Inequality and Asset Prices during Sudden Stops [†]

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Abstract

This paper studies the cross-sectional dimension of Fisher's debt-deflation mechanism that triggers endogenous financial crises of the Sudden Stop type. Analyzing micro-data from Mexico, we show that this mechanism's cross-sectional dimension has macroeconomic implications that operate via two opposing effects, making the role of inequality during crises ambiguous. We propose a small open economy, asset-pricing model with heterogeneous-agents and aggregate risk to measure the effects of inequality on the severity of financial crises. In contrast to the representative-agent framework, the model with heterogeneity generates persistent current account reversals with smaller drops in asset prices and larger drops in consumption driven by the most leveraged households. Moreover, calibrating the heterogeneous-agents model to an advanced economy where the dividend risk, which drives wealth inequality, is one-half of the benchmark emerging-markets model, larger debt positions are supported, and Sudden Stop crises are less severe, as observed in the data.

JEL CLASSIFICATION: D31, E21, E44, F32, F41, G01.

KEY WORDS: Inequality, Sudden Stops, Debt-deflation, Asset-pricing, Household Leverage.

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1 Introduction

In the last 40 years, 58 financial crises have occurred in both emerging and developed economies of the Sudden Stop type, each characterized by episodes of a large reversal in the current account deficit.[1] The occurrence of these crises has led to a vast literature that studies Sudden Stops using models with financial frictions but assuming a representative-agent framework. In this paper, we argue that inequality in wealth and leverage across households plays an important role in determining the aggregate effects of a financial crisis.[2] Specifically, an economy's aggregate exposure to tighter financial conditions depends on the share of financially vulnerable households defined as those that end up constrained when the crisis happens. Sudden Stops are characterized by large declines in asset prices, which affect households differently depending on their balance sheet. For example, micro-data evidence from Mexico (an open economy commonly used to study Sudden Stops) shows that during the 2009 crisis, households in the top decile of leverage decreased their expenditures by 6.2% while households in the bottom decile increased their expenditures by 5.4%. Moreover, the asset holdings of wealthy households with leverage in the bottom eight deciles increased 59.4% while wealthy households in the top two deciles of leverage fire-sold and decreased the most their assets during the crisis. Hence, studying only aggregate dynamics misses the fact that financial crises do not affect all households in the same way and that inequality has aggregate implications.

This paper addresses this issue by examining the cross-sectional dimension of the debt-deflation mechanism introduced by Fisher (1933). This mechanism works as follows. After a *negative*[3] aggregate shock that tightens the financial conditions of the economy, financially constrained agents sell part of their collateralizable assets, which puts downward pressure on asset prices. As asset prices drop, (possibly more) finan-

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[1] See Bianchi and Mendoza (2020) for a recent survey and review of the stylized facts of Sudden Stops.

[2] Figure 6 shows descriptive evidence that emerging economies are more unequal than advanced economies, and that Sudden Stop episodes are more severe in more unequal economies.

[3] Commonly studied negative shocks in small open economy models are an increase in the international interest rate, a decrease in total factor productivity, a drop in the terms of trade, or an ad-hoc tightening of the financial conditions of the economy. In this paper, the financial tightening shock will be an increase in the international interest rate.

cially constrained agents have to sell a larger asset position, which causes feedback that puts additional downward pressure on asset prices, and this, in turn, further tightens aggregate financial conditions. This paper posits that the cross-sectional dimension of the debt-deflation mechanism matters for macro dynamics of Sudden Stops via two opposing effects: First, a crisis-dampening effect that weakens the debt-deflation mechanism because unconstrained wealthy households can buy the depressed assets fire-sold by financially constrained households. Second, a crisis-amplifying effect that strengthens the debt-deflation mechanism because of financially vulnerable households that become credit-constrained as asset prices fall. As aggregate financial conditions tighten, such households also have to sell assets, increasing the downward pressure on asset prices. Because these two cross-sectional effects constitute opposing forces, the role of the cross-section and inequality during crises is quantitatively ambiguous. Hence, this paper conducts a quantitative investigation of the degree to which the severity of Sudden Stop crises is affected by inequality in an economy.

To shed light on the empirical relevance of these issues, we examine a panel household survey for Mexico that provides evidence of the dampening and amplifying cross-sectional effects. Moreover, we test – and reject – the individual complete-market hypothesis. These results support our decision to use a heterogeneous-agent framework to study financial crises and cross-sectional dynamics in households' consumption and portfolio choice

Then, the paper conducts a quantitative analysis of the effect of wealth inequality on Sudden Stops. To this end, we propose a small open economy, asset-pricing Bewley model with debt and assets, an endogenous occasionally-binding loan-to-value (LtV) collateral constraint, and aggregate risk. At the individual level, markets are incomplete, and households face both idiosyncratic labor and dividend income risk. The combination of the dividend risk with an imperfect debt market (the LtV constraint) generates an *asset-wealth trade-off*: more asset holdings relax the individual collateral constraint and allow for better consumption smoothing (reducing consumption volatility) but also, more asset holdings increase the divided risk exposure which leads to higher income volatility of the household (increasing consumption volatility), incentivizing additional precautionary savings. This trade-off makes high-dividend asset-rich households deleverage faster than low-dividend households, producing an

empirically plausible leverage ratio distribution with wealthy unconstrained households that face non-degenerate portfolio choices.

In a version of the model calibrated to an emerging economy (Mexico), the quantitative analysis shows that the dampening effect dominates and asset prices drop less in heterogeneous-agents economies. In contrast to the representative-agent framework, the model generates persistent current account reversals with larger drops in consumption driven by the most leveraged households, consistent with the data. Moreover, calibrating the model to an advanced economy where the dividend risk variance is one-half of the benchmark emerging-markets model, the average net foreign debt position is 8.5 percent larger, and during a crisis, consumption drops 1.1 percentage points less, and asset prices drop 0.4 percentage points less. Hence, the model predicts that in economies with lower dividend return variance, income inequality is lower, the economy supports larger debt positions and Sudden Stop crises are less severe, as observed in the data.

After reviewing the literature in Section 2, in Section 3 we describe the empirical evidence that supports the cross-sectional effects of the debt-deflation mechanism. The proposed model is described in Section 4. Section 5 describes the cross-sectional effects through the lens of the model. Section 6 presents the quantitative analysis and Section 7 concludes.

2 Related Literature

This paper contributes to three strands in the economics literature. In the first strand, Sudden Stop crises have been studied using representative-agent models with financial frictions. For instance, Mendoza (2010) studies Sudden Stops in a standard representative firm-agent real business cycle model augmented with a debt-deflation mechanism. He introduces a loan-to-value collateral constraint that generates a pecuniary externality, reflecting that agents do not internalize how their decisions today affect the equilibrium *Tobin's Q* price of capital that tightens or loosens the debt capacity. In a related paper, Mendoza and Smith (2006) study the debt-deflation mechanism in a small open economy with a representative-agent that trades domestic equity with a foreign investor. In their model, the combination of a collateral

constraint and equity trading costs can produce realistic Sudden Stops. Our paper complements both studies, yet it differs fundamentally from them because we study the cross-sectional dimension of the debt-deflation mechanism. To this end, we introduce market incompleteness at the individual level and study how the distribution of households along bonds, assets, and individual productivities affects the asset's price, portfolio choices, and consumption dynamics during crises.

