}{R_0^*}, F^N(h_0^N, A_0^N), \pi_0 \right) - \beta R_0^* u_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*), 0\right) \right],\end{aligned} \tag{27}$$

where the continuation value for the central bank is given by (24) and we denote by ϑ_0, η_0, and μ_0 the corresponding Lagrange multipliers.

Remarks on the central bank problem. Three important observations from this problem are worth making. *First, the central bank cannot generally achieve the first-best allocation*. Because sticky prices for intermediate goods make inflation costly, the central bank may not be able to achieve a zero labor wedge and zero inflation simultaneously. In particular, from (26), we can see that if the natural wage deviates from the sticky wage and the central bank implements zero inflation, employment will deviate from the efficient level.[17]

Second, the only foreign variable that appears in the central bank problem is the world real rate. The reason is that although foreign monetary policies can alter the exchange rate vis-à-vis the domestic country, the domestic central bank can alter these movements by varying the nominal rate.[18] Because the presence of the world real rate reflects an intertemporal channel, we refer to it as the "financial channel of international spillovers."

Third, the trade balance not only affects the tradable resources available for consumption but

[17]Notice that the efficient level of employment for a small open economy (i.e., the one associated with a zero labor wedge) is generally different from $\bar{h}$. As we will see below, the two always coincide in the Nash equilibrium (i.e., when $b_1^* = 0$).

[18]A common policy argument in discussions on spillovers is that foreign monetary policy tightening leads to an appreciation of the foreign currency and export inflation abroad. This view is misguided according to our model. A country can always offset the effects on the domestic currency price of tradables by adjusting the interest rate in the same direction as foreign economies.

also affects the last two implementability constraints. As we will see, this will imply that a central bank will attempt to modify the trade balance through monetary policy to achieve a better menu of output and inflation.

Optimality conditions. The first-order necessary condition with respect to b_1^* yields[19]

$$\eta_0 = \left[\delta_0 - \phi^T + \sigma_0 \phi^T\right] u_T(c_0^T, c_0^N, \pi_0)\mu_0, \tag{28}$$

where δ_0 is given by (A.5) in Appendix A.2 and satisfies $\delta_0 > 1$. Condition (28) implies that the Lagrange multipliers on households' Euler equation (5) and households' intra-temporal allocation of hours worked (25) have the same sign. To understand why, suppose the central bank perceives a positive shadow value from raising the ratio of non-tradable employment to tradable employment (that is, $\eta_0 > 0$). Notice that if households were to borrow more, the increase in consumption would lead to higher demand for tradables and non-tradables. Higher demand for non-tradables implies higher hours employed in the non-tradable sector (while hours in the tradable sector are independent of domestic demand conditions).[20] Therefore, a higher level of borrowing would result in more hours in the non-tradable sector relative to those in the tradable sector (relaxing the constraints for the central bank). From the perspective of the central bank of the small open economy, this implies that a positive shadow value from higher non-tradable to tradable hours is associated with a positive shadow value from higher household borrowing.

Optimality with respect to h_0^T and h_0^N delivers a *targeting rule* for the small open economy described in the proposition below.

Proposition 1 (Targeting Rule)**.** *The optimal monetary policy for a single country targets*

$$\sum_{i=T,N} \delta_0^i \alpha^i \phi^i \, \tau_0 = \chi(1 + \psi_b b_1^*) \sum_{i=T,N} \delta_0^i (1 - \alpha^i)\phi^i \, \Theta(\pi_0)\pi_0, \tag{29}$$

where $\Theta(\pi_0) > 0$ *with* $\Theta(0) = \frac{\varepsilon}{\varepsilon + \chi}$ *defined in* (A.6)*, and* $1 + \psi_b b_1^* > 0$ *with* $sign(\psi_b) = sign(\alpha^N - \alpha^T)$ *defined in* (A.14)*. Moreover,* δ_0^N *and* δ_0^T *are positive coefficients defined in* (A.10) *and* (A.9)*, and recall also that* τ_0 *stands for the labor wedge.*

Proof. In Appendix A.2 □

[19] We proceed under the assumption that the first-order conditions are both necessary and sufficient for optimality. When we solve the model numerically, we verify this to be the case.

[20] For given monetary policy, employment of tradables remains actually fixed. This is because tradable employment depends only on the wage in units of tradables, and the price of tradables in the small open economy is determined by the law of one price.

Equation (29) characterizes how the central bank trades off the labor wedge, τ_0, with CPI inflation, π_0. There are two novel elements relative to standard open economy targeting rules. First, the relevant measure of inflation for welfare is CPI inflation. This contrasts with much of the literature where the central bank targets PPI inflation (see, e.g., Itskhoki and Mukhin, 2023). Second, the level of the trade balance affects the relative weight on inflation.[21] Below, we will delve into the incentives for an individual central bank to manage the trade balance and show how this depends crucially on the difference in labor intensities $\alpha^N - \alpha^T$.

Given that $1 + \psi_b b_1^* > 0$, the proposition reveals that under optimal policy, only one of two scenarios can emerge: either the economy faces overheating and deflation ($\tau_0 < 0$ and $\pi_0 < 0$), or it faces a recession and positive inflation ($\tau_0 > 0$ and $\pi_0 > 0$). To understand the intuition, consider the possibility that a central bank faces a recession and deflation. In that case, by lowering the nominal interest rate and allowing for higher prices, the central bank can narrow the output gap and reduce deflation. By the same token, if there is a positive output gap and inflation is above the target, it would be optimal to raise the policy rate, as this would help lower inflation and take output closer to the efficient level. From (29), it is also clear that if inflation cost was zero, $\chi = 0$, the central bank would set the labor wedge to zero and implement the first-best allocation for any combination of shocks.

Trade-balance management. When households borrow, they equate the marginal benefits of consuming today to the marginal costs of repaying tomorrow, as given by (5). However, by (25), a central bank also perceives that changes in international borrowing (and thus changes in the trade balance) affect the reallocation of hours worked across sectors, which in turn affects inflation. In particular, the perceived social marginal benefit of the reallocation of hours worked η_0 across sectors is given by

$$\eta_0 = \chi \frac{\phi^N \phi^T}{\sum_i \delta_0^i \alpha^i \phi^i} (\alpha^N - \alpha^T) \Theta(\pi_0) \pi_0. \tag{30}$$

An important takeaway from condition (30) is that the sign of η_0 (and thus μ_0) depends on the difference in labor intensity across sectors $\alpha^N - \alpha^T$ and the sign of inflation. If we assume that non-tradables are more labor intensive ($\alpha^N > \alpha^T$), when the economy has high

[21]The existing open-economy literature that builds on the workhorse model of Gali and Monacelli (2005) has shown that non-cooperative monetary policies are generally outward-looking and tradeoff the output gap with inflation and deviations of the terms-of-trade from the efficient level, where the latter arises from the country's monopoly power in the supply of its own good (see, for example, De Paoli, 2009 or Corsetti et al., 2010, among others). However, the level of the trade balance does not affect the tradeoff between these objectives.

inflation, the central bank in the small open economy would like to reallocate labor towards the more labor-intensive sector (i.e., $\eta_0 > 0$ and thus $\mu_0 > 0$). As discussed above, when a sector is more labor intensive, prices respond relatively less to a change in production in that sector. Therefore, starting from a situation with high inflation, the central bank can achieve a reduction in inflation by shifting employment towards the more labor-intensive sector. On the other hand, if inflation is negative, the central bank internalizes that a reallocation of hours away from the more labor-intensive sector (non-tradables) towards the less labor-intensive sector (tradables) would help raise inflation towards the target and improve welfare.[22]

Condition (30) also implies that when the two sectors are equally labor intensive, $\alpha^N = \alpha^T$, the central bank does not perceive any social benefit from changing the composition of hours between the tradable sector and non-tradable sector. It also therefore follows that households' borrowing choices are socially optimal, from the perspective of the central bank.

Given how the trade balance affects inflation, the key question then is how monetary policy shapes households' borrowing decisions. This is, in fact, a point related to the classic Marshall-Lerner condition, which is said to hold when a depreciation of the exchange rate leads to an increase in the trade surplus. We can derive the following *generalized* Marshall-Lerner condition:

Lemma 2 (Generalized Marshall-Lerner Condition)**.** *In response to a domestic monetary expansion, the change in the trade balance satisfies*

$$\frac{db_1^*}{dR_0} < 0 \iff \sigma_0 < \widetilde{\sigma} \equiv 1 - \frac{\alpha^T}{\Upsilon(\pi_0) \sum_i \alpha^i \phi^i}$$

with $\Upsilon(\pi_0) > 0$ *and* $\Upsilon(0) = 1$.

Proof. In Appendix A.3 □

The lemma generalizes existing results in the literature to a situation with multi-sector production.[23] Whether an expansionary monetary policy expands the trade balance

[22]When $\alpha^N < \alpha^T$, the signs of both Lagrange multipliers are reverted.

[23]The classic Marshall-Lerner condition, derived originally in a partial equilibrium setting, posits that the trade surplus increases in response to a depreciation if the sum of the (static) elasticities of exports and imports to exchange rates exceed one. We note here that we express it in terms of bonds and the nominal rate, but this is equivalent since the trade balance equals b_1^* and a decrease in R depreciates the exchange rate e through (6). In addition, we also note that it is well understood that in a dynamic general equilibrium model, the effects depend on intertemporal considerations (see Bianchi and Coulibaly, 2021, for a decomposition).

depends on the elasticities of substitution and labor intensities in the two sectors. If the tradable sector were an endowment, $\alpha^T = 0$, we would obtain the familiar result that the trade surplus increases in response to a fall in the nominal rate (i.e., $db_1^*/dR_0 < 0$) if and only if the intertemporal elasticity of substitution was lower than the intratemporal elasticity of substitution between tradables and non-tradables (which in this case is assumed to be one).[24] In our model with endogenous production in the tradable sector, the lower interest rate expands tradable output and thus is an additional force towards a trade surplus. Therefore, to obtain a decrease in net exports in response to a lower nominal interest rate, the intertemporal elasticity of substitution must be lower. In addition, it also follows that if $\alpha^T \geq \alpha^N$, a monetary expansion increases the trade surplus for *any* intertemporal elasticity of substitution. Intuitively, a higher α^T implies that tradable output responds more to an increase in the price of tradables (for a given wage), and through consumption smoothing, this means a higher trade surplus.

We highlight that the empirical literature does not offer conclusive evidence on whether a monetary expansion increases or decreases the trade surplus. As we will see, whether the Marshall-Lerner condition holds or not turns out to be key for the results.[25]

Takeaway. To summarize, the key takeaway of this section is that by influencing the trade balance, the central bank can improve its output-inflation tradeoff when labor intensities differ between the tradable and non-tradable sectors. Moreover, whether the central bank would like to stimulate capital inflows or capital outflows depends on the sign of inflation and the difference in labor intensities.

3.2 Nash Equilibrium

In the previous section, we characterized the optimal policy for the central bank of a small open economy for an arbitrary world real rate. We can now define a Nash equilibrium as the outcome when all central banks are simultaneously maximizing the welfare of their representative household and the market for the global real asset clears. Notice that because all countries are identical, we can focus on the symmetric Nash equilibrium.

[24] Much of the literature focuses on the Cole-Obstfeld parameterization with unitary elasticities of substitution where capital flows do not respond to changes in nominal rates.

[25] For example, Boyd, Caporale and Smith (2001) argue that the Marshall-Lerner condition holds in the long run, while Boehm, Levchenko and Pandalai-Nayar (2023) argue that it fails in the short run. Other studies, such as Dong (2017), argue that whether the Marshall-Lerner condition holds or not depends on the precise methodology used.

We let $\mathcal{U}(R_0, R_0^*)$ denote the lifetime utility of the representative household in a competitive equilibrium where the central bank sets the nominal rate to R_0 and the world real rate is R_0^*. In addition, we let $\mathcal{R}^*(R_0)$ denote the equilibrium world real rate when all countries set R_0. We define the Nash equilibrium as follows.

Definition 2 (Nash Equilibrium)**.** The nominal interest rate in the Nash equilibrium is such that

$$R_0 = \operatorname{argmax}_x \ \mathcal{U}(x, \mathcal{R}^*(R_0)).$$

That is, the Nash equilibrium corresponds to the outcome when every central bank is playing its best response to other central bank policies. By symmetry, in any Nash equilibrium, there are no capital flows, and exchange rates are constant. Replacing $b_1^* = 0$ in the targeting rule (29), we arrive at

$$\tau_0 = \chi \psi_0^{NE} \Theta(\pi_0)\pi_0, \quad \text{with } \psi_0^{NE} \equiv \frac{\sum_{i=T,N} \delta_0^i (1-\alpha^i)\phi^i}{\sum_{i=T,N} \delta_0^i \alpha^i \phi^i}. \tag{31}$$

Toward a characterization of the differences between the Nash equilibrium and the cooperative monetary policy, we define a measure of *output gaps* as the deviations of employment relative to the first-best levels:

$$\widehat{h}_t^N \equiv \frac{h_t^N}{\bar{h}_t^N} - 1, \qquad \widehat{h}_t^T \equiv \frac{h_t^T}{\bar{h}_t^T} - 1.$$

From (25), we can see that using $b_1^* = 0$, we obtain that the ratio of employment in the two sectors in the Nash equilibrium, $\frac{h_0^N}{h_0^T}$, equals the employment ratio in the efficient allocation. Moreover, this implies that the output gaps in the tradable and non-tradable sectors are equalized: $\widehat{h}_t^T = \widehat{h}_t^N = \widehat{h}_t$.