A second strand of the literature focuses on asset prices in closed economies with individual incomplete markets. Aiyagari and Gertler (1991) study asset prices and particularly the equity premium puzzle (see Mehra and Prescott (1985)) in a closed economy with two assets (bonds and stocks), adjustment costs, and individual labor income risk. The authors conclude that the difference in relative adjustment costs between assets and the need to trade assets for consumption smoothing – introduced by the individual market incompleteness – can generate a spread between the return on bonds and stocks. Heaton and Lucas (1996), who study an economy with two types of agents, income risk, adjustment costs, short-sales constraints, and debt constraints, find that the adjustment costs can generate higher equity premiums. Studying the excess volatility in asset prices that a loan-to-value constraint causes, Aiyagari and Gertler (1999) explains price volatility in a model with limited heterogeneity. There are only two representative-agents in their environment: a household and a trader. When the trader is constrained, the multiplier in the collateral constraint is active for the whole population of traders. This behavior translates into higher volatility in asset prices. More recently, Storesletten, Telmer, and Yaron (2007) show that in a life-cycle model, the effects of idiosyncratic labor risk are quantitatively significant if the idiosyncratic risk becomes more volatile during economic contractions. They further demonstrate that idiosyncratic risk inhibits inter-generational risk sharing, imposing a disproportionate share of aggregate risk on the wealthy middle-aged cohorts who demand an equity premium for their exposure to this risk. In their setting, the young cohorts do not hold equity to avoid the counter-cyclical volatility risk. Our paper differs but complements this strand of the literature since although we study a small open economy, the risky domestic asset is closed to foreign investors and behaves as a closed economy. This framework allows analyzing the distributional effects of an endogenous occasionally-binding constraint that introduces a pecuniary externality.

We introduce a loan-to-value collateral constraint which, together with the uninsurable dividend risk, generates an *asset-wealth trade-off* from holding additional assets. This trade-off generates unconstrained wealthy households which endogenously have a diversified portfolio. Moreover, we show that in our setting, the equity premium can be decomposed into a constraint effect, a risk effect, a trading cost effect that is expected to be close to zero, and a short-sales effect. The trading cost effect will only be non-zero because of the combination of the collateral constraint and the trading cost function. Hence, most risk compensation proceeds from the LtV constraint and individual risk.

A third strand studies the macroeconomy accounting for individual heterogeneity, a line of inquiry begun with the pioneering work of Krusell and Smith (1997), who developed quantitative tools to analyze economies in which the market clearing price is a function of the distribution of agents (and not only of the mean aggregate state) with individual incomplete markets and aggregate risk. Mendoza, Quadrini, and Rios-Rull (2009) examine how global imbalances can be precipitated by the integration of economies that have different financial markets development. They study the transition path after an unexpected integration of economies and analyze the global balance sheet and equilibrium interest rates. In a related paper, Kaplan and Violante (2014) study households with access to two types of assets that differ in their liquidity. Guerrieri and Lorenzoni (2017) study the transition path in a closed economy that experienced an unexpected tightening in the exogenous debt limit. Also, in a recent working paper, Huo and Ríos-Rull (2016) examine the effect of asset prices in a closed economy without aggregate risk and study the transition after an unexpected shock in the financial conditions. In contrast, we study the general equilibrium in a small open economy with aggregate risk and individual labor and dividend productivities. This setup, augmented with an individual loan-to-value collateral constraint, allows us to analyze the cross-sectional dimension of the debt-deflation mechanism and the pecuniary externality that it generates. Finally, in a series of recent empirical papers that study the relationship between income inequality and crises, Bordo and Meissner (2012) and Morelli and Atkinson (2015) study the predictive power of rising income inequality on financial crises without finding conclusive evidence. One exception is Kumhof, Rancière, and Winant (2015), who propose a model to study the effect

of changes in the top income distribution on household leverage and crises. Lastly, Guntin, Ottonello, and Perez (2020) use micro-data to asses individual consumption changes in episodes of large aggregate consumption adjustments. The authors argue that consistent with the permanent income hypothesis, households with high income and liquid assets adjust their consumption severely during such episodes. The present paper complements but differs fundamentally from these papers because it studies a model with ex-ante homogeneous agents with ex-post heterogeneity and uses this heterogeneous-agent framework to study Sudden Stops and the cross-sectional dynamics in the consumption and portfolio choice of households. Moreover, we document the importance of leverage and not only the liquidity of assets. In particular, we find that during a Sudden Stop, households with high leverage adjust the most their consumption.

3 The Cross-Sectional Effects in the Data

This section first describes the data used to show that the cross-sectional effects of the debt-deflation mechanism are empirically relevant. Then, sorting the households according to their net wealth and leverage ratio, we obtain the changes in their individual asset values and consumption during the 2009 Sudden Stop crisis. The results show that the households in the top decile of wealth and top decile of leverage ratio fire-sold the most their assets while the low-leveraged households increased their asset holdings.

3.1 Description of the data

We use data from The Mexican Family Life Survey (MxFLS) for the three available waves: 2002, 2005, and 2009. The MxFLS is a longitudinal household survey that collected information from a representative sample of approximately 8,400 households in 150 localities throughout Mexico. The survey covers information on expenditures, income, assets, and liabilities.[4] The MxFLS is representative at the national, urban-

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[4]To the best of our knowledge, this survey is the only publicly available data source that covers information about the households' stock of assets and liabilities.

rural, and regional level.[5] The sample selection criterion we used corresponds to the households that answered the survey in all three waves. The resulting sub-sample corresponds to 78% of the households in 2005.

Table 1 shows the mean net wealth, the portfolio decomposition, and the leverage ratio in 2005 ordered by deciles of the net wealth distribution. The leverage ratio is defined as the household's total debt over the sum of the household's assets. As the second and third rows show, Mexican households' wealth is mostly in physical assets (real estate and other durable goods). Although the proportion of debt decreases as households have higher net wealth, as we can see from the last two rows of the table, there are leveraged and non-leveraged households in each of the deciles. The next subsection will analyze the asset and consumption dynamics for households grouped by their level of leverage ratio and net wealth.

Table 1: Mean net wealth and its composition by deciles in 2005

	I	II	III	IV	V	VI	VII	VIII	IX	X
Net Wealth	-507	761	2,564	5,368	9,184	14,451	20,524	29,512	45,067	204,855
Assets										
Real Estate (%)	-103.6	24.2	46.9	69.6	76.9	80.9	82.5	82.8	82.1	75.1
Other (%)	-68.5	88.3	49.5	30.7	23.4	19.8	15.8	14.2	14.2	9.3
Financial (%)	-10.7	9.7	12	7.5	4.5	4.9	3.4	5.3	6.3	16.8
Debt (%)	282.8	-22.2	-8.3	-7.7	-4.9	-5.6	-1.7	-2.3	-2.6	-1.2
Leverage Ratio										
Mean	0.75	0.05	0	0.01	0.01	0.03	0	0	0	0
p90	1.69	0.38	0.17	0.16	0.12	0.09	0.04	0.05	0.06	0.04
p10	0	0	0	0	0	0	0	0	0	0

Notes: Ordered by deciles of net wealth in dollars of 2005. Source: MxFLS.

3.2 Stylized Facts: Differentiated Individual Effects

Mexico, as almost any other open economy, experienced a severe Sudden Stop crisis in 2009. Aggregate data shows a current account reversal of 1.5 percentage points relative to GDP, a 7% drop in per capita consumption, and house prices 4% below the pre-crisis trend in 2010 (for an overview of the aggregate time series see Appendix A). Moreover, the MxFLS survey shows that from 2005 to 2009, the sum of the

[5]For a detailed description of the survey see Rubalcava and Teruel (2006) and Rubalcava and Teruel (2013).

households' gross asset values dropped 1%. At the household level, however, the crisis had different effects depending on the composition of their balance sheets.