In the following section, we will examine the optimal policy under cooperation and show how a global planner would choose a different *level* of output and inflation compared to the Nash equilibrium.

4 Monetary Policy under Cooperation

We evaluate in this section whether coordination calls for tighter or looser monetary policy relative to the Nash equilibrium. We define the *optimal cooperative monetary policy* as the outcome of a planner's problem that chooses the interest rates on behalf of all countries to maximize average welfare.[26]

4.1 Optimal Policy Problem

Because all countries are identical, the optimal monetary policy maximizes the welfare of any given country. The problem of the global planner consists of choosing $\{h_0^N, h_0^T, \pi_0\}$ to maximize (2). In contrast to the problem for a small open economy (27), the planner now internalizes that in equilibrium, the market for the global asset must clear, which implies that $c_0^T = F^T(h_0^T, A_0^T)$.

We can write the associated Lagrangian as follows:

$$\mathcal{L} = u\left(F^T(h_0^T, A_0^T), F^N(h_0^N, A_0^N), \pi_0\right) - \kappa_0(h_0^T + h_0^N) \tag{32}$$
$$+\vartheta_0\left[\left(1+\frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0 - \frac{W}{w_0^n P_{t-1}}\prod_{i=T,N}\left(\frac{h_0^i}{\bar{h}_0^i}\right)^{(1-\alpha^i)\phi^i} + 1\right] + \eta_0\left[\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N} - 1\right].$$

Optimality with respect to h_0^T and h_0^N yield the optimal targeting rule described in the Lemma below.

Proposition 2. *The optimal cooperative monetary policy targets:*

$$\tau_0 = \chi\psi^{GP}\Theta(\pi_0)\pi_0, \quad \textit{with } \psi^{GP} \equiv \frac{\psi_0^{NE}}{1+(\alpha^N-\alpha^T)(\sigma_0-\widetilde{\sigma})\Delta} \quad \textit{and } \Delta > 0. \tag{33}$$

Proof. In Appendix A.4 □

Comparing equation (33) with (31), we can see that cooperation prescribes a different weight on the output gap. In particular, whether the planner puts more weight on inflation or output than individual central banks depends on the product of two sufficient statistics, the difference in labor intensities, $\alpha^N - \alpha^T$, and the response of the trade balance to an expansionary policy—that is, the sign of $\sigma_0 - \widetilde{\sigma}$.

When labor intensities are equal across sectors, $\alpha^N = \alpha^T$, the planner and central banks in the Nash equilibrium put the same weight on output (for any value of σ_0). The intuition for this result is that when the two sectors have the same labor intensity, the social and private marginal benefits of borrowing are aligned, as we explained above. This can be

[26] One can interpret the cooperation regime as a monetary union.

seen more clearly by combining (28) and (30), which yields

$$u_T(c_0^T, c_0^N, \pi_0)\mu_0 = \Delta \frac{\alpha^N - \alpha^T}{\alpha^N} \tau_0 h_0^N, \tag{34}$$

where recall that $\Delta > 0$. It is immediate from this condition that $\mu_0 = 0$ when $\alpha^T = \alpha^N$ regardless of the value of the labor wedge.

Consider instead the case where $\alpha^N > \alpha^T$. If the economy faces positive inflation $\pi_0 > 0$—in which case it is also in a recession, $\widehat{h}_0 < 0$, as explained in Section 3.1—the central bank from every small open economy would like to reallocate employment towards non-tradables and induce more household borrowing (i.e., $\eta_0 > 0$ and $\mu_0 > 0$). Insofar as the generalized Marshall-Lerner condition holds, this implies that central banks restrict monetary policy to attract capital inflows and run a trade deficit. This results in a larger output contraction relative to the global planner that internalizes that capital flows would be zero in equilibrium.

The next proposition leverages this insight to formally compare the levels of employment and nominal rates in the Nash equilibrium and the cooperative equilibrium.

Proposition 3 (Under-tightening or Over-tightening)**.** *Let $\widehat{h}_0^{NE}$ and $\widehat{h}_0^{GP}$ denote the output gap in the Nash equilibrium and in the cooperative equilibrium and R_0^{NE} and R_0^{GP} the corresponding interest rates. We have that*

i) $\widehat{h}_0^{NE}$ and $\widehat{h}_0^{GP}$ have the same sign;

ii) the employment levels satisfy

$$\widehat{h}_0^{NE} > \widehat{h}_0^{GP} \iff (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{w}_0 < 0,$$

where $\widehat{w}_t \equiv \frac{W_t}{w_t^n P_{t-1}} - 1$ represents the wage gap;

iii) the interest rates satisfy

$$R_0^{NE} < R_0^{GP} \iff (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{h}_0^{NE} > 0.$$

Proof. In Appendix A.6 □

The proposition highlights that if the Nash equilibrium faces a recession, the economy under optimal cooperative monetary policy does as well (and conversely, for the case of overheating). Although the sign of the output gap under cooperation mirrors that of the

Nash equilibrium, the level of the output gap (and inflation) is different. In particular, whether the Nash equilibrium displays over-tightening ($R_0^{NE} > R_0^{GP}$) or under-tightening ($R_0^{NE} < R_0^{GP}$) depends on a set of *sufficient statistics*: the differences in labor intensity, the response of the trade balance to a monetary expansion and the sign of the output gap. In particular, when non-tradables are more labor intensive and the generalized Marshall-Lerner condition holds, we have over-tightening if the economy is in a recession (and under-tightening if the economy is overheated). Insofar as non-tradables are more labor intensive, the economy can also display under-tightening in a recession when the generalized Marshall-Lerner condition fails. Table 1 presents the taxonomy with all possible cases.[27]

(a) Marshall-Lerner holds $\sigma > \widetilde{\sigma}$

	$\alpha^N > \alpha^T$	$\alpha^N < \alpha^T$
Recession $\pi > 0$	Over-tightening	Under-tightening
Overheating $\pi < 0$	Under-tightening	Over-tightening

(b) Marshall-Lerner fails $\sigma < \widetilde{\sigma}$

	$\alpha^N > \alpha^T$	$\alpha^N < \alpha^T$
Recession $\pi > 0$	Under tightening	N/A
Overheating $\pi < 0$	Over-tightening	N/A

Table 1: Over-tightening or under-tightening?

In related work, Fornaro and Romei (2023) study optimal monetary policy coordination in a model with exogenous inflation costs where the spillovers operate through the world real rate, as in our model. In their analysis, they assume $\alpha^N = 1, \sigma_0 = 1$ and find that countries put too little weight on the output gap in response to a recessionary shock, in line with our general characterization.

Illustration. Figure 2 presents an illustration of the cooperative and non-cooperative equilibrium for three different wage gaps. The x-axis presents the output gap (which recall is the same for tradables and non-tradables), and the y-axis presents inflation. The red and blue downward-sloping curves illustrate the inflation-output tradeoff (IO) for the Nash equilibrium and cooperative equilibrium, respectively, as defined by the targeting rules in equations (31) and (33). The Phillips curve, depicted by the green upward sloping

[27]As mentioned above, if $\alpha^T > \alpha^N$, the generalized Marshall-Lerner is always satisfied, hence, the N/A in the last column of panel (b).

curves is common to both equilibria and is defined by (26) (noting again that $\widehat{h}_0^T = \widehat{h}_0^N$). The intersection of the two curves represents the equilibrium.

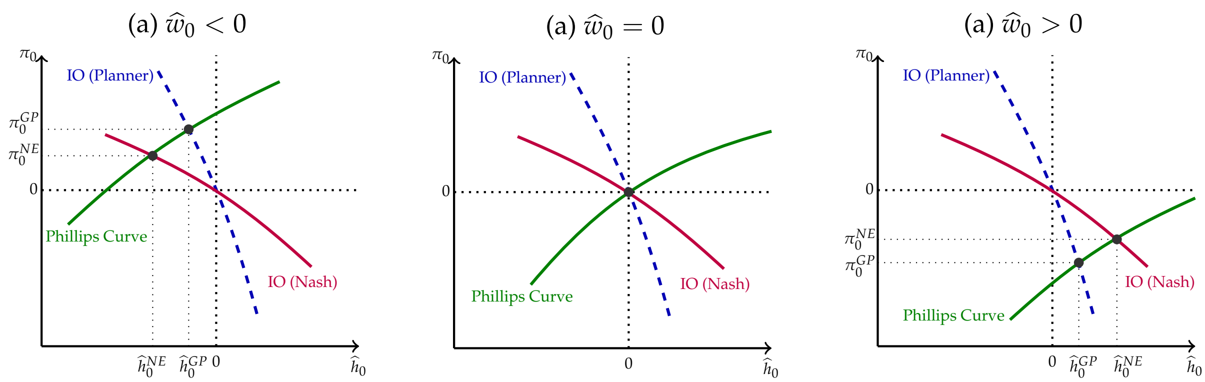

Figure 2: Nash equilibrium vs. cooperative equilibrium for $\alpha^N > \alpha^T$ and $\sigma_0 > \widetilde{\sigma}$

Note: IO stands for Inflation-Output trade-off. IO (Nash) and IO (Planner) correspond respectively to (31) and (33). Phillips curve corresponds to (26), where we used $\widehat{h}_0^T = \widehat{h}_0^N = \widehat{h}_0$.

The plot considers the case where non-tradables are more labor intensive and the generalized Marshall-Lerner condition holds (i.e, $\alpha^N > \alpha^T$ and $\sigma_0 > \widetilde{\sigma}$). In line with Proposition 2, the slope for IO curve under cooperation is steeper than for the Nash equilibrium, reflecting that the planner puts more weight on the output gap. The figure displays three panels, which vary depending on the sign of the wage gap: negative wage gap (panel [a]), zero wage gap (panel [b]), and positive wage gap (panel [c]). Starting from the middle, we can see that the allocations under cooperative and non-cooperative monetary policy coincide and equal the first-best allocation. That is, the intersection of the three curves goes through the ideal point (0,0). When the wage gap is negative (panel [a]), both economies feature a recession. Because the planner puts more weight on output and less weight on inflation, the planner allows for more inflation and faces a small recession. Finally, when the wage gap is positive (panel [c]), the planner allows for more deflation and reduces the degree of overheating in the labor market.

4.2 Inspecting the Mechanism

To delve deeper into the gains from coordination, we consider the dual formulation of the planner problem

$$\max_{\boldsymbol{R}_0} \int U_k(R_{k,0}, \mathcal{R}_0^*(\boldsymbol{R}_0))dk,$$

where recall that $\mathcal{R}^*(\boldsymbol{R}_0)$ denote the equilibrium world real rate when all countries set $\boldsymbol{R}_0$. The optimality condition for the nominal rate for the planner yields

$$\frac{\partial \mathcal{U}_k(R_{k,0}, \mathcal{R}_0^*)}{\partial R_0^k} + \frac{d\mathcal{R}_0^*}{d\boldsymbol{R}_0}\frac{\partial \mathcal{U}_k}{\partial R_0^*} = 0. \tag{35}$$

In contrast to the Nash equilibrium, where each country sets the nominal rate to maximize its own welfare, implying that $\frac{\partial \mathcal{U}_k}{\partial R_{k,0}} = 0$, the social planner instead realizes that changing nominal rates alters the real rate, and in turn, changes in the real rate affect welfare in other countries. The second term in (35) indicates that to understand how the planner would deviate from the non-cooperative equilibrium, we must take into account two crucial considerations: how welfare changes with R_0^* and how R_0^* changes with $\boldsymbol{R}_0$. We proceed now to analyze these spillover effects.

Consider first the effects of an infinitesimal change in the world real rate. We have that evaluated at the Nash equilibrium, the welfare effects are given by

$$\left.\frac{\partial \mathcal{U}_k}{\partial R_0^*}\right|_{R_0^*=R_0^{*NE}} = -\frac{h_0^N}{\alpha^N \phi^N}\frac{\Delta}{R_0^*}(\alpha^N - \alpha^T)\tau_0. \tag{36}$$

This expression follows from an envelope condition.[28] It shows that the first-order effects of changes in the world real rate on welfare are determined by the output gap and the differences in labor intensity. In particular, welfare goes up when interest rates rise if the sign of the product of the output gap and the difference in labor intensity $\alpha^N - \alpha^T$ is positive. In a nutshell, countries benefit from lower real interest rates if they face a recession and non-tradables are more labor intensive (or if they face overheating and non-tradables are less labor intensive). When individual countries set their monetary policy, they do not internalize the general equilibrium effects on the world real rate and how this affects welfare in other countries.

Following the results from Lemma 2, we can infer how monetary policy affects the world real rate. Using the results of that lemma and market clearing in the world asset market, $b_1^* = 0$, we obtain[29]

$$\sigma_0 \frac{d\mathcal{R}_0^*}{R_0^*} = (\sigma_0 - \widetilde{\sigma})\frac{dR_0}{R_0}. \tag{37}$$

When $\sigma_0 > \widetilde{\sigma}$, a monetary policy expansion in one country raises its trade balance. When all countries simultaneously expand their monetary policy, the world real rate must fall

[28] Appendix A.5 provides the derivation.