Supporting evidence of the cross-sectional effects:
The dampening cross-sectional effect comes from the unconstrained wealthy households that can buy the depressed assets fire-sold by the financially constrained households during a crisis. Table 2 shows the median change in the real estate owned by households sorted out according to their net wealth and leverage ratio in 2009.[6] Wealthy households correspond to the top decile of net wealth, and the financially constrained households correspond to the top decile of the leverage ratio.[7] As shown in the table, the real estate held by wealthy unconstrained households (top right cell) increased by 59.4% while the rest of households experienced drops in their asset holdings. Hence, this evidence supports the dampening effects coming from the cross-sectional dimension: wealthy unconstrained agents take advantage of the depressed prices and increase their asset positions.

Assuming that there were no creation or destruction of real estate, then it must be the case that since the assets held by the unconstrained wealthy agents increased, they were necessarily buying assets from someone else. Hence, other households were selling their assets. Since the amplifying effect comes from the households that are close to becoming financially constrained, and once the mechanism is triggered, they end up financially constrained and strengthen the downward pressure on asset prices. The magnitude of the numbers in the table suggests that the wealthy financially constrained – the households in the top deciles according to the net wealth and to the leverage ratio – fire-sold the most their assets putting downward pressure on their prices. Furthermore, wealthy financially vulnerable – the households in the top decile according to the net wealth and in the ninth decile according to the leverage ratio – also ended up fire-selling their assets as the financial conditions tightened. Hence, this evidence supports the amplifying effects coming from the cross-sectional dimension: financially vulnerable agents end up constrained and decrease their asset positions, increasing downward pressure on asset prices.

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[6]The survey data corresponds to the value of real estate. To obtain the quantity change, we deflated the value change with an aggregate house price index.

[7]In the calibration of the model, we will set the leverage limit equal to the leverage of the 90th percentile.

Table 2: Median % Real Estate Change 2005-09

| Leverage Ratio | Net Wealth | |
	I-IX (Non-Wealthy)	X (Wealthy)
I-VIII	-1.1	59.4
IX	-1.9	-15.0
X	-1.4	-36.5

Notes: Ordered by deciles in 2009. Source: MxFLS.

Additionally, in Table 3 we show the median change in the consumption of the households according to their leverage ratio in 2005. During the crisis, households that in 2005 were in the top decile of leverage decreased by 6.2% their consumption. These households were the most affected by the crisis since right before the crisis happened, they were the most exposed to changes in the financial conditions of the economy. In contrast to the declines in consumption of the high leveraged households, the ones in the first decile of leverage, that mostly had no debt and are net savers, increased their consumption by 5.4%. Households that were moderately leveraged – second to ninth deciles – increased their consumption by less than the non-leveraged households supporting a potential snowball effect: as the financial conditions tightened because financially constrained agents fire-sold their assets, financially vulnerable households ended up constrained. Moreover, these dynamics are different during normal years. In the first column of the table, we can see that households in the second to ninth deciles of leverage are the ones that slightly decreased their expenditures. While households in the top and bottom deciles of leverage increased their expenditures by 3.8% and 1.9%, respectively.

3.3 Stylized Facts: Heterogeneous Consumption Dynamics

In this subsection, we give evidence that the households have heterogeneous consumption dynamics and that the modeling choice of a heterogeneous-agent framework is supported by the data. Following Jappelli and Pistaferri (2017), we perform a test of the complete-market hypothesis for Mexico. Under complete markets, changes in individual consumption depend only on aggregate fluctuations common to all indi-

Table 3: Median % Consumption Change

| Leverage Ratio	Normal Times 2002-05	Crisis Times 2005-09
I	1.9	5.4
II-IX	-1.2	3.6
X	3.8	-6.2

Notes: Ordered by deciles in 2002 (first column) and 2005 (second column). Source: MxFLS.

viduals. To perform the test, we estimate the following regression

$$\Delta \log c_t^i = \beta \Delta \log C_t + \delta \Delta \log y_t^i + u_t^i, \tag{1}$$

where c_t^i is the household i consumption in C_t is the aggregate consumption in year t and y_t^i is the household i income in year t. We reject at 1% significance level the joint test of $\beta = 1$ and $\delta = 0$. The point estimates with standard errors in parenthesis are $\beta = 0.41$ (0.16) and $\delta = 0.04$ (0.006). Which are similar to the evidence from Thailand presented in Townsend (1995). Moreover, as we can see in Figure 1 changes in consumption vary across households both in normal and crisis years. However, during the crisis, there is a larger negative mass and a more concentrated distribution.

Figure 1: Household Distribution of Annualized Percent Change in Consumption
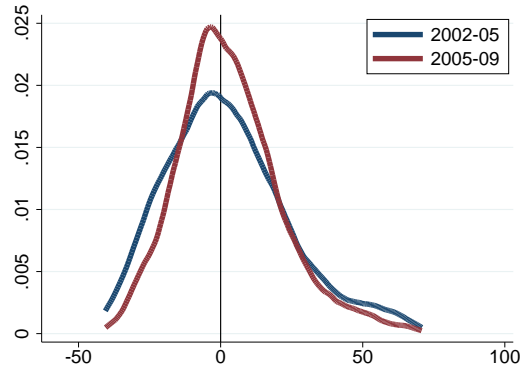
Notes: The distributions are truncated at the top and bottom 1%. Source: MxFLS.

Additionally, Table 4 and Figure 2 show how the households' leverage ratio distribution changed before and during the crisis. Between 2002 and 2005, before the

crisis, the share of net saver households increased by 1.7 percentage points and the share of financially constrained households decreased by 2.3 percentage points. Then, between 2009 and 2005, as the crisis unfolds and aggregate liquidity is reduced, we see a large decline of 5 percentage points in the share of net saver households. Such a decline could be the consequence of households having to use part of their savings to smooth consumption during the crisis. Also, in the same period, the share of financially constrained households increased by 1.7 percentage points as a result of the tightening financial conditions.

Table 4: Distribution of Households in %

	2002	2005	2009
Savers (leverage ratio ≤ 0)	12.5	14.2	9.2
Indebted not constrained (leverage ratio $\in (0, 0.168)$)	75.2	75.8	79.1
Financially constrained (leverage ratio ≥ 0.168])	12.3	10.0	11.7

Notes: The leverage ratio level considered to be the threshold between financially constrained an indebted unconstrained households is 0.168 and corresponds to the 90th percentile of its distribution in 2005. Source: MxFLS.

Figure 2: Negative Leverage Ratio Histogram
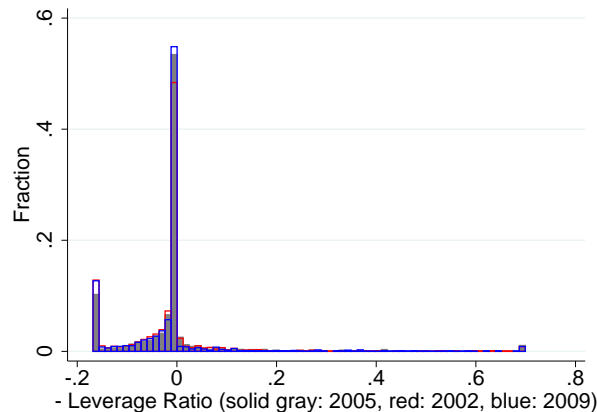
Notes: The leverage ratio corresponds to the total debts over the total assets of the household. Positive values of the negative leverage ratio correspond to households with net savings and negative values correspond to households with net debts. The distribution is truncated at -0.168 which is the 90th percentile of the leverage ratio distribution in 2005 and at the top 1%. Source: MxFLS.