[29] Equation (37) uses (5), (6), (11), and (25), with market clearing for global assets $b_1^* = 0$.

to clear the asset market. Conversely, when $\sigma_0 > \widetilde{\sigma}$, a monetary expansion leads to an increase in the world real rate.

Putting together (36) and (37), we can now trace the sign of the second term in the planner's optimality (35). In sum, in a Nash equilibrium, central banks use monetary policy to steer capital flows and improve their output-inflation stability tradeoff. In general equilibrium, however, capital flows net out to zero, and the global economy ends up with a distorted inflation-output outcome. Whether the planner finds it optimal to expand or tighten monetary policy relative to the Nash equilibrium depends on how monetary policy impacts the world real rate and whether individual central banks benefit from lower or higher real rates.

4.3 The Need for Cooperation

In this section, we analyze the importance of cooperation by inspecting how individual countries would unilaterally deviate from the coordinated solution. We will work with a linear-quadratic approximation to the policy problem around the efficient allocation and provide a simple graphical representation.

Linearizing the equilibrium conditions for a single small open economy around the efficient allocation, we obtain the following system:[30]

$$\widehat{b}^* = a_1 \left[-(\sigma - \widetilde{\sigma})\, \widehat{R} + \sigma\, \widehat{R}^* \right] \tag{MP}$$

$$\frac{\chi + \varepsilon}{\varepsilon}\, \pi = \widehat{w} + \sum_{i=T,N} (1 - \alpha^i)\phi^i\, \widehat{h}^N + a_2 \Big[(\sigma - \widetilde{\sigma}) \sum_{i=T,N} \alpha^i \phi^i\, \widehat{h}^N + \widehat{R}^* \Big] \tag{AS}$$

with $\widehat{b}^* = \frac{b_1^*}{R_0^* F^T(\bar{h}_0^T, A_0^T)}$. Given a nominal rate $\widehat{R}$ and a world real rate $\widehat{R}^*$, the outcomes for $(\widehat{b}^*, \widehat{h}^N, \widehat{\pi})$, are fully determined by (MP), (AS), and[31]

$$a_2(\sigma - \widetilde{\sigma}) \sum_{i=T,N} \alpha^i \phi^i\, \widehat{h}^N = \widehat{b}^* - a_2 \widehat{R}^*. \tag{38}$$

A second-order approximation of the objective function around the efficient allocation

[30](AS) combines linearized (25) and (26), where $a_2 \equiv \left[\delta + (\sigma - 1)\left(\phi^T + \alpha^N \phi^N\right)\right]^{-1} > 0$. (MP) combines linearized (5) and (6), and uses (13) and (17), where $a_1 \equiv \sum_i \alpha^i \phi^i \left[(\delta - \alpha^T)\left(\delta + (\sigma - 1)\sum_i \alpha^i \phi^i\right)\right]^{-1} > 0$.

[31]Equation (38) is obtained by linearizing the Euler equation (5) and using (25).

gives rise to the following welfare-based loss function:[32]

$$\mathbb{L} \equiv \frac{1}{2}\left[\left(1+(\sigma-1)\sum_{i=T,N}\alpha^i\phi^i\right)\sum_{i=T,N}\alpha^i\phi^i\left(\widehat{h}^N\right)^2+\chi\left(\pi\right)^2+\left(\delta-\phi^T+\sigma\phi^T\right)\phi^T\left(\widehat{b}^*\right)^2\right]. \quad (39)$$

Under this linear-quadratic setting, the problem of a central bank is to minimize (39) subject to (MP), (AS), and (38).

Figure 3 presents a graphical illustration. The top panels illustrate the case where the Marshall-Lerner condition holds, and the bottom panels illustrate the case where it fails. Let us focus on the former. The lines with elliptical shapes in panel (b) represent the indifference curve, as given by (39), where we replace $\widehat{b}^*$ with (38). The slope of the indifference curve is governed by the relative cost of inflation and output. Notice that the sign of the slope changes when inflation or the output gap changes sign. The ideal point (0,0) is illustrated with the gray, solid dot. As the indifference curves get closer to the ideal point (0,0), the level of utility increases.

Point G represents the point chosen by the global planner who internalizes the effects of monetary policy on the world real rate. This point lies in the aggregate supply for the world economy:[33]

$$\frac{\chi+\varepsilon}{\varepsilon}\,\pi = \widehat{w} + \sum_{i=T,N}(1-\alpha^i)\phi^i\,\widehat{h}^N. \quad (\text{AS}_W)$$

Because a change in aggregate output affects the world real interest rate, the aggregate supply faced by the global planner is different from the one faced by the small open economy. Under the assumption that non-tradables are more labor intensive, $\alpha^N > \alpha^T$, the (AS_W) curve is flatter than (AS) when the generalized Marshall-Lerner condition holds.

The green dotted line AS' represents the aggregate supply curve for an individual central bank that takes as given the world real rate in the cooperative equilibrium. Point E' corresponds to the menu of inflation and output gap that the central bank would choose. As one can see in the figure, this point is tangent to the indifference curve that is closest to the ideal point $(0,0)$. The tangency point between (AS) and the indifference curve represents the optimal solution for an individual central bank.

However, point E' is not an equilibrium as it captures a situation where only an individual central bank deviates. When the generalized Marshall-Lerner condition holds,

[32]See Appendix A.9 for the derivation of the loss function.

[33]Equation (AS_W) follows directly from combining (38) with $\widehat{b}^* = 0$ and (AS).

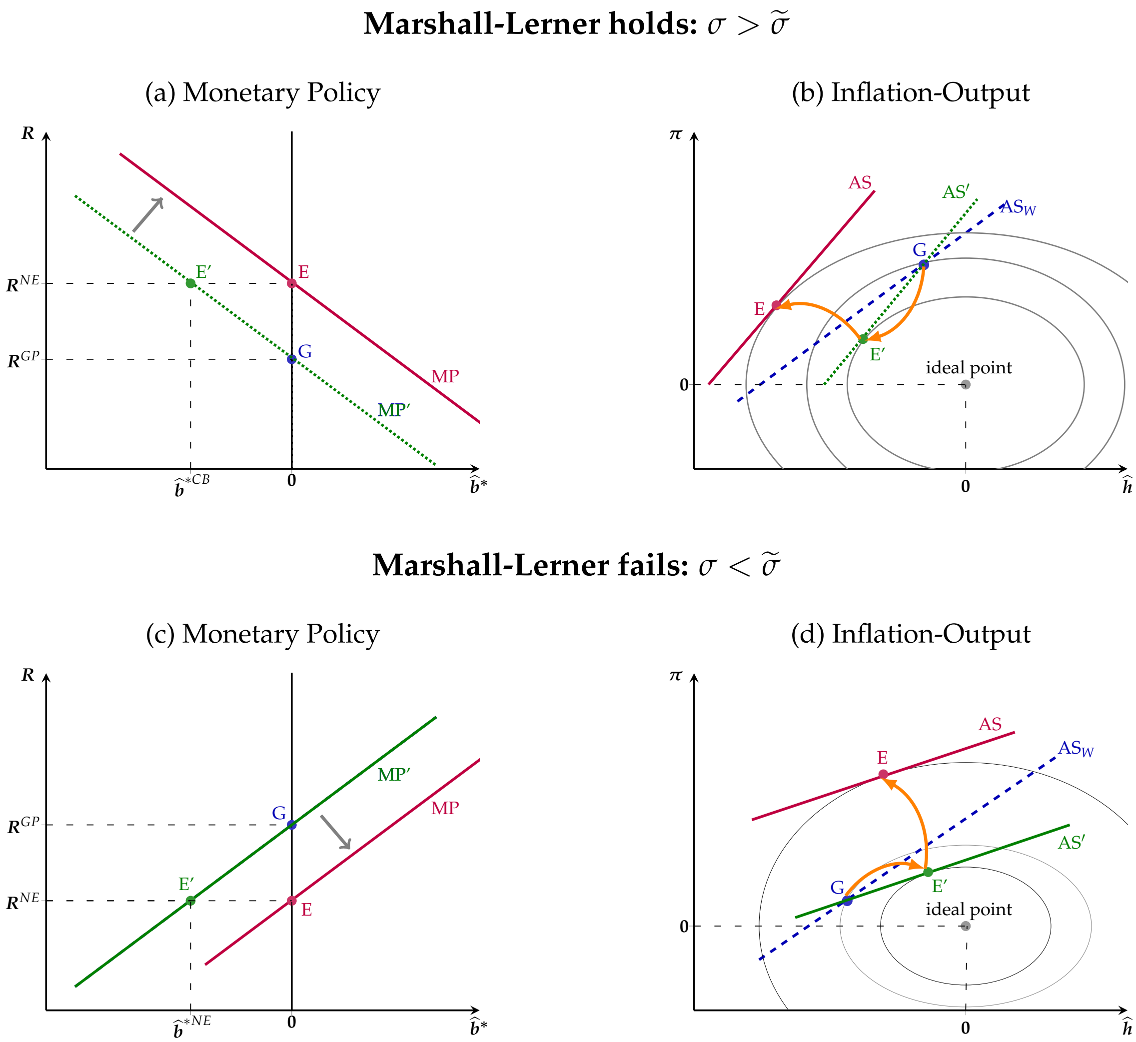

Figure 3: The Need for Coordination

Note: The figure presents cases where $\widehat{w} > 0$ and parameters are such that $\alpha^N > \alpha^T$. The top (bottom) panels present the case of over-tightening (under-tightening).

the higher nominal interest rate chosen by individual central banks would give rise to a trade deficit. In general equilibrium, this means that the world real rate must go up. Graphically, this means the curve MP in panel (a) shifts to the right to the point where $\widehat{b}_1^* = 0$. In addition, once the world real rate goes up, the AS curve faced by an individual central bank shifts up and to the left, as illustrated in panel (b). The result is that the Nash equilibrium ends up at the point E, further away from the ideal point. Compared to the cooperative outcome, the Nash equilibrium ends up with a larger recession and lower inflation.

When the generalized Marshall-Lerner condition fails, we can see in panel (d) that the Nash equilibrium ends up at a point with a smaller recession but higher inflation. As explained above, central banks now lower the nominal rate relative to the global planner to increase capital inflows. The result is an equilibrium with a lower output gap and higher inflation.

4.4 Anticipated Shocks: A Case of Prudential Undertightening

Until now, we considered an economy that faces a sudden shock that creates an output-inflation tradeoff at $t=0$. In this section, we consider the possibility of a future shock. This extension allows us to examine a situation where central banks may be using monetary policy to affect their net foreign asset position and improve their output-inflation tradeoff in the future. To highlight the key mechanism, we focus for simplicity on the case where the Phillips curve is static.[34]

We consider an initial situation where the economy is at the first-best and at period $t=-1$. Agents anticipate a shock to the economy at period $t=0$. Let us start with the analysis of the non-cooperative solution. The problem the central bank faces at $t=-1$ is analogous to the one described in (27), with the difference that now the continuation value is *not* the one associated with the flexible wage allocation. The individual central bank can still achieve the efficient allocation at $t=-1$, given that the shock will hit at $t=0$. However, the central bank internalizes that by changing its net foreign asset position, it will improve the output-inflation tradeoff at $t=0$, when the shock hits.

Under the assumption that $\alpha^N > \alpha^T$, the central bank perceives an extra benefit from raising its net foreign asset position if the shock tomorrow leads to a recession (and an extra marginal cost if the shock tomorrow leads to overheating). In turn, to the extent that $\sigma_{-1} > \widetilde{\sigma}$, the central bank would cut the nominal rate if the shock tomorrow led to a recession (and increase the nominal interest rate if the shock tomorrow led to overheating).

On the other hand, the anticipation of the shock has no effect on the optimal monetary policy under cooperation at period $t=-1$. That is, the planner sets the nominal rate to achieve the efficient allocation at $t=-1$. Intuitively, the desire to accumulate net foreign asset position for individual countries is a zero-sum game. When central banks depart from the efficient allocation at $t=-1$, they end up worsening the allocation without any future gains.

These insights are summarized in the next proposition.

Proposition 4. *Consider $\widehat{w}_{-1} = 0$. Then,*

[34] This can be microfounded by assuming that the cost of changing prices for the intermediate good firms is a function of the previous period's average price index: $\frac{\chi}{2}\left(\frac{p_{jt}}{p_{t-1}}-1\right)^2 P_t q_t$, as in Bilbiie (2024).

i) the optimal monetary policy under cooperation features $\widehat{h}_{-1} = \pi_{-1} = 0$*;*

ii) the Nash equilibrium features

a) $\widehat{h}_{-1} > 0$ *and* $\pi_{-1} > 0$ *if* $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}_0 > 0$

b) $\widehat{h}_{-1} < 0$ *and* $\pi_{-1} < 0$ *if* $(\sigma_{-1} - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}_0 < 0$*.*

Proof. In Appendix A.7 □

A feature of our environment with anticipated shocks is that *countries can now experience both overheating labor markets and high inflation*. This is an interesting feature because a common characteristic of New Keynesian models is that the central bank faces unemployment and high inflation or overheating and low inflation.

The implications of cooperation for policy rates are summarized in the following corollary.

Corollary 1 (Prudential under-tightening)**.** *Suppose countries anticipate a recession at* $t = 0$*. Then,*

$$R_{-1}^{NE} < R_{-1}^{GP} \iff (\alpha^N - \alpha^T)(\sigma_{-1} - \widetilde{\sigma})\widehat{h}_{-1}^{NE} > 0.$$

Proof. In Appendix A.8 □

Our sufficient statistics therefore remain valid in the presence of anticipated shocks. That is, the extent to which there is over- or under-tightening depends on the product of the difference in labor intensity, $\alpha^N - \alpha^T$, the response of the trade balance to a monetary expansion, $\sigma_{-1} - \tilde{\sigma}$, and the sign of the output gap.

The inefficiency of the non-cooperative outcome can be referred to as a problem of "prudential under-tightening." That is, by attempting to increase the future net foreign asset position, with a prudential goal, central banks will conduct a monetary policy that inefficiently boosts output when there is an expected recession (and inefficiently depresses output when there is an expectation of overheating).

4.5 Quantitative Gains from Monetary Policy Coordination

We evaluate in this section the quantitative gains from monetary policy coordination. Specifically, we examine the differences in output and inflation achieved in the cooperative and non-cooperative equilibrium and the welfare implications.

Parameter values. We calibrate the model using advanced economies as a reference. The time period is a year. Households' utility function has the constant relative risk-aversion form $U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, with $\sigma = 5$. Following Schmitt-Grohé and Uribe (2016), we set the labor intensity in the non-tradable sector to $\alpha^N = 0.75$. We set the weight on tradable consumption in the CES function to $\phi^T = 0.26$, which implies a share of non-tradable output of 75%, in the range of observed values in the data. The labor intensity in the tradable sector is set to match an aggregate labor share of 2/3, which implies $\alpha^T = 0.43$.[35] The discount factor β is set to 0.96, which implies a steady-state value for the world real interest rate of 4%. Finally, we set the elasticity of substitution among differentiated varieties ε to 7.66, corresponding to an 11.5% net markup, in the range found by Diewert and Fox (2008). We set χ so that the slope of the linearized Philips in our model coincides with the slope in the corresponding Calvo model with prices adjusting on average every 3 quarters, in line with Nakamura and Steinsson (2008). This calibration implies $\chi = 11.4$. [36]

Quantitative results. We consider a range of shocks to the disutility of labor, as a proof of concept.[37] We assume the shock hits at $t = 0$ and is anticipated at $t = -1$. Figure 4 presents the results. The figure plots the output gap, inflation, and welfare in periods $t = -1$ and $t = 0$.

Let us discuss first the effects at $t = 0$, which are illustrated in the bottom panels. If labor disutility falls at $t = 0$, the efficient level of output increases, which implies that the natural wage falls below the sticky wage and the economy faces an inefficiently low level of output given the initial monetary policy. In the Nash equilibrium, central banks respond by loosening monetary policy in order to mitigate the recession, and this policy gives rise to inflation. Under the constellation of parameters considered, we have over-tightening: individual central banks do not lower interest rates sufficiently relative to the cooperative solution. As a result, countries face a deeper recession in the Nash equilibrium and a lower inflation rate. As shown in panels (d) and (e) of Figure 4, the difference in output and inflation can reach approximately two and one-half percentage points.

Next, we analyze the effects at $t = -1$. To be in a better position to manage the recession

[35] The aggregate labor share is given by $\frac{W_t(h_t^T + h_t^N)}{P_t^T y_t^T + P_t^N y_t^N} = \alpha^T \phi^T + \alpha^N \phi^N$.

[36] The slope of the linearized Philips curve is $\frac{\varepsilon}{\chi+\varepsilon} \sum_i (1 - \alpha^i)\phi^i$ in our model and $\frac{(1-\theta)(1-\beta\theta)\sigma}{\theta} \sum_i (1 - \alpha^i)\phi^i$ in the corresponding Calvo model, where θ is the probability of a price adjustment at a given quarter. Thus, we have $\chi = \frac{\varepsilon\theta}{(1-\theta)(1-\beta\theta)\sigma} - \varepsilon$.

[37] A shock to κ does not affect allocations in the Nash equilibrium for a *given monetary policy*. However, it does affect the efficient allocation, and thus monetary policy responds.

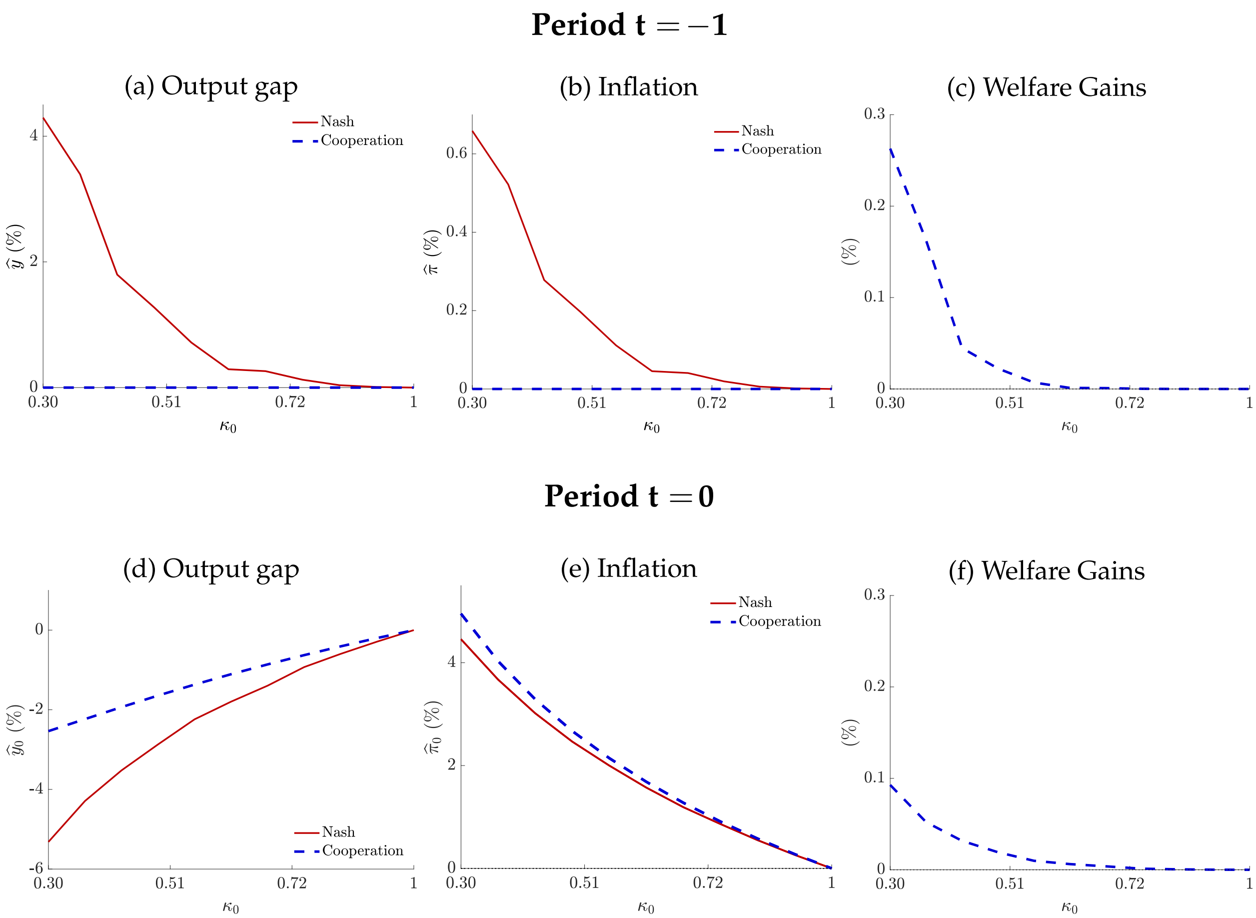

Figure 4: Cooperation versus Nash Equilibrium

Note: The shock considered is a decrease in κ_0. The parameter values are $\alpha^N = 0.75$, $\alpha^T = 0.43$, $\phi^T = 0.26$, $\beta = 0.96$, $\varepsilon = 10$, $\sigma = 5$, $\chi = 11.4$. Under this parameterization, Marshall-Lerner holds. Welfare gains are measured in consumption equivalence in terms of current consumption.

at $t = 0$, central banks seek to prudentially increase their trade surplus so as to have a higher NFA position. In this case, we have under-tightening: whereas the planner keeps policy rates unchanged and continues to stabilize the output gap and inflation at $t = -1$, in the Nash equilibrium, central banks cut the nominal rate, giving rise to an overheated labor market and positive inflation. As panels (a) and (b) show, the output gap can reach 4% and inflation 0.6%.

Finally, we analyze the welfare gains from cooperation. Panel (c) presents the percentage increase in consumption at $t = -1$ that would make households indifferent between remaining in the Nash equilibrium and moving to the cooperative equilibrium, assuming that at $t = 0$ the economy is in the cooperative equilibrium. Panel (f) presents the analogous consumption variation at $t = 0$. The key takeaway is that there are significant welfare gains from cooperation for moderately large shocks. Moreover, as it turns out, the gains from correcting under-tightening at $t = -1$ tend to be larger than the gains from correcting over-tightening at $t = 0$.

4.6 Monetary Coordination throughout History

Our theoretical framework offers a comprehensive taxonomy of the possible constellations that can lead to coordinated efforts towards either more expansionary or more contractionary monetary policies.[38] This taxonomy is useful for understanding a long history of coordinated monetary policy arrangements. As highlighted by Bordo (2021) and Frankel (2016), history shows numerous instances of coordinated efforts to adjust monetary policy towards either a more expansionary or a more contractionary stance.

After the abandonment of the gold standard during World War I, concerted efforts were made to return to parity with gold, as stated by the Financial Commission of the 1922 Genoa Economic and Monetary Conference. However, the Great Depression prompted most countries to once again forsake the gold standard, leading to the so-called currency wars, where nations sought to maintain depreciated exchange rates to gain competitive advantages. The post-World War II era saw the creation of the Bretton Woods system, which established a system of fixed exchange rates pegged to the U.S. dollar. Following the collapse of Bretton Woods and subsequent oil price shocks, central banks sought to coordinate less restrictive monetary policies. The Plaza Accord of 1985 exemplified this effort, as advanced central banks aimed to induce a depreciation of the dollar amid a recessionary context. In the aftermath of the global financial crisis and the COVID crisis, there have been numerous calls for coordinated actions to address *reverse* currency wars.[39]

5 Extensions

The model presented can accommodate a variety of configurations and applications. In this section, we provide three key extensions.[40]

[38]Recall that when the economy faces only shocks in the current period, in equilibrium, the economy can experience either a recession and high inflation or overheated labor markets and deflation, as summarized in Table 1. However, the dynamic considerations of future shocks imply that the economy the economy can simultaneously experience overheated labor markets and inflation, or recession and deflation.

[39]For instance, Rajan (2015) famously argued that “international monetary cooperation has broken down” following the rise in long-term U.S. rates after the announcement of future tapering of quantitative easing. See also Obstfeld (2022) for further discussion.

[40]For simplicity, the targeting rule under these extensions is derived assuming that the objective function is separable in inflation. See Appendix B for details.

CES aggregate. In our baseline framework, we consider a unitary elasticity of substitution between tradables and non-tradables. We now generalize the consumption of the composite to allow for a CES aggregator with elasticity $1/\gamma$. Our core findings remain essentially unchanged. As we show in Appendix B.1, the only difference in the condition required for the trade balance to increase in response to a monetary expansion. Namely, the Marshall-Lerner condition dictates that the trade balance increases in response to a monetary expansion if and only if $\sigma_0 > \gamma\widetilde{\sigma}$. That is, a lower elasticity of substitution between goods necessitates a lower elasticity of intertemporal substitution for the trade balance to increase. Intuitively, as the intra-temporal elasticity of substitution rises, a depreciation leads to larger expenditure switch from tradables towards non-tradables. Consequently, consuming fewer tradables means that more tradable output is available for export, leading to an increase in the trade balance.

Imperfect labor mobility. In our baseline framework, we assume that households are indifferent between working in the tradable and non-tradable sectors. We now show that with imperfect substitutability of labor, our key sufficient statistic results remain.

We assume that aggregate hours worked is a CES composite of n_t^T and n_t^N:

$$n_t = \left[\frac{1}{2}\left(n_t^T\right)^{1+\frac{1}{\xi}} + \frac{1}{2}\left(n_t^N\right)^{1+\frac{1}{\xi}}\right]^{\frac{\xi}{\xi+1}}, \tag{40}$$

where $\xi \geq 0$ denotes the elasticity of substitution.