Finally, we complement the evidence from the MxFLS with the Income and Ex-

penditure Household Survey (ENIGH). This survey is cross-sectional and is done every two years. In Figure 3 we show the Gini coefficient for consumption, and we can see that during the crisis, consumption inequality decreased more than the pre-crisis trend. This evidence is in line with the higher concentration documented in Figure 1.

Figure 3: Consumption Gini Coefficient
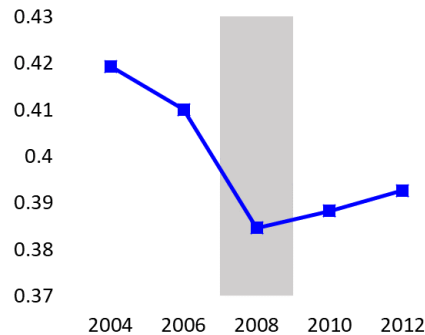
Notes: Source: ENIGH.

Having documented stylized facts about households' cross-section, we describe the proposed model that accounts for the households' balance sheet heterogeneity in the next section.

4 Model

4.1 Environment

The proposed framework is a Bewley model of a small open economy with international bonds, domestic equity, and an endogenous occasionally binding constraint. Time is discrete and infinite $t = 0, ..., \infty$. The economy is populated by a unit measure of households. There are two financial assets: a one-period risk-free international bond that the households can trade with the rest of the world and a risky domestic asset (land) that is only tradable between the households and is subject to a trading cost.[8] Borrowing is subject to a loan-to-value (LtV) collateral constraint by which the

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[8]The assumption of only domestic trading could be relaxed to allow foreign ownership up to a certain percentage of the shares in the economy. With an exogenous stochastic foreign demand for

households' international debt cannot exceed a fraction of the market value of their assets, i.e., the domestic asset is collateralizable.[9] Regarding the financial market's structure in the economy, markets are incomplete at the aggregate and individual levels. With respect to the aggregate risk, the economy is subject to an aggregate shock that determines the international interest rate. Concerning the individual risk, the households face non-insurable idiosyncratic labor income risk and dividend income risk. The latter risk means that households buy ex-ante identical shares of the risky domestic asset but get ex-post heterogeneity in the return. Evidence of a similar individual return on wealth is documented by Fagereng et al. (2020) and related individual capital income risk has been used by Angeletos (2007), Mendoza, Quadrini, and Rios-Rull (2009), Benhabib, Bisin, and Zhu (2011) and Hubmer, Krusell, and Smith Jr (2020). The combination of the dividend risk with an imperfect debt market (the LtV constraint) generates an *asset-wealth trade-off*: more asset holdings relax the collateral constraint and allow for better consumption smoothing (reducing consumption volatility) but also, more asset holdings increase the divided risk exposure which leads to higher income volatility of the household (increasing consumption volatility), incentivizing additional precautionary savings. This *asset-wealth trade-off* will be studied in Section 5.1.

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domestic shares, asset prices could become more volatile.

[9]The micro-foundations of the collateral constraint are similar to the ones presented by Bianchi and Mendoza (2018) extended for an economy with non-insurable idiosyncratic risk. Specifically, the LtV constraint is derived from an incentive compatibility constraint resulting from a limited enforcement problem. In an economy where debt contracts are signed with creditors in a competitive environment and households can always switch to another creditor at any point in time. At the beginning of the period credit and asset markets open, production happens and households choose b_{t+1}^i with price R_t^{-1} and a_{t+1}^i with price q_t. Then, markets close, and households decide to divert the resources from the credit and default. Local competitive financial intermediaries monitor costlessly who diverts resources and seize a fraction κ of the household asset holdings, which are $q_t a_{t+1}^i$. After defaulting, the household regains access to credit markets instantaneously and repurchases the assets that investors sell in open markets at a price q_t. In this environment, a household that borrows $-R_t^{-1} b_{t+1}^i$ and engages in diversion activities gains $-R_t^{-1} b_{t+1}^i$ and loses $\kappa q_t a_{t+1}^i$. Hence, households repay if and only $-R_t^{-1} b_{t+1}^i \leq \kappa q_t a_{t+1}^i$.

4.2   Households

There is a continuum unit measure of households. Each household $i \in [0,1]$ maximizes:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t^i) \right], \tag{2}$$

where c_t^i is consumption of household i, $\beta \in (0,1)$ is the common discount factor and the utility function, $u(\cdot)$, has a common constant relative risk aversion (CRRA) form. Households have access to the international bond market and the domestic asset market. However, since debt markets are imperfect, only secured-debt is available and household's domestic assets serve as collateral. At the beginning of the period, each household holds b_t^i risk-free international bonds, a_t^i shares of the risky domestic asset that has an endogenous price q_t and pays a dividend d_t^i. The household receives labor endowment income w_t^i and uses funds to buy consumption goods c_t^i, bonds to carry for the next period at an exogenous price equal to the inverse of the gross international rate R_t and asset holdings to carry for next period facing a quadratic trading cost of the form $\Phi(a_{t+1}^i, a_t^i) = \frac{\phi}{2}(a_{t+1}^i - a_t^i)^2$. This cost reflects that trading the domestic asset requires a higher level of financial knowledge relative to the bond market and that physical assets are relatively less liquid than bonds. The household's budget constraint is

$$c_t^i + R_t^{-1} b_{t+1}^i + q_t(a_{t+1}^i + \Phi(a_{t+1}^i, a_t^i)) = w_t^i + a_t^i(q_t + d_t^i) + b_t^i. \tag{3}$$

Households face a loan-to-value constraint that limits their ability to leverage foreign debt on domestic asset holdings. Next period debt (negative bonds) can not exceed a constant fraction κ of the market value of asset holdings. The collateral constraint is

$$R_t^{-1} b_{t+1}^i \geq -\kappa q_t a_{t+1}^i. \tag{4}$$

In addition, there is a short-sales constraint on the asset $a_{t+1}^i \geq 0$.[10] Note that the portfolio choice problem is well defined given the combination of the trading costs in

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[10]The short-sales constraint is needed to ensure that the state space of asset holdings is compact and that the LtV constraint is not irrelevant. If unlimited short selling of assets were possible, households could always undo the effect of Equation 4.

the asset market and the loan-to-value debt constraint.