Given that hours worked in the tradable sector and in the non-tradable sector are not perfect substitutes, wages need not be equal across the two sectors. We denote by W^N and W^T the prevailing sticky wages at date $t = 0$ in the tradable and the non-tradable sectors, respectively. The ratio of hours in a small open economy is given by

$$\frac{h_0^N}{h_0^T} = \frac{W^T}{W^N}\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\left[1 - \frac{b_1^*}{R_0 F^T(h_0^T, A_0^T)}\right],$$

from which it follows that in any symmetric competitive equilibrium, the output gaps in the two sectors are proportional. As we show in Appendix B.2, the optimal targeting rule in the Nash equilibrium and under cooperation continues to be given by (31) and (33), and the sufficient statistics determining the relative weights on inflation are the same as in our baseline framework.

Oil shocks. In our baseline framework, labor is the only factor of production. One may wonder whether in a setup with multiple inputs, the result on over- or under-tightening may depend on the elasticity of the other factors or the general curvature of the production function. In this section, we incorporate oil as an intermediate input and show that labor intensity remains the key sufficient statistic.

We assume that households in each country are endowed with M_t units of oil, which are used as intermediate inputs for production and can be exchanged with the rest of the world without any trade costs. Market clearing in the oil market is given by $m_t^T + m_t^N = M_t$. We think of a reduction in M_0 as an "oil shock."[41] The production function is given by $F^i(h_t^i, m_t^i, A_t^i)$, and we denote the oil intensity in each sector by ζ^i:

$$\zeta^i \equiv \frac{d \log F^i(h_t^i, m_t^i, A_t^i)}{d \log m_t^i}.$$

As detailed in Appendix B.3, (31) and (33) remain the optimal targeting rules. However, the relative weights on inflation are now determined by

$$\frac{\psi_0^{NE}}{\psi^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\, \Delta_m,$$

with $\Delta_m > 0$ given by (B.22). The difference in labor intensity across sectors remains a key sufficient statistic, as in our baseline model. Importantly, the difference in the intensity of oil in production across sectors is irrelevant to whether central banks over- or under-tighten in the Nash equilibrium. The takeaway is that the relevant factor intensity is the one corresponding to the *sticky price factor.*

6 Conclusion

This paper develops a general theory of monetary policy coordination under financial integration. In contrast to the traditional focus on terms of trade externalities, we highlight a pecuniary externality operating through the global capital market. Specifically, individual countries fail to internalize how their monetary policy decisions impact the global real interest rate and, consequently, the ability of foreign central banks to stabilize output and inflation.

[41] Auclert, Monnery, Rognlie and Straub (2023) show that coordinating on a tighter monetary policy is desirable from the perspective of oil importer countries to reduce their import prices. These terms of trade manipulation motives are absent in our setup.

We identify three sufficient statistics that determine whether the Nash equilibrium exhibits over-tightening or under-tightening: the output gap, sectoral differences in labor intensity, and the response of the trade balance to a nominal depreciation of the exchange. Our characterization is independent of the specific shocks driving the economy and provides general guidelines for concrete policy discussions on monetary policy coordination.

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APPENDIX

A Proofs

A.1 Proof of Lemma 1

The proof follows directly from rearranging (18) and (19) and the specification of the utility function. □

A.2 Proof of Proposition 1

The first-order conditions with respect to h_0^N, h_0^T, π_0, and b_1^* are given by

$$F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N, \pi_0)\tau_0 = \frac{\phi^N(1-\alpha^N)}{h_0^N}\left[1 + \left(1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0\right]\vartheta_0 + \frac{\eta_0}{h_0^N} - F_h^N(h_0^N, A_0^N)u_{TN}(c_0^T, c_0^N, \pi_0)\mu_0 \quad \text{(A.1)}$$

$$F_h^T(h_0^T, A_0^T)u_T(c_0^T, c_0^N, \pi_0)\tau_0 = \frac{\phi^T(1-\alpha^T)}{h_0^T}\left[1 + \left(1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0\right]\vartheta_0 - \frac{\eta_0}{h_0^T} - F_h^T(h_0^T, A_0^T)u_{TT}(c_0^T, c_0^N, \pi_0)\mu_0 \quad \text{(A.2)}$$

$$\left[1 + \left(1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0\right]\vartheta_0 = c_0 U'(c_0)\left[1 + \frac{\phi^T}{c_0^T}(1-\sigma_0)\mu_0\right]\chi\Theta(\pi_0)\pi_0 \quad \text{(A.3)}$$

$$\eta_0 = \left[\delta_0 + (\sigma_0 - 1)\phi^T\right]u_T\left(c_0^T, c_0^N, \pi_0\right)\mu_0 \quad \text{(A.4)}$$

where

$$\delta_0 \equiv 1 + R_0^* c_0^T\left[\frac{1}{u_{T,1}}\frac{-du_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*), 0\right)}{db_1^*}\right] \quad \text{(A.5)}$$

$$\Theta(\pi_0) \equiv \frac{1 + \left[1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2\right]\pi_0}{\left[1 + \frac{\chi}{\varepsilon}(1+\pi_0)(1+3\pi_0)\right]\left(1 - \frac{\chi}{2}\pi_0^2\right)} > 0 \quad \text{(A.6)}$$

Under the assumption that consumption policy functions, $\mathcal{C}^T(b_1^*)$ and $\mathcal{C}^N(b_1^*)$, are increasing in initial wealth b_1^*, it is straightforward to see that $\delta_0 > 1$. To see why $\Theta(\pi_0) > 0$, notice that $1 + (1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2)\pi_0 > 0$ by (26) and $1 - \frac{\chi}{2}\pi_0^2 = \frac{q_0}{c_0} > 0$. Moreover,

$1+\frac{\chi}{\varepsilon}(1+\pi_0)(1+3\pi_0) = 1+\frac{\chi}{\varepsilon}(1+\pi_0)^2+\frac{\chi}{\varepsilon}(1+2\pi_0)$ and $1+2\pi_0 > 0$ follows from the fact, in the symmetric equilibrium, for $p_t \leq \frac{1}{2}p_{t-1}$, the marginal revenue of the firm (net of adjustment cost) is increasing in p_t. Thus, $p_t \leq \frac{1}{2}p_{t-1}$ cannot be optimal implying that $p_t > \frac{1}{2}p_{t-1}$.

Using (A.4) and (21), we can rewrite (A.1) and (A.2) to obtain

$$F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N, \pi_0)\tau_0 = \frac{\phi^N(1-\alpha^N)}{h_0^N}c_0 U'(c_0)\,\chi\Theta(\pi_0)\pi_0 + \frac{\eta_0}{h_0^N}\delta_0^T \tag{A.7}$$

$$F_h^N(h_0^N, A_0^N)u_N(c_0^T, c_0^N, \pi_0)\tau_0 = \frac{\phi^T(1-\alpha^T)}{h_0^T}c_0 U'(c_0)\,\chi\Theta(\pi_0)\pi_0 + \frac{\eta_0}{h_0^T}(\delta_0^N+\delta_0^b b_1^*) \tag{A.8}$$

where we define

$$\delta_0^T \equiv \frac{\delta_0+(\sigma_0-1)\left[1-(1-\alpha^N)\phi^N\left(1+\chi\Theta(\pi_0)\pi_0\right)\right]}{\delta_0+(\sigma_0-1)\phi^T} > 0 \tag{A.9}$$

$$\delta_0^N \equiv \frac{\delta_0-\alpha^T+(\sigma_0-1)(1-\alpha^T)\phi^T\left(1+\chi\Theta(\pi_0)\pi_0\right)}{\delta_0+(\sigma_0-1)\phi^T} > 0 \tag{A.10}$$

$$\delta_0^b \equiv \alpha^T\frac{1-\phi^T+\sigma_0\phi^T}{\delta_0-\phi^T+\sigma_0\phi^T} > 0 \tag{A.11}$$

$\delta_0^b > 0$ follows directly from $\delta_0 > 1$. To see why $\delta_0^T > 0$ and $\delta_0^N > 0$, note that $\delta_0 + (\sigma_0 - 1)\phi^T > 0$ from $\delta_0 > 1$. To see that the numerators of (A.10) and (A.9) are positive, note that there are increasing in $\sigma_0 > 0$. Then, consider $\sigma_0 \to 0$ and use $1+\chi\Theta(\pi_0)\pi_0 < \frac{1}{\sum_i(1-\alpha^i)\phi^i}$ from (33) and $\tau_0 < 1$. Furthermore, it is worth noting from (A.10) and (A.11), and using (17), that $\delta_0^N + \frac{\delta_0^b}{R_0^* c_0^T}b_1^* > 0$.

Finally, we combine (A.7) and (A.8) and arrive at

$$\left[\alpha^N\phi^N\left(\delta_0^N+\frac{\delta_0^b}{R_0^*c_0^T}b_1^*\right)+\alpha^T\phi^T\delta_0^T\frac{F^T(h_0^T,A_0^T)}{c_0^T}\right]\tau_0$$
$$= \left[(1-\alpha^N)\phi^N\left(\delta_0^N+\frac{\delta_0^b}{R_0^*c_0^T}b_1^*\right)+(1-\alpha^T)\phi^T\delta_0^T\right]\chi\Theta(\pi_0)\pi_0 \tag{A.12}$$

Using (17), $\frac{F^T(h_0^T,A_0^T)}{c_0^T} = 1+\frac{b_1^*}{R_0^*c_0^T}$, we can rewrite (A.12) as

$$\sum_{i=T,N}\alpha^i\phi^i\delta_0^i\,\tau_0 = (1+\psi_b b_1^*)\sum_{i=T,N}(1-\alpha^i)\phi^i\delta_0^i\,\chi\Theta(\pi_0)\pi_0 \tag{A.13}$$

which corresponds to (A.13), and where

$$\psi_b = (\alpha^N - \alpha^T)\frac{\phi^T\phi^N\delta_0^N\delta_0^T}{\sum_i(1-\alpha^i)\phi^i\delta_0^i\sum_i\alpha^i\phi^i\delta_0^i}\cdot\frac{\delta_0^b}{R_0^*c_0^T}. \tag{A.14}$$

From (A.14), $\text{sign}(\psi_b) = \text{sign}(\alpha^N - \alpha^T)$. Moreover, we have that $1+\psi_b b_1^* > 0$ which follows directly from (A.12) and the fact that the terms multiplying τ_0 and π_0 are positive.

□

A.3 Proof of Lemma 2

Using (5) and (25), we obtain

$$u_T\left(F^T(h_0^T,A_0^T)-\frac{b_1^*}{R_0^*},F^N\left(\left(1-\frac{b_1^*/R_0^*}{F^T(h_0^T,A_0^T)}\right)\frac{\alpha^N\phi^N}{\alpha^T\phi^T}h_0^T,A_0^N\right),\pi_0\right) = \beta R_0^* u_T\left(\mathcal{C}^T(b_1^*),\mathcal{C}^N(b_1^*),0\right)$$

Differentiating this expression with respect to R_0 and evaluating at $b_1^*=0$, we get

$$-\left[\alpha^T+(\sigma_0-1)(\alpha^T\phi^T+\alpha^N\phi^N)\right]\frac{1}{h_0^T}\frac{dh_0^T}{dR_0}-(1-\sigma_0)\frac{\chi\pi_0}{1-\frac{\chi}{2}\pi_0^2}\frac{d\pi_0}{dR_0} + \left[1+(\sigma_0-1)(\phi^T+\alpha^N\phi^N)\right]\frac{1}{R_0^*c_0^T}\frac{db_1^*}{dR_0} = \frac{1-\delta_0}{R_0^*c_0^T}\frac{db_1^*}{dR_0} \tag{A.15}$$

where we use $u_{TT}(c_0^T,c_0^N,\pi_0) = -\frac{[1+(\sigma_0-1)\phi^T]u_T(c_0^T,c_0^N,\pi_0)}{c_0^T}$ and $u_{TN} = -\frac{(\sigma_0-1)\phi^N u_T(c_0^T,c_0^N,\pi_0)}{c_0^N}$ and δ_0 defined in (A.5). Next, we use (25) and differentiate (26) to obtain

$$\left[1+\frac{\chi}{\varepsilon}(1+\pi_0)(1+3\pi_0)\right]\frac{\chi}{\varepsilon}\frac{d\pi_0}{dR_0} = \left[1+\left(1+\frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0\right]\frac{\sum_i(1-\alpha^i)\phi^i}{h_0^T}\frac{dh_0^T}{dR_0} \tag{A.16}$$

Then, we use (11) to express (6) as $R_0 = R_0^*\frac{W_1F_h^T(h_0^T,A_0^T)}{F_h^T(h_1^T,A_1^T)W}$. We differentiate this expression with respect to R_0 and evaluate it at $b_1^*=0$ to get

$$1 = -(1-\alpha^T)\frac{R_0}{h_0^T}\frac{dh_0^T}{dR_0} + (1-\delta_0)\frac{R_0}{R_0^*c_0^T}\frac{db_1^*}{dR_0} \tag{A.17}$$

where we use $F_h^T(h_1^T,A_1^T) = \frac{\kappa_1}{u_T(\mathcal{C}^T(b_1^*),\mathcal{C}^N(b_1^*),0)}$ by $\tau_1=0$ and δ_0 defined in (A.5).