Lastly, the income of the households is composed of an idiosyncratic and an aggregate part like in Benhabib, Bisin, and Zhu (2015). The individual wage takes the form $w_t^i = \epsilon_t^{i,w} w$ and the individual rate of return $d_t^i = \epsilon_t^{i,d} d$. Where $\{\epsilon_t^{i,w}, \epsilon_t^{i,d}\}$ correspond to the idiosyncratic risk components which will be specified in the next subsection, and $\{w, d\}$ correspond to the aggregate, exogenous, and constant components.[11]

4.3 Exogenous Stochastic Processes

The economy is exposed to only one aggregate shock. The process for the international interest rate is $R_t = \epsilon_t^R \bar{R}$ and $\log(\epsilon_t^R) = \rho_R \log(\epsilon_{t-1}^R) + \eta_t^R$ with $\eta_t^R \sim \mathcal{N}(0, \sigma_R^2)$. Regarding the individual shocks, the individual wage takes the form $w_t^i = \epsilon_t^{i,w} \bar{w}$ and $\log(\epsilon_t^{i,w}) = \rho_w \log(\epsilon_{t-1}^{i,w}) + \eta_t^{i,w}$ with $\eta_t^{i,w} \sim \mathcal{N}(0, \sigma_w^2)$, and the individual dividend takes the form $d_t^i = \epsilon_t^{i,d} \bar{d}$ and $\log(\epsilon_t^{i,d}) = \rho_d \log(\epsilon_{t-1}^{i,d}) + \eta_t^{i,d}$ with $\eta_t^{i,d} \sim \mathcal{N}(0, \sigma_d^2)$. Note that the idiosyncratic labor and dividend risk that the households face do not have aggregate implications on the returns:

$$\int_0^1 d_t^i \, \mathrm{d}i = \int_0^1 \epsilon_t^{i,d} d \, \mathrm{d}i = d \quad \text{and} \quad \int_0^1 w_t^i \, \mathrm{d}i = \int_0^1 \epsilon_t^{i,w} w \, \mathrm{d}i = w.$$

4.4 Closing the domestic asset market

The domestic asset is in constant positive fix net supply equal to $\bar{K}$ and in equilibrium it is equal to the total asset holdings (demand) of the households. Hence, market-clearing in the asset market requires: $\int_0^1 a_t^i \, \mathrm{d}i = \bar{K}$ for every t.

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[11]The structure of the income endowments is similar to an economy in which households supply 1 unit of labor inelastically, and production is done with a competitive constant returns to scale production function that only demands aggregate labor, and pays competitive wages w to each household. Additionally, households have an "Ak" production function that uses their individual assets to produce and households obtain dividends d from such production. In the end, households supply effective units of labor and assets so returns are multiplied by the idiosyncratic shocks.

4.5   Recursive Formulation

To characterize the problem of the agents and the equilibrium in recursive form we start by defining the states of the economy. Households are heterogeneous in their current holding of bonds, assets, idiosyncratic labor and dividend productivity. The individual states are: $(b, a, \epsilon^w, \epsilon^d)$. We need to keep track of both the individual bonds and assets given the asset trading costs and the imperfect debt market. Let $\Omega(b, a, \epsilon^w, \epsilon^d)$ be the endogenous distribution of households according to their bonds, assets and individual productivities. Regarding aggregate states, to forecast asset prices, the households need to know the distribution of wealth. Hence, the aggregate states correspond to the endogenous distribution Ω, and the exogenous shock to the international interest rate ϵ^R. Letting the superscript $\prime$ correspond to the variables in the next period, the recursive problem of a household becomes:

$$v(\underbrace{b, a, \epsilon^w, \epsilon^d}_{\text{Ind. states}}, \underbrace{\Omega, \epsilon^R}_{\text{Agg. states}}) = \max_{\{c, b', a' \geq 0\}} u(c) + \beta \mathbb{E}[v(b', a', \epsilon^{w'}, \epsilon^{d'}, \Omega', \epsilon^{R'})] \quad s.t.$$

$$c + R(\epsilon^R)^{-1} b' + q(\Omega, \epsilon^R)(a' + \Phi(a', a)) = \epsilon^w w + a(q(\Omega, \epsilon^R) + \epsilon^d d) + b, \quad \text{multiplier } \lambda(\cdot)$$

$$R(\epsilon^R)^{-1} b' \geq -\kappa q(\Omega, \epsilon^R) a', \quad \text{multiplier } \mu(\cdot)$$

$$\Phi(a', a) = \frac{\phi}{2}(a' - a)^2$$

$$\Omega' = H^\Omega(\Omega, \epsilon^R), \tag{5}$$

where $H^\Omega(\cdot)$ corresponds to the aggregate law of motion of the distribution of households.

4.5.1   Definition of a Recursive Competitive Equilibrium

Let the individual bond and asset holdings be elements $(b, a) \in [\underline{b}, \bar{b}] \times [0, \bar{a}] \equiv \mathcal{S}$ and the individual productivities be elements $(\epsilon^w, \epsilon^d) \in \{\epsilon_1^w, ..., \epsilon_{N_w}^w\} \times \{\epsilon_1^d, ..., \epsilon_{N_d}^d\} \equiv \mathcal{E}^I$. Let $\mathcal{M}$ be the set of probability measures of the set $\mathcal{S} \times \mathcal{E}^I$ and the aggregate shocks be elements $\epsilon^R \in \{\epsilon_1^R, ..., \epsilon_{N_R}^R\} \equiv \mathcal{E}^A$. Finally, let the function $\pi(\epsilon'|\epsilon)$ be the exogenous Markov transition probability of next period shocks take the realization ϵ' conditional on the shocks in the current period being ϵ, where $\epsilon = (\epsilon^w, \epsilon^d, \epsilon^R) \in \mathcal{E}^I \times \mathcal{E}^A$. Now we can define a recursive competitive equilibrium.

Definition 1. *A recursive competitive equilibrium in this economy is given by a value function $v : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \to \mathbb{R}$, policy functions for the household $c : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \to \mathbb{R}$, $b' : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \to \mathbb{R}$ and $a' : \mathcal{S} \times \mathcal{E}^I \times \mathcal{M} \times \mathcal{E}^A \to \mathbb{R}$, domestic asset pricing function $q : \mathcal{M} \times \mathcal{E}^A \to \mathbb{R}$, and an aggregate law of motion $H^\Omega : \mathcal{M} \times \mathcal{E}^A \to \mathcal{M}$ such that:*

1. *Given the asset pricing function and the aggregate law of motion, the value function v satisfies the household's Bellman equation [5](#) and c, a', b' are the associated policy functions.*

2. *For all $\Omega \in \mathcal{M}$ and all $\epsilon^R \in \mathcal{E}^A$, the asset market clears:*
$$\int\limits_{\mathcal{S} \times \mathcal{E}^I} a \, \mathrm{d}\Omega = \int\limits_{\mathcal{S} \times \mathcal{E}^I} a'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, \mathrm{d}\Omega = \bar{K}.$$

3. *For all $\Omega \in \mathcal{M}$ and $\epsilon^R \in \mathcal{E}^A$, the aggregate resource constraint is satisfied:*
$$\int\limits_{\mathcal{S} \times \mathcal{E}^I} c(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, \mathrm{d}\Omega + R(\epsilon^R)^{-1} \int\limits_{\mathcal{S} \times \mathcal{E}^I} b'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R) \, \mathrm{d}\Omega$$
$$+ q(\Omega, \epsilon^R) \int\limits_{\mathcal{S} \times \mathcal{E}^I} \Phi(a'(b, a, \epsilon^w, \epsilon^d, \Omega, \epsilon^R), a) \, \mathrm{d}\Omega = w + \int\limits_{\mathcal{S} \times \mathcal{E}^I} a\epsilon^d d \, \mathrm{d}\Omega + \int\limits_{\mathcal{S} \times \mathcal{E}^I} b \, \mathrm{d}\Omega.$$

4. *The aggregate law of motion is generated by the exogenous Markov process π and the policy functions b' and a' as described below:*

   Let $(\epsilon^w, \epsilon^d) = \epsilon^I$ and $\epsilon^R = \epsilon^A$ and define the transition function $Q_{\Omega, \epsilon^A} : \mathcal{S} \times \mathcal{E}^I \times \mathcal{B}(\mathcal{S}) \times \mathcal{B}(\mathcal{E}^I) \to [0, 1]$, where $\mathcal{B}(\cdot)$ is the corresponding Borel set, by

$$Q_{\Omega, \epsilon^A}(b, a, \epsilon^I, \mathscr{S}, \mathscr{E}^I) =$$
$$\begin{cases} \sum\limits_{\epsilon^{I\prime} \in \mathscr{E}^I, \epsilon^{A\prime} \in \mathcal{E}^A} \pi(\epsilon^{I\prime}, \epsilon^{A\prime} | \epsilon^I, \epsilon^A), & \text{if } (b'(b, a, \epsilon^I, \Omega, \epsilon^A), a'(b, a, \epsilon^I, \Omega, \epsilon^A)) \in \mathscr{S}. \\ 0, & \text{otherwise.} \end{cases}$$

   Then, for any $\mathscr{S} \in \mathcal{B}(\mathcal{S})$ and any $\mathscr{E}^I \in \mathcal{B}(\mathcal{E}^I)$ the aggregate law of motion is given by

$$\Omega'(\mathscr{S}, \mathscr{E}^I) = (H^\Omega(\Omega, \epsilon^A))(\mathscr{S}, \mathscr{E}^I) = \int\limits_{\mathcal{S} \times \mathcal{E}^I} Q_{\Omega, \epsilon^A}(b, a, \epsilon^I, \mathscr{S}, \mathscr{E}^I) \, \mathrm{d}\Omega.$$

5 The Cross-Sectional Effects in the Model

In this section, we study the cross-sectional effects on the credit and equity channel of the economy.

5.1 Market Incompleteness and Risk Exposure

The households are exposed to two sources of non-insurable idiosyncratic risk that have different equilibrium implications. Note that the standard Bewley non-insurable persistent labor income risk ϵ^w, together with the constant aggregate labor income endowment assumption implies a fixed labor risk exposure. This means that the exposure to the labor earnings risk is independent of the households' decisions. In contrast, the idiosyncratic persistent dividend productivity, ϵ^d, allows the households to change future risk exposure by changing the next period holdings of the asset.

This varying dividend risk exposure, combined with the loan-to-value collateral constraint, generates an *asset-wealth trade-off*. To see this, first, note that when households are in an adverse state, they can smooth consumption in two ways: by lowering their bond holdings b' (if these are already negative, this means borrow more) or by reducing their asset holdings a'. Given the financial frictions in the debt market (see Equation 4), to have credit capacity and hence borrow, the household needs first to buy domestic assets. Note that although the current dividend return is given since the current asset holdings are fixed in the current period (they are an individual state variable), the household chooses how much future exposure to have by choosing the next period asset holdings a'. Because the flow income of the household is given by $FI(a, \epsilon^w, \epsilon^d) = \epsilon^w w + a\epsilon^d d$, with independent idiosyncratic risks its variance is $\mathbb{V}[FI(a, \epsilon^w, \epsilon^d)] = w^2\sigma^2_{\epsilon^w} + a^2 d^2\sigma^2_{\epsilon^d}$ which is a convex function with respect to the asset holdings. This translates into more income volatility for asset-rich households. This property of the flow income generates the following trade-off from getting more assets:

1. Households get higher debt capacity that allows higher smoothing and reduces consumption volatility since $R(\cdot)^{-1}b'(\cdot) \geq -\kappa q(\cdot)a'(\cdot)$, incentivizing lower precautionary savings.

2. Households get higher future income risk that increases consumption volatility, incentivizing higher precautionary savings.

In equilibrium, indebted asset-poor households increase their debts as they increase their assets, and for households with high dividend returns, when they become asset-rich, they start deleveraging (precautionary saving motives kick in) and some end up being savers due to the increasing income risk.[12] This behavior generates unconstrained wealthy households which endogenously have a diversified portfolio: asset-rich households end up holding both positive international bonds and domestic assets.

Similar trade-offs have been studied in the literature but through different mechanisms. Mendoza, Quadrini, and Rios-Rull (2009) find that an individual investment shock (similar to an individual dividend shock) makes agents lower their debt positions as they increase their net wealth. The outcome for asset-rich households is the same but for different reasons. Because we introduce the shock with persistence (theirs is an *iid* shock) the households with a negative dividend shock want to lower their bond position (or increase debts if negative) as the asset position increases. Moreover, in our paper, introducing the LtV constraint and the individual non-trivial portfolio choice problem makes asset-poor households increase their debts as they increase their assets. In another study, Benhabib, Bisin, and Zhu (2011) show that idiosyncratic capital returns determine the properties of the right tail of the wealth distribution in a Bewley economy. Their theoretical result is in line with the *asset-wealth trade-off* described above since asset-rich households that get a positive dividend shock will increase their net wealth by two sources: by buying more assets and by increasing their bond position (or decreasing their debt if the bond position is negative). Hence the share of wealthy households and the wealth inequality increase. However, again, the combination of the dividend risk with the LtV constraint allows the model to generate an empirical plausible distribution of constrained households, financially vulnerable households that hold debt, and households with positive bond positions (savers).

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[12]See the Online Appendix for a graphical analysis of the policy functions done for the calibrated stationary model of Section 6.2.

5.2 Financial Premia

In this subsection, we study the effects that the households' balance sheet heterogeneity introduces. Specifically, we analyze the cross-sectional dimension of the debt-deflation mechanism in terms of the external financing premium and equity premium at the individual and aggregate levels. For simplicity, we omit the state variables and re-introduce the superscript i to identify household-specific variables. Let λ^i, μ^i and ψ^i be the multipliers on the budget constraint, the collateral constraint, and the short-sales constraint, respectively, and let $\tilde{\mu}^i = \frac{\mu^i}{\lambda^i}$ and $\tilde{\psi}^i = \frac{\psi^i}{\lambda^i}$.

Similar to the analysis done by Mendoza and Smith (2006) but for an economy with heterogeneous-agents, from the first-order conditions of household i's problem we obtain an Euler Equation for individual bonds:

$$\lambda^i R^{-1} - \mu^i R^{-1} = \beta \mathbb{E}[\lambda^{i\prime}] \quad \Rightarrow$$

$$0 < 1 - \tilde{\mu}^i = \beta R \mathbb{E}\left[\frac{\lambda^{i\prime}}{\lambda^i}\right] \leq 1 \quad \text{since } \lambda^i > 0, \mu^i \geq 0 \text{ and } \tilde{\mu}^i = \frac{\mu^i}{\lambda^i} \in [0,1).$$

Let the individual expected effective interest rate be the inverse of the individual stochastic discount factor $\mathbb{E}[R^{i,eff}] = \mathbb{E}[SDF^i]^{-1} = \mathbb{E}\left[\beta \frac{\lambda^{i\prime}}{\lambda^i}\right]^{-1}$. Then, from the above Euler Equation we get an individual expected external financing premium on debt:

$$\mathbb{E}[R^{i,eff}] - R = R \frac{\tilde{\mu}^i}{1 - \tilde{\mu}^i} \geq 0. \tag{6}$$

This individual premium reflects the fact that when the constraint binds ($\tilde{\mu}^i > 0$), the household would want to borrow more than what the collateral constraint allows. Also, note that it is increasing on $\tilde{\mu}^i$. This means that as the constraint tightens, the household would be willing to pay an interest rate higher than R for more debt.