Finally, we substitute (A.16) and (A.17) into (A.15) and arrive at

$$-\frac{db_1^*}{dR_0} = \frac{R_0^*c_0^T}{R_0}\frac{\alpha^T+(\sigma_0-1)\left[\alpha^T\phi^T+\alpha^N\phi^N-\sum_i(1-\alpha^i)\phi^i\chi\Theta(\pi_0)\pi_0\right]}{(\delta_0-\alpha^T)[\delta_0+(\sigma_0-1)(\phi^T+\alpha^N\phi^N)]} \tag{A.18}$$

Defining $\Upsilon(\pi_0) \equiv 1 - \frac{\sum_i (1-\alpha^i)\phi^i}{\sum_i \alpha^i \phi^i} \chi \Theta(\pi_0)\pi_0$, we have that (A.18) becomes

$$-\frac{db_1^*}{dR_0} = \frac{R_0^* c_0^T}{R_0} \frac{\alpha^T + (\sigma_0 - 1)\Upsilon(\pi_0)(\alpha^T \phi^T + \alpha^N \phi^N)}{(\delta_0 - \alpha^T)[\delta_0 + (\sigma_0 - 1)(\phi^T + \alpha^N \phi^N)]} \tag{A.19}$$

Notice that $\Upsilon(\pi_0) > 0$ by (33) and $\tau_0 < 1$. The result in Lemma 2 then follows directly from (A.19), where it should be noticed that the denominator is positive because $\delta_0 > 1$. □

A.4 Proof of Proposition 2

The first-order condition of the global planning problem (32) with respect to π_0 yields

$$\left[1 + \frac{\chi}{\varepsilon}(1+\pi_0)(1+3\pi_0)\right] \vartheta_0 = c_0 U'(c_0) \frac{\chi \pi_0}{1 - \frac{\chi}{2}\pi_0^2} \tag{A.20}$$

We then combine the optimality conditions for h_0^N and h_0^T along with (21) and (A.20) to get

$$\tau_0 = \chi \psi^{GP} \Theta(\pi_0)\pi_0, \quad \text{with } \psi^{GP} \equiv \frac{\sum_i (1-\alpha^i)\phi^i}{\sum_i \alpha^i \phi^i} \tag{A.21}$$

Taking the ratio $\frac{\psi_0^{NE}}{\psi^{GP}}$, with ψ_0^{NE} defined in (31), we obtain the expression of ψ^{GP} in (33) with $\Delta = \phi^T \phi^N [(\delta_0 - \phi^T + \sigma_0 \phi^T) \sum_i \delta_0^i (1-\alpha^i)\phi^i]^{-1} > 0$. □

A.5 Derivation of (36)

Applying the envelope theorem to the central bank's problem (27), we get

$$\frac{dV_0}{dR_0^*} = -\frac{b_1^*}{(R_0^*)^2} u_T(c_0^T, c_0^N, \pi_0) - \frac{b_1^*}{(R_0^*)^2 c_0^T} \eta_0 - \frac{1}{R_0^*} u_T(c_0^T, c_0^N, \pi_0)\mu_0$$

Evaluating it at $R_0^* = R_0^{*NE}$, we arrive at

$$\left.\frac{dV_0}{dR_0^*}\right|_{R_0^* = R_0^{*NE}} = -\frac{1}{R_0^*} u_T(c_0^T, c_0^N, \pi_0)\mu_0$$

We then combine it with (34) to obtain (36).

A.6 Proof of Proposition 3

Proof of item (i). Consider π_0 and $\widehat{h}_0$ solution of the following system of equations

$$\underbrace{\tau\left(\widehat{h}_0,\pi_0\right)-\chi\psi\,\Theta(\pi_0)\pi_0}_{\Gamma(\widehat{h}_0,\pi_0)}=0 \tag{A.22}$$

$$\left[1+\frac{\chi}{\varepsilon}(1+\pi_0)^2\right]\pi_0=(1+\widehat{w}_0)\left(1+\widehat{h}_0\right)^{\sum_i(1-\alpha^i)\phi^i}-1 \tag{A.23}$$

where $\psi>0$ and $\tau(h_0^N,\pi_0)$ is given by

$$\tau_t(\widehat{h}_t,\pi_t)\equiv 1-\frac{\kappa_t}{F_h^N\left((1+\widehat{h}_t)\bar{h}_t^N,A_t^N\right)u_N\left(F^T\left((1+\widehat{h}_t)\bar{h}_t^T,A_t^T\right),F^N\left((1+\widehat{h}_t)\bar{h}_t^N,A_t^N\right),\pi_t\right)} \tag{A.24}$$

Note that $\widehat{h}_0^{NE}$ and π_0^{NE} (respectively $\widehat{h}_0^{GP}$ and π_0^{GP}) are solutions of (A.22) and (A.23) for $\psi=\psi_0^{NE}>0$ (respectively $\psi=\psi^{GP}>0$).

From (A.22), we have $\Gamma(0,0)=0$. Because $\frac{\partial\Gamma(\widehat{h}_0,\pi_0)}{\partial\widehat{h}_0}<0$ and $\frac{\partial\Gamma(\widehat{h}_0,\pi_0)}{\partial\pi_0}<0$, it follows that either (i) $\widehat{h}_0<0$ and $\pi_0>0$ or (ii) $\widehat{h}_0>0$ and $\pi_0<0$.

Suppose that $\widehat{w}_0>0$, then from (A.23) it has to be that $\widehat{h}_0<0$. This is because if $\widehat{h}_0>0$ then $\widehat{\pi}_0<0$ which implies that the right-hand side of (A.23) is a positive number while the left-hand side is negative. Thus $\widehat{h}_0>0$ cannot be a solution. Similarly, if $\widehat{w}_0<0$, then from (A.23) it has to be that $\widehat{h}_0<0$. Finally for $\widehat{w}_0=0$ we have that $\widehat{h}_0=\pi_0=0$. Thus,

$$\text{sign}(\widehat{h}_0)=-\text{sign}(\widehat{w}_0)\quad\text{for any }\psi>0 \tag{A.25}$$

It follows from (A.25) that $\text{sign}(\widehat{h}_0^{NE})=\text{sign}(\widehat{h}_0^{GP})$.

Proof of item (ii). We start by expressing (A.22) as $\mathcal{T}(\widehat{h},\psi)=0$ where

$$\mathcal{T}(\widehat{h},\psi)\equiv\tau\left(\widehat{h}_0,\pi(\widehat{h}_0)\right)-\chi\psi\,\Theta(\pi(\widehat{h}_0))\pi(\widehat{h}_0)$$

and $\pi(\widehat{h}_0)$ is given by (A.23), and recall that $\mathcal{T}(\widehat{h}_0^{NE};\psi_0^{NE})=0$ and $\mathcal{T}(\widehat{h}_0^{GP};\psi^{GP})=0$. Notice also that because $\frac{\partial\mathcal{T}(\widehat{h},\psi)}{\partial\widehat{h}}<0$, we have that $\widehat{h}_0^{NE}>\widehat{h}_0^{GP}\Leftrightarrow\mathcal{T}(\widehat{h}_0^{NE},\psi^{GP})<0$.

Letting τ_0^{NE} denotes τ_0 in the Nash equilibrium, we have that

$$\mathcal{T}\left(\widehat{h}_0^{NE},\psi^{GP}\right)=\underbrace{\mathcal{T}(\widehat{h}_0^{NE},\psi_0^{NE})}_{=0}+\frac{\psi^{GP}}{\psi_0^{NE}}\Delta(\sigma_0-\widetilde{\sigma})(\alpha^N-\alpha^T)\,\tau_0^{NE} \tag{A.26}$$

by which $\widehat{h}_0^{NE} > \widehat{h}_0^{GP} \Leftrightarrow (\sigma_0 - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{h}_0^{NE} > 0 \Leftrightarrow (\sigma_0 - \widetilde{\sigma})(\alpha^N - \alpha^T)\widehat{w}_0 < 0$ where we used $\text{sign}(\widehat{h}_0) = -\text{sign}(\widehat{w}_0)$ from (A.25).

Proof of item (iii). By (5) and (6) with $\pi_1 = 0$, we have $U'(c_0) = \beta R_0 U(c_1)$. In the Nash equilibrium $c_t^T = F^T(h_t^T, A_t^T)$ and the nominal rate is given by

$$R_0 = \frac{U'\left(\left(1 - \frac{\chi}{2}\pi_0^2\right) q\left(F^T\left(\frac{\alpha^T\phi^T}{\alpha^N\phi^N}h_0^N, A_0^T\right), F^N\left(h_0^N, A_0^N\right)\right)\right)}{\beta U'\left(q\left(F^T\left(\bar{h}_1^T, A_1^T\right), F^N\left(\bar{h}_1^N, A_1^N\right)\right)\right)}$$

where we also used (25) with $b_1^* = 0$. Totally differentiating this equation we obtain

$$\frac{dR_0}{dh_0^N} = -\sigma_0 \Upsilon(\pi_0)\left[\alpha^T\phi^T + \alpha^N\phi^N\right]\frac{R_0}{h_0^N} < 0 \tag{A.27}$$

where $\Upsilon(\pi_0) > 0$ is defined in (A.19). From (A.27), we have $R_0^{GP} > R_0^{NE} \iff \widehat{h}_0^{GP} < \widehat{h}_0^{NE}$. Combined with (A.26), we get $R_0^{GP} > R_0^{NE} \iff (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\widehat{h}_0^{NE} > 0$. □

A.7 Proof of Proposition 4

Given that the global planning problem is static, the solution to the problem date $t = -1$ (the targeting rule) is given by (A.21) where variables at $t = 0$ are replaced with variables at $t = -1$. The solution to the planner's problem is given by the targeting rule and the Phillips curve at $t = -1$ with $\widehat{w}_{-1} = 0$. That is,

$$\tau\left(\widehat{h}_{-1}, \pi_{-1}\right) - \chi\psi^{GP}\,\Theta(\pi_{-1})\pi_{-1} = 0 \tag{A.28}$$

$$\left[1 + \frac{\chi}{\varepsilon}(1 + \pi_{-1})^2\right]\pi_{-1} = \left(1 + \widehat{h}_{-1}\right)^{\sum_i (1-\alpha^i)\phi^i} - 1 \tag{A.29}$$

where $\tau(\widehat{h}_{-1}, \pi_{-1})$ is the labor wedge at $t = -1$ given (A.24).For $\widehat{h}_{-1} = \pi_{-1} = 0$ we have $\tau(\widehat{h}_{-1}, \pi_{-1}) = 0$ and therefore (A.28) and (A.29) are satisfied.

Next, we turn to the solution under Nash. The Lagrangian associated with the central bank's problem at $t = -1$ is analogous to (27) where the continuation value $V_0(b_0^*)$ solves (27) for a given b_0^*. Optimality condition for h_{-1}^T and h_{-1}^N combined with the envelope condition for $V_0(b_0)$ yields the following targeting rule

$$-\tau_{-1} + \chi\psi^{NE}\,\Theta(\pi_{-1})\pi_{-1} = \beta\Lambda\left(\sigma_{-1} - \widetilde{\sigma}\right)u_T(c_0^T, c_0^N)\mu_0, \tag{A.30}$$

with $\Lambda > 0$ and where μ_0 satisfies (34). Combining (A.30) with (A.29) and (34), we get

$$\mathcal{T}(\widehat{h}_{-1}) = -\beta\Lambda\frac{\Delta}{\alpha^N}h_0^N(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T)\tau_0 \tag{A.31}$$

where

$$\mathcal{T}(\widehat{h}_{-1}) \equiv \tau\left(\widehat{h}_{-1}, \pi(\widehat{h}_{-1})\right) - \chi\psi^{NE}\,\Theta(\pi(\widehat{h}_{-1}))\pi_{-1}$$

with $\pi(\widehat{h}_{-1})$ satisfying (A.29). We have that $d\mathcal{T}(\widehat{h}_{-1})/d\widehat{h}_{-1} < 0$ with $\mathcal{T}(0) = 0$. Therefore, $\widehat{h}_{-1} > 0$ if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T)\widehat{h}_0 > 0$. Moreover, for $\widehat{h}_{-1} > 0$, we have by (26) that $\widehat{\pi}_{-1} > 0$. Conversely when $\widehat{h}_{-1} < 0$ we have that $\widehat{\pi}_{-1} < 0$. □

A.8 Proof of Corollary 1

Suppose $\widehat{h}_0^N < 0$. Note that in cooperation solution features $\widehat{h}_{-1}^N = 0$. By (A.31) the Nash equilibrium coincides with the cooperation solution if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T) = 0$. Furthermore, the Nash equilibrium features under-tightening $\widehat{h}_{-1}^N > 0$ if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T) > 0$ or equivalently if and only if $(\sigma_{-1}-\widetilde{\sigma})(\alpha^N-\alpha^T)\widehat{h}_{-1}^N > 0$. □

A.9 Derivation of the Loss Function

The second-order Taylor expansion of

$$\mathcal{U}_0 \equiv u\left(F^T\left(h_0^T, A_0^T\right) - \frac{b_1^*}{R_0^*}, F^N\left(h_0^N, A_0^N\right), \pi_0^2\right) - \kappa\left(h_0^T + h_0^N\right) + \beta V\left(b_1^*\right)$$

around the first-best allocation yields

$$\begin{aligned}\mathcal{U}_0 - \bar{\mathcal{U}} = \frac{\kappa_0\bar{h}_0^T}{\alpha^T\phi^T}\Bigg\{ &-\left[1+(\sigma_0-1)\sum_i\alpha^i\phi^i\right]\alpha^T\phi^T\frac{1}{2}\left(\widehat{h}_0^T\right)^2 - \left[1+(\sigma_0-1)\sum_i\alpha^i\phi^i\right]\alpha^N\phi^N\frac{1}{2}\left(\widehat{h}_0^N\right)^2 \\ &+\left[1+(\sigma_0-1)\sum_i\alpha^i\phi^i\right]\alpha^T\phi^T\frac{1}{2}\left(\widehat{h}_0^N-\widehat{h}_0^T\right)^2 + \left[1+(\sigma_0-1)\sum_i\alpha^i\phi^i\right]\alpha^T\phi^T\frac{b_1^*}{R_0^*F(\bar{h}_0^T, A_0^T)}\widehat{h}_0^N \\ &-\left[\delta_0+(\sigma_0-1)\phi^T\right]\phi^T\frac{1}{2}\left(\frac{b_1^*}{R_0^*F(\bar{h}_0^T, A_0^T)}\right)^2 - \frac{\chi}{2}\left(\pi_0\right)^2\Bigg\}\end{aligned} \tag{A.32}$$

where we used (20) and $V'(b_1^*) = u_T(c_1^T, c_1^N, 0)$. δ_0 is defined in (A.5). We then substitute the linearized (25), that is $\widehat{h}_0^T = \widehat{h}_0^N + \frac{b_1^*}{R_0^*F(\bar{h}_0^T, A_0^T)}$, into (A.32) and rearrrange it to get (39).