Similarly, from the first-order conditions of household i's problem we obtain the Euler Equation for individual assets:

$$q(\lambda^i(1 + \Phi_1^i) - \kappa \mu^i) - \psi^i = \beta \mathbb{E}[\lambda^{i\prime}(q' + d^{i\prime} - q' \Phi_2^{i\prime})],$$

where Φ_j^i corresponds to the partial derivative with respect to argument j. Let

$\tilde{d}^{i,\prime} = d^{i\prime} - q'\Phi_2^{i\prime}$ and the individual return on the asset be $\tilde{R}^{i,q} = \left(\frac{q' + \tilde{d}^{i,\prime}}{q}\right)$. Then, from the above Euler Equation we get an individual expected equity premium:

$$\mathbb{E}[\tilde{R}^{i,q}] - R = \frac{R\left((1-\kappa)\tilde{\mu}^i - \mathbb{COV}[SDF^i, \tilde{R}^{i,q}] + \Phi_1^i - \tilde{\psi}^i\right)}{1 - \tilde{\mu}^i}. \tag{7}$$

As in Mendoza and Smith (2006) but at an individual level, in Equation 7 we see a direct positive effect in the individual equity premium coming from the collateral constraint: as $\tilde{\mu}^i$ increases, the individual equity premium increases by an additive term that multiplies $R(1-\kappa)$ and by a multiplicative factor $(1/(1-\tilde{\mu}^i))$ that affects the whole premia. Also, there is a positive risk effect coming from the covariance term that will become more negative due to the precautionary savings.[13] Lastly, there is an ambiguous effect coming from the marginal trading costs. This last effect is expected to be negative for financially constrained households since when $\tilde{\mu}^i > 0$, the household will sell assets to smooth consumption and $a^{i\prime} < a^i \Rightarrow \Phi_1^i < 0$. When the constraint binds, a larger equity premium reflects that buying an extra unit of the asset provides an additional benefit since this additional unit also relaxes the constraint. However, this additional benefit is imperfect since κ fraction of the assets is pledgeable as collateral.

The aggregate expected equity rate of return, $\mathbb{E}[R^q]$, can be obtained by first integrating the individual expected asset returns over all the households:

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[13]This risk effect also includes the next period's marginal trading cost effect that is expected to increase the precautionary motives. The intuition for this is the following. Note that the household that next period gets a high divided return will buy more shares, hence $a^{i\prime\prime} > a^{i\prime} \Rightarrow \Phi_2^{i\prime} < 0 \Rightarrow \tilde{d}^{i,\prime} > d^{i\prime}$, effectively the individual dividend risk increases due to the trading costs.

$$\int_0^1 \mathbb{E}[\tilde{R}^{i,q}]\,\mathrm{d}i = \int_0^1 \left(\mathbb{E}\left[\frac{q' + \tilde{d}^{i\prime}}{q}\right]\right)\mathrm{d}i = \mathbb{E}\left[\int_0^1 \left(\frac{q' + \tilde{d}^{i\prime}}{q}\right)\mathrm{d}i\right] = \mathbb{E}\left[\frac{q'}{q} + \frac{1}{q}\int_0^1 \tilde{d}^{i\prime}\,\mathrm{d}i\right] =$$

$$\mathbb{E}\left[\frac{q'}{q} + \frac{1}{q}\int_0^1 d^{i\prime}\,\mathrm{d}i - q'\Phi_2^{i\prime}\,\mathrm{d}i\right] = \mathbb{E}\left[\frac{q'}{q} + \frac{1}{q}\underbrace{\int_0^1 d^{i\prime}\,\mathrm{d}i}_{=d'=d} + \frac{1}{q}\int_0^1 q'\phi(a^{i\prime\prime\prime} - a^{i\prime})\,\mathrm{d}i\right] =$$

$$\mathbb{E}\left[\frac{q'}{q} + \frac{1}{q}d' + \frac{q'\phi}{q}\left(\underbrace{\int_0^1 a^{i\prime\prime\prime}\,\mathrm{d}i}_{=\bar{K}} - \underbrace{\int_0^1 a^{i\prime}\,\mathrm{d}i}_{=\bar{K}}\right)\right] = \mathbb{E}\left[\frac{q' + d'}{q}\right] \equiv \mathbb{E}\left[R^q\right].$$

Then, we use the expected returns derived in Equation 7 to obtain a decomposition of the aggregate expected equity premium. Assuming that fraction $\bar{I} \in [0, 1]$ of households are credit constrained and without loss of generality sorting constrained households from 0 to $\bar{I}$ we obtain the following result:

$$\mathbb{E}[R^q] - R = \underbrace{R(1-\kappa)\int_0^{\bar{I}} \frac{\tilde{\mu}^i}{1 - \tilde{\mu}^i}\,\mathrm{d}i}_{\text{Constraint Effect: } +\bar{I} \text{ and } +\tilde{\mu}} - \underbrace{R\int_0^1 \frac{\mathbb{COV}[SDF^i, \tilde{R}^{i,q}]}{1 - \tilde{\mu}^i}\,\mathrm{d}i}_{\text{Risk Effect: ``+''}}$$

$$+ \underbrace{R\int_0^1 \frac{\Phi_1^i}{1 - \tilde{\mu}^i}\,\mathrm{d}i}_{\text{Trading Cost Effect: ``}\approx 0\text{''}} - \underbrace{\frac{R}{q}\int_0^1 \frac{\tilde{\psi}^i}{1 - \tilde{\mu}^i}\,\mathrm{d}i}_{\text{Short-Sales Effect: ``}-\text{''}}.$$

This expression shows that the aggregate excess returns can be decomposed into four effects. First, a positive direct effect coming from the measure of constrained households and from how "strong" the constraint binds. Second, the risk effect coming from the covariance between the individual stochastic discount factor and the individual return on the equity (note that the integral becomes a weighted average of the covariances with larger weights on constrained households since $\tilde{\mu}^i > 0 \Rightarrow$

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$1/(1 - \tilde{\mu}^i) > 1)$. Since constrained households are expected to have more negative covariances due to the increased individual consumption volatility and the precautionary savings behavior, we expect a positive risk effect. Third, the trading cost effect, again, the weighted average puts more weight on constrained households, and since $\int_0^1 \Phi_1^i \, di = 0$ we can expect the aggregate effect to be close to zero and decreasing with respect to ϕ. This trading cost effect comes from the interaction of the collateral constraint and the trading cost function since if there are no constrained households, this term becomes zero. Fourth, a short-sales effect that decreases the equity premium since households with a binding short-sales constraint increase the marginal gain of additional asset holdings and has no effect on the marginal benefit of saving in assets.

Finally, the debt-deflation cross-sectional effects in the risk premium are:

1. Dampening effect: having more unconstrained wealthy households reduces the equity premium by having a smaller risk effect since they are better able to smooth consumption.

2. Amplifying effect: having more financially vulnerable households increases the equity premium due to a larger constraint effect (larger $\bar{I}$) and by having a larger risk effect since these constrained households have more consumption volatility.