B Proofs of Extensions

To simplify the analysis, we assume that households' preferences are separable between consumption and inflation. In particular, we assume that they are described by

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t^T, c_t^N\right) - \kappa_t h_t - \frac{\chi}{2}(\pi_t)^2 \right], \tag{B.1}$$

which can be seen as a second-order approximation around $\pi_t = 0$ of the indirect utility of households. More specifically, the first-order approximation around $\pi_t = 0$ yields

$$u\left(c_t^T, c_t^N, \pi_t\right) - \kappa_t h_t \approx u(c_t^T, c_t^N) - \kappa_t h_t - \frac{\chi_t}{2}(\pi_t)^2,$$

where $\chi_t = c_t U'(c_t)\chi$. (B.1) considers the case where $\chi_t = \bar{\chi}$ constant.[42]

B.1 Elasticity of Substitution

This section extends the baseline model with CES aggregators. Households' preferences are described by (B.1) where the consumption good c_t is now a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a CES aggregator

$$c_t = \left[\sum_{i \in \mathcal{S}} \phi^i \left(c_t^i\right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

The budget constraint of households is identical to the one in the baseline model. The household's optimality condition with respect to c_t^T and c_t^N (8) is now given by

$$\frac{P_t^N}{P_t^T} = \frac{\phi^N}{\phi^T} \left(\frac{c_t^T}{c_t^N} \right)^{\gamma} \tag{B.2}$$

Using (B.2), we can express the share of expenditures in tradables $\tilde{\phi}_t^T \equiv P_t^T c_t^T / (P_t c_t)$ as $\tilde{\phi}_t^T = \phi^T (c_t^T / c_t)^{1-\gamma}$. and the share of expenditures in non-tradables is $\tilde{\phi}_t^N = 1 - \tilde{\phi}_t^T$. The remaining optimality conditions of the household's problem are (5), (6) (and (4) for $t > 0$) while for firms, (11), (12) continue to hold. Combining (B.2) with (11) and (12) we obtain

$$\frac{h_t^N}{h_t^T} = \frac{\alpha^N \tilde{\phi}_t^N}{\alpha^T \tilde{\phi}_t^T} \left[1 - \frac{b_1^*}{R_0^* F^T(h_0^T, A_0^T)} \right] \tag{B.3}$$

[42]Note that under $U(c) = \log(c)$, we have $c_t U'(c_t) = 1$ and thus $\chi_t = \chi$.

While using (18) and (19), the optimal ratio of hours in the first-best allocation becomes

$$\frac{\bar{h}_t^T}{\bar{h}_t^N} = \frac{\alpha^T \tilde{\phi}_t^T}{\alpha^N \tilde{\phi}_t^N}$$

which corresponds to the employment ratio in a competitive symmetric equilibrium for any monetary policy. Therefore, in any symmetric competitive equilibrium, the output gaps in the tradable and non-tradable sectors are proportional, and to a first-order

$$\widehat{h}_t^N = \Xi\, \widehat{h}_t^T, \quad \text{where } \Xi \equiv \frac{1-\alpha^T+\alpha^T\gamma}{1-\alpha^N+\alpha^N\gamma} > 0. \tag{B.4}$$

In the lemma below, we summarize the effects of monetary policy on the trade balance.

Lemma B.1 (Generalized Marshall-Lerner Condition)**.** *The response of the trade balance to a domestic monetary expansion satisfies* $-\frac{db_1^*}{dR_0} > 0 \iff \sigma_0 > \gamma\widetilde{\sigma}$ *where* $\widetilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T\phi^T+\Xi\alpha^N\phi^N}$.

Proof. Proceeding similarly as in Appendix A.3 by combining (5), (6), (11), (25) we get

$$\left[\delta_0+(\sigma_0-1)(\alpha^T\phi^T+\Xi\alpha^N\phi^N)\right](\delta_0-\alpha^T)db_1^* = -\frac{R_0^*}{R_0}c_0^T\left[\alpha^T+(\sigma_0-\gamma)(\alpha^T\phi^T+\Xi\alpha^N\phi^N)\right]dR_0$$

Thus $-\frac{db_1^*}{dR_0} > 0 \iff \alpha^T\gamma + (\sigma_0-\gamma)(\alpha^T\phi^T+\Xi\alpha^N\phi^N) > 0$. Defining $\widetilde{\sigma} \equiv 1 - \frac{\alpha^T}{\alpha^T\phi^T+\Xi\alpha^N\phi^N}$, we obtain that $-db_1^*/dR_0 > 0$ if and only if $\sigma_0 > \gamma\widetilde{\sigma}$. □

Note that, given preferences, the consumer price index P_t now satisfies

$$P_t = \left[\sum_{i\in\mathcal{S}} (\phi^i)^{\frac{1}{\gamma}} (P_t^i)^{1-\frac{1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}$$

Thus, using the definition of the natural wage we can express the inflation gap as

$$\left(1+\frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0 = \frac{W}{w_0^n P_{t-1}}\left[\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}\left(F_h(h_t^i,A_t^i)\right)^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}\left(F_h(\bar{h}_t^i,A_t^i)\right)^{\frac{\gamma}{1-\gamma}}}\right]^{\frac{\gamma}{\gamma-1}} - 1 \tag{B.5}$$

The Lagrangian associated with the central bank's problem can be written as follows

$$\mathcal{L} = u\left(F^T(h_0^T, A_0^T) - \frac{b_1^*}{R_0^*}, F^N(h_0^N, A_0^N)\right) - \frac{\chi}{2}(\pi_0)^2 - \kappa_0(h_0^T + h_0^N) + \beta V_1(b_1^*)$$
$$+\vartheta_0\left[\left(1+\frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0 - \frac{W}{w_0^n P_{t-1}}\left(\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(h_t^i, A_t^i))^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(\bar{h}_t^i, A_t^i))^{\frac{\gamma}{1-\gamma}}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right]$$
$$+\eta_0\left[\left(1 - \frac{b_1^*}{R_0^* F(h_0^T, A_0^T)}\right)\frac{\alpha^N \tilde{\phi}_0^N}{\alpha^T \tilde{\phi}_0^T}\frac{h_0^T}{h_0^N} - 1\right]$$
$$+\mu_0\left[u_T\left(F^T(h_0^T, A_0^T), F^N(h_0^N, A_0^N)\right) - \beta R_0^* u_T\left(\mathcal{C}^T(b_1^*), \mathcal{C}^N(b_1^*)\right)\right]$$

Optimality condition for b_1^* yields $\eta_0 = \left[\delta_0 + (\sigma_0\gamma^{-1} - 1)\tilde{\phi}_0^T\right] u_T(c_0^T, c_0^N)\mu_0$ where δ_0 is given by (A.5). Using this equation and combining the first-order conditions for h_0^T and h_0^N, we obtain the following targeting rule in the Nash equilibrium (where $b_1^* = 0$):

$$\tau_0 = \chi\psi^{NE}\,\Theta(\pi_0)\pi_0 \quad \text{with} \quad \psi_0^{NE} = \frac{\sum_{i=T,N}\delta_0^i(1-\alpha^i)\tilde{\phi}_0^i}{\sum_{i=T,N}\delta_0^i\alpha^i\tilde{\phi}_0^i} \tag{B.6}$$

where $\tau_t^T = \tau_0^T = \tau_0$ is defined in (22), and $\delta_0^T > 0$ and $\delta_0^N > 0$ are given by

$$\delta_0^T \equiv 1 + \alpha^N(\gamma - 1) + \frac{(\sigma_0 - \gamma)\alpha^N\tilde{\phi}_0^N}{\delta_0 - \tilde{\phi}_0^T + \sigma_0\gamma^{-1}\tilde{\phi}_0^T}$$
$$\delta_0^N \equiv 1 + \alpha^T(\gamma - 1) - \alpha^T\frac{\gamma + (\sigma_0 - \gamma)\tilde{\phi}_0^T}{\delta_0 - \tilde{\phi}_0^T + \sigma_0\gamma^{-1}\tilde{\phi}_0^T}$$

To see why $\delta_0^T > 0$ and $\delta_0^N > 0$, notice that for $\sigma > \gamma$ this is trivial. For $\sigma < \gamma$, it can be shown that δ_0^T and δ_0^N are increasing in γ, and we have $\lim_{\gamma\to 0}\delta_0^T > 0$ and $\lim_{\gamma\to 0}\delta_0^N > 0$.

Under cooperation, the Lagrangian associated with the planner problem is given by

$$u\left(F^T(h_0^T, A_0^T) - \frac{b_1^*}{R_0^*}, F^N(h_0^N, A_0^N)\right) - \frac{\chi}{2}(\pi_0)^2 - \kappa_0(h_0^T + h_0^N)$$
$$+\vartheta_0\left[\left(1+\frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0 - \frac{W}{w_0^n P_{t-1}}\left(\frac{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(h_t^i, A_t^i))^{\frac{1-\gamma}{\gamma}}}{\sum_{i=T,N}(\phi^i)^{\frac{1}{\gamma}}(F_h(\bar{h}_t^i, A_t^i))^{\frac{\gamma}{1-\gamma}}}\right)^{\frac{\gamma}{\gamma-1}} + 1\right] + \eta_0\left[\frac{\alpha^N\tilde{\phi}_0^N}{\alpha^T\tilde{\phi}_0^T}\frac{h_0^T}{h_0^N} - 1\right]$$

The targeting rule, which combined the first-order condition, for h_0^T and h_0^N is given by

$$\tau_0 = \chi \psi^{GP}\, \Theta(\pi_0)\pi_0 \quad \text{with } \psi^{GP} = \frac{\sum_{i=T,N} \delta_x^i (1-\alpha^i)\tilde{\phi}_0^i}{\sum_{i=T,N} \delta_0^i \alpha^i \tilde{\phi}_0^i} \tag{B.7}$$

with $\delta_x^T = 1 + \alpha^N(\gamma - 1)$ and $\delta_x^N = 1 + \alpha^T(\gamma - 1)$. Taking the ratio between the relative weights in the targeting rules (B.6) and (B.7), we arrive at

$$\frac{\psi_0^{NE}}{\psi^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \gamma\widetilde{\sigma})\, \Delta, \quad \text{with } \Delta \equiv \frac{\tilde{\phi}_0^T \tilde{\phi}_0^N}{(\delta_0 - \tilde{\phi}_0^T + \sigma_0 \gamma^{-1} \tilde{\phi}_0^T) \sum_i \delta_0^i (1-\alpha^i)\tilde{\phi}_0^i} > 0.$$

Therefore, our key sufficient statistic results remain unchanged. The only difference with our baseline framework is that the condition required for the trade balance to increase in response to a monetary expansion is now given by $\sigma > \gamma\widetilde{\sigma}$ where $\widetilde{\sigma}$ is the threshold in our baseline framework (where $\gamma = 1$) and γ is the elasticity of substitution between goods.

B.2 Imperfect Labor Mobility

In this section, we extend the baseline model with imperfect labor mobility. Households' preferences are given by (B.1) where aggregate hours worked n_t is now a composite of hours worked in the tradable sector and in the non-tradable sector according to (40). The budget constraint of households is identical to the one in the baseline model. The household's optimality condition with respect to c_t^T and c_t^N is given by (8), while the optimal labor supply decisions for $t > 0$ (4) now satisfy

$$\frac{W_t^N}{P_t^N} = \frac{\kappa_t}{u_N(c_t^T, c_t^N)} \left(\frac{n_t^N}{n_t}\right)^{\frac{1}{\xi}}, \qquad \frac{W_t^T}{P_t^T} = \frac{\kappa_t}{u_T(c_t^T, c_t^N)} \left(\frac{n_t^T}{n_t}\right)^{\frac{1}{\xi}}. \tag{B.8}$$

where W_t^T and W_t^N are the nominal wages in the tradable and non-tradable sectors. The remaining optimality conditions of households are (5), (6). For firms, optimality conditions (11), (12) now become $P_t^i F_h(h_t^i, A_t^i) = W_t^i$ for $i = T, N$ which combined with (8) yields

$$\frac{h_0^N}{h_0^T} = \frac{W^T}{W^N} \frac{\alpha^N \phi^N}{\alpha^T \phi^T} \left[1 - \frac{b_1^*}{R_0 F^T(h_0^T, A_0^T)}\right], \tag{B.9}$$

while the employment ratio in the efficient allocation is

$$\frac{\bar{h}_0^N}{\bar{h}_0^T} = \left[\frac{\alpha^N \phi^N}{\alpha^T \phi^T}\right]^{\frac{\xi}{1+\xi}}. \tag{B.10}$$

Before turning to comparing the targeting rules, we find it useful to describe the natural wage under imperfect labor mobility.

The natural wage and the labor wedge. Define $\widetilde{w}_t = \frac{(W_t^T)^{\phi^T}(W_t^N)^{\phi^N}}{P_t}$. The natural wage, that is the wage, in units of the final good, that would prevail in equilibrium if prices and wages were flexible, satisfy

$$\widetilde{w}_t^n = \prod_{i=T,N} \left(\alpha^i A_t^i\right)^{\phi^i} \left(\bar{h}_t^i\right)^{-(1-\alpha^i)\phi^i}. \tag{B.11}$$

where we use (10), (11), (12) and $\chi = 0$. Given that $\pi_{t+1} = 0$, (B.11) and (10) can be used to express the level of inflation as

$$\left(1 + \frac{\chi}{\varepsilon}(1+\pi_0)^2\right)\pi_0 = \frac{\widetilde{W}}{\widetilde{w}_0^n P_{t-1}} \left(\frac{h_0^T}{\bar{h}_0^T}\right)^{(1-\alpha^T)\phi^T} \left(\frac{h_0^N}{\bar{h}_0^N}\right)^{(1-\alpha^N)\phi^N} - 1. \tag{B.12}$$

The nominal wages in period 0 are fixed at arbitrary values W^T and W^N. To simplify the analysis, we assume that $\frac{W^N}{W^T} = \left(\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\right)^{1/(1+\xi)}$ which ensures that the labor wedges are equalized across sectors, $\tau_0^T = \tau_0^N$, where (similar to (22)) the labor wedges are defined as

$$\tau_t^i = 1 - \frac{1}{F_h^i(h_t^i, A_t^i)u_i(c_t^T, c_t^N)}\kappa_t \left(\frac{h_t^i}{h_t}\right)^{\frac{1}{\xi}}. \tag{B.13}$$

Targeting rules. The Lagrangian for the central bank problem is thus analogous to (27) where aggregate hours are now given by (40) and the Phillips curve is given by (B.12).

Given preferences given by (B.1), the optimality condition for π_0 (A.3) simplifies to $(1+2\pi_0)\vartheta_0 = c_0 U'(c_0)\frac{\varepsilon\pi_0}{1-\frac{\chi}{2}\pi_0^2}$ which combined with the optimality conditions for h_0^N and h_0^T which yield (A.13) with $\delta_\pi^i = 0$. In the Nash equilibrium where $b_1^* = 0$, we obtain

$$\tau_0 = \chi\psi^{NE}\,\Theta(\pi_0)\pi_0, \quad \text{with } \psi_0^{NE} \equiv \frac{\sum_{i=T,N}\delta_0^i(1-\alpha^i)\phi^i}{\sum_{i=T,N}\delta_0^i\alpha^i\phi^i}, \tag{B.14}$$

with $\delta_0^i = \tilde{\delta}_0^i > 0$ given by (A.10) and (A.9). The Lagrangian associated with the global planner's problem is analogous to (32) with aggregate hours now given by (40). After combining the optimality conditions for h_0^T and h_0^N, we obtain

$$\tau_0 = \chi\psi^{GP}\,\Theta(\pi_0)\pi_0, \quad \text{with } \psi^{GP} \equiv \frac{\sum_{i=T,N}(1-\alpha^i)\phi^i}{\sum_{i=T,N}\alpha^i\phi^i}.$$

Taking the ratio of the relative weights we arrive at

$$\frac{\psi_0^{NE}}{\psi^{GP}} = 1 + (\alpha^N - \alpha^T)(\sigma_0 - \widetilde{\sigma})\,\Delta.$$

with Δ defined in A.4. The difference in the weight that central banks put on inflation in the Nash equilibrium and under cooperation is the same as in the baseline. Our key sufficient statistic remains unchanged.

B.3 Oil Shock

This section extends the model to incorporate oil as an intermediate input. We assume that households receive an endowment of oil which can be sold to firms domestically and abroad. The law of one price is assumed to hold in the market for oil, that is $P_{mt} = e_t P_{mt}^*$ where P_{mt} and P_{mt}^* are the domestic and the world price of oil, and e_t is the effective exchange rate. Combined with the law of one price for tradables, this implies that $\frac{P_{mt}}{P_t^T} = \frac{P_{mt}^*}{P_t^{T*}}$. We can thus express households' budget constraint as

$$P_t^T c_t^T + P_t^N c_t^N + \frac{b_{t+1}}{R_t} + \frac{P_t^T b_{t+1}^*}{R_t^*} = W_t(n_t^T + n_t^N) + \varphi_t + P_{mt} M_t + b_t + P_t^T b_t^*,$$

where M_t is the supply of oil in the domestic economy. The production functions are given by $F^i(h_t^i, m_t^i, A_t^i)$ with α^i, and ζ^i denoting the intensity of labor and oil respectively. At the optimum, the demand for labor is analogous to (11)-(12) and given by $P_t^i F_h(h_t^i, m_t^i, A_t^i) = W_t$ for all $i \in \mathcal{S}$; while their demand for oil is given by

$$m_t^T = \frac{\zeta^T}{\alpha^T}\frac{W_t}{P_{mt}} h_t^T, \quad m_t^N = \frac{\zeta^N}{\alpha^N}\frac{W_t}{P_{mt}} h_t^N \tag{B.15}$$

The Lemma below describes the allocation of oil in any symmetric competitive equilibrium.

Lemma B.2. *In any symmetric competitive equilibrium, the allocation of intermediate oil inputs is efficient and given by*

$$m_t^N = \frac{\zeta^N \phi^N}{\sum_{i=T,N} \alpha_m^i \phi^i} M_t, \quad m_t^T = \frac{\zeta^T \phi^T}{\sum_{i=T,N} \alpha_m^i \phi^i} M_t \tag{B.16}$$

Proof. The proof combines the ratio of the two equations in (B.15) with (25), together with $b_1^* = 0$ and market clearing for oil $m_t^T + m_t^N = M_t$. □

Denoting by $\overline{m}_0^T$ and $\overline{m}_0^N$ the allocation in (B.16), the Lagrangian associated with the global planning problem can be expressed as

$$u\left(F^T(h_0^T,\overline{m}_0^T,A_0^T),F^N(h_0^N,\overline{m}_0^N,A_0^N)\right)-\kappa_0(h_0^T+h_0^N)-\frac{\chi}{2}(\pi_0)^2$$
$$+\vartheta\left[\frac{\chi}{\varepsilon}(1+\pi_0)\pi_0-\frac{W}{W_0^n}\left(\frac{F_h\left(\bar{h}_0^T,\overline{m}_0^T,A_0^T\right)}{F_h\left(h_0^T,\overline{m}_0^T,A_0^T\right)}\right)^{\phi^T}\left(\frac{F_h\left(\bar{h}_0^N,\overline{m}_0^N,A_0^N\right)}{F_h\left(h_0^N,\overline{m}_0^N,A_0^N\right)}\right)^{\phi^N}+1\right]+\eta\left[\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N}-1\right].$$

Notice that the allocation of oil is independent of policy. The targeting rule under cooperation, which combines the optimality condition for h_0^T and h_0^N, is therefore identical to (33) and given by

$$\tau_0=\chi\psi^{GP}\,\Theta(\pi_0)\pi_0\quad\text{with}\;\;\psi^{GP}=\frac{\sum_i(1-\alpha^i)\phi^i}{\sum_i\alpha^i\phi^i}\tag{B.17}$$

where

$$\tau_0\equiv F_h^N(h_0^N,\overline{m}_0^N,A_0^N)\,u_N\left(F^T\left(\frac{\alpha^T\phi^T}{\alpha^N\phi^N}h_0^N,M_0-\overline{m}_0^N,A_0^T\right),F^N\left(h_0^N,\overline{m}_0^N,A_0^N\right)\right)-\kappa_0$$

We now turn to deriving the targeting rule in the Nash equilibrium. Combining (B.15) with (25), the allocation of oil input across sectors in a small open economy is given by

$$m_t^T(b_{t+1}^*)=\frac{\zeta^T\phi^T(1-\widehat{b}_{t+1}^*)}{\zeta^N\phi^N+\zeta^T\phi^T(1-\widehat{b}_{t+1}^*)}M_t\;\;\text{and}\;\;m_t^N(b_{t+1}^*)=\frac{\zeta^N\phi^N}{\zeta^N\phi^N+\zeta^T\phi^T(1-\widehat{b}_{t+1}^*)}M_t\tag{B.18}$$

with $\widehat{b}_{t+1}^*\equiv\frac{b_{t+1}^*}{R_t^*F^T(h_t^T,m_t,A_t^T)}$. Using (B.18), we can express the Lagrangian associated with the central bank problem as

$$\mathcal{L}=u\left(F^T(h_0^T,m^T(b_1^*),A_0^T)-\frac{b_1^*}{R_0^*},F^N(h_0^N,m^N(b_1^*),A_0^N)\right)$$
$$-\kappa_0(h_0^T+h_0^N)-\frac{\chi}{2}(\pi_0)^2+\beta V_1\left(b_1^*\right)+\eta_0\left[\left(1-\frac{b_1^*}{R_t^*F^T(h_t^T,m_t,A_t^T)}\right)\frac{\alpha^N\phi^N}{\alpha^T\phi^T}\frac{h_0^T}{h_0^N}-1\right]$$
$$+\vartheta_0\left[\frac{\chi}{\varepsilon}(1+\pi_0)\pi_0-\frac{W}{W_0^n}\left(\frac{F_h\left(\bar{h}_0^T,\overline{m}_0^T,A_0^T\right)}{F_h\left(h_0^T,m^T(b_1^*),A_0^T\right)}\right)^{\phi^T}\left(\frac{F_h\left(\bar{h}_0^N,\overline{m}_0^N,A_0^N\right)}{F_h\left(h_0^N,m^N(b_1^*),A_0^N\right)}\right)^{\phi^N}+1\right]$$
$$+\mu_0\left[u_T\left(F^T(h_0^T,m^T(b_1^*),A_0^T)-\frac{b_1^*}{R_0^*},F^N(h_0^N,m^N(b_1^*),A_0^N)\right)-\beta R_0^*u_T\left(\mathcal{C}^T(b_1^*),\mathcal{C}^N(b_1^*)\right)\right]$$

The optimality condition with respect to b_1^* is given by

$$\eta_0 = \left[\delta_0^m + (\sigma_0 - 1)\phi^T\right] u_T(c_0^T, c_0^N)\mu_0 \tag{B.19}$$

where on the competitive equilibrium path, δ_0^m is given by

$$\delta_0^m = \delta_0 + \frac{\zeta^T\phi^T \cdot \zeta^N\phi^N}{\zeta^T\phi^T + \zeta^N\phi^N} + \chi\left(\phi^T\frac{F_{hm}^T}{F_h^T} + \phi^N\frac{F_{hm}^N}{F_h^N}\right)(1+\pi_0)\pi_0 \tag{B.20}$$

Notice by (B.18) and (B.16) that in the Nash equilibrium where $\widehat{z}_0 = 0$, the allocation of oil is optimal. Moreover, the optimality condition for h_0^N and h_0^T are akin to (A.1) and (A.2) where δ_0 is replaced with δ_0^m. As a result, the targeting rule in the Nash equilibrium is

$$\tau_0 = \chi\psi^{NE}\,\Theta(\pi_0)\pi_0 \quad \text{with} \;\; \psi_0^{NE} = \frac{\sum_i \delta_0^i(1-\alpha^i)\phi^i}{\sum_i \delta_0^i\alpha^i\phi^i} \tag{B.21}$$

where δ_0^N and δ_0^T satisfy (A.10) and (A.9) where δ_0 is replaced with δ_0^m. Taking the ratio of the relative weights on inflation in (B.17) and (B.21), we arrive at

$$\frac{\psi_0^{NE}}{\psi^{GP}} = 1+(\alpha^N-\alpha^T)(\sigma_0-\widetilde{\sigma})\,\Delta_m, \quad \text{with } \Delta_m \equiv \frac{\phi^T\phi^N}{\left(\delta_0^m-\phi^T+\sigma_0\phi^T\right)\sum_i \delta_0^i(1-\alpha^i)\phi^i} > 0, \tag{B.22}$$

which confirms that the difference in the weight that the central banks put on inflation in the Nash equilibrium and under cooperation is the same as in the baseline framework.