Note that the precautionary behavior introduced by the *asset-wealth trade-off*, under empirically suitable high persistence of the dividend risk, generates unconstrained households. Hence, in the stationary equilibrium the measure of financially constrained households is $\bar{I} < 1$. Intuitively, when households get a high individual dividend return, they accumulate more assets. Since the individual risk is sufficiently persistent, this gives households enough time to become asset-rich and the dividend risk exposure is high enough such that the precautionary savings motive makes households deleverage and become unconstrained. In the next section, we use the model as a measurement device to quantitatively study the cross-sectional effects of a Sudden Stop episode.

6 Quantitative Analysis

This section presents the quantitative results of the model. Due to the computational intensity of the solution method, we calibrate the parameters using the stationary model without aggregate risk.[14] To calibrate the model, we use data for Mexico. Table 6.1 shows the calibrated parameters.

6.1 Calibration

Regarding the set of parameters that are calibrated outside of the model, we set the household's risk aversion $\nu = 2$ which is a value common in the literature. The collateral debt fraction κ equal to 0.168 which is the 90th percentile of the leverage ratio dsitribution in 2005. Lastly, the net asset supply is normalized at 1. Then, we calibrate by simulation the discount factor $\beta = 0.90$ to match the average net foreign asset position relative to GDP for Mexico equal to -35% and the trading cost parameter ϕ equal to 2.7 to obtain an average transaction cost of 5% which is consistent with the estimates from Aiyagari and Gertler (1999).

To estimate the exogenous earning process we apply the methodology described in Krueger, Mitman, and Perri (2016) using Mexican data.[15] First, we estimate a Mincer log-earnings equation with time fixed effects

$$\log(Y_{a,t}^i) = \beta' X_{a,t}^i + D_t + y_{a,t}^i \ , \tag{8}$$

where each observation corresponds to an individual i, with quarterly age a and in quarter t. $Y_{a,t}^i$ corresponds to the annual income of the person, the vector of controls $X_{a,t}^i$ includes a cubic polynomial on age, dummy variables for the education level and a dummy variable that identifies if the worker is in the informal sector. Finally, D_t

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[14]Since the economy has an endogenous occasionally-binding constraint, the household's policy functions are expected to be highly non-linear, and a global solution method is needed. We use the *FiPIt* algorithm proposed by Mendoza and Villalvazo (2020) to solve the household's problem combined with the stochastic-simulation approach by Maliar, Maliar, and Valli (2010) and Krusell and Smith (1997) to solve the aggregate uncertainty problem.

[15]There is a vast literature on the estimation of the labor income risk (see Meghir and Pistaferri (2004), Storesletten, Telmer, and Yaron (2004), Guvenen (2007), Heathcote, Storesletten, and Violante (2010)).

Table 5: Parameters		
Parameter	Value	Source or Target
Calibrated outside of the model		
ν Risk aversion	2	Common in the literature
κ Debt fraction of collateral	0.168	90th percentile lev. ratio in 2005
$\bar{K}$ Net asset supply	1	Normalization
Calibrated by simulation		
β Discount factor	0.90	Average NFA/GDP ratio of -35%
ϕ Trading cost	2.7	Average transaction cost of 5%
Individual labor income risk		
w Average wage	0.072	See Section 6.1
ρ_w Autocorrelation	0.906	
σ_w Std. dev. (%)	19.8	
Individual dividend income risk		
d Average dividend yield	0.0445	See Section 6.1
ρ_d Autocorrelation	0.905	
σ_d Std. dev. (%)	61.8	
Aggregate interest rate risk		
R Average interest rate	1.054	See Section 6.1
ρ_R Autocorrelation	0.905	
σ_R Std. dev. (%)	1.9	

corresponds to the time fixed effects dummy variables. After running the regression, we obtain the residuals $y_{a,t}^i$ and assume the income risk follows a stationary process with a persistent and transitory component. The stationarity assumption allows us to drop the time dimension and the income risk model becomes

$$y_a^i = z_a^i + \epsilon_a^i$$
$$z_a^i = \rho_w z_{a-1}^i + \eta_a^{i,w}$$
$$\eta_a^{i,w} \sim (0, \sigma_w^2), \quad z_0^i \sim (0, \sigma_{z_0}^2), \quad \epsilon_a^i \sim (0, \sigma_\epsilon^2). \tag{9}$$

Now the objective is to estimate the vector of parameters $\theta = (\rho_w, \sigma_w^2, \sigma_{z_0}^2, \sigma_\epsilon^2)$.

These parameters are identified with the following theoretical moments:

$$\begin{aligned}
\rho_w =& \frac{\mathbb{COV}[y_a^i, y_{a-2}^i]}{\mathbb{COV}[y_{a-1}^i, y_{a-2}^i]}, \\
\sigma_\epsilon^2 =& \mathbb{V}[y_{a-1}^i] - \rho^{-1}\mathbb{COV}[y_a^i, y_{a-1}^i], \\
\sigma_w^2 =& \mathbb{V}[y_{a-1}^i] - \mathbb{COV}[y_a^i, y_{a-2}^i] - \sigma_\epsilon^2, \\
\sigma_{z_0}^2 =& \mathbb{V}[y_0^i] - \sigma_\epsilon^2 .
\end{aligned} \tag{10}$$

We use data from the National Survey of Employment and Occupation (ENOE) to do an over-identified GMM estimation with an identity weighting matrix.[16] The ENOE survey is a quarterly household rotating panel with a representative sample of 120,000 households that started in 2005-I. Every household is interviewed for 5 consequently quarters and each quarter 20% of the sample is replaced. As the standard practice in the literature, our sample selection criteria are individuals with ages between 20 and 60, males, and with positive earnings. Table 6 shows the estimated parameters and compares them with the literature's estimation done for the US.

Table 6: Annual Income Process Estimates

	Mexico Benchmark	Mexico Formal Employment	US (a)	US (b)	US (c)
ρ_w	0.906	0.922	0.999	0.988	0.970
σ_w^2	0.039	0.038	0.017	0.015	0.038

Notes: The results for Mexico correspond to data from the ENOE survey from 2005-I to 2014-IV. The estimates are annualized following Krueger, Mitman, and Perri (2016). Column (a) corresponds to Storesletten, Telmer, and Yaron (2004), (b) to Guvenen (2009) , and (c) to Krueger, Mitman, and Perri (2016).

We find that the estimated persistence of the income risk process is smaller, and the variance is larger for Mexico compared to the US. A reason for this difference could come from the informal market structure that is common in emerging economies (Leyva and Urrutia (2020)). The Mexican labor market is characterized by having

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[16]Note that to just-identify the parameters we only need data for ages $(a, a-1, a-2)$. Since we are using data for 160 quarterly-ages the system is over-identified.

a high informality rate in which more than 50% of informal employment. Since the informal sector is relatively more flexible than the formal sector, it could create a less permanent effect of idiosyncratic shocks. Moreover, Gomes, Iachan, and Santos (2020) find that informality is associated with more volatile earnings. Finally, the combination of a large informal sector and the lack of unemployment insurance could also cause a higher income risk.[17] To explore this reason, in the second column, we show the results from the estimation done with a subsample of only formal employment. As expected, the difference narrows, although the change is small. Given that we do not explore specific heterogeneity in the labor markets in the model, we still use as a benchmark the results from the first column that include all the employment. Lastly, the discrete labor income risk process is approximated using a symmetric 2-state Markov chain using a simple persistence rule following Mendoza (2010). The discretized risk takes the values $\epsilon^w \in \{\epsilon_L^w = 0.80, \epsilon_H^w = 1.20\}$ and the probability that the next period realization of the shock is the same as the current period is $Pr[\epsilon^{w'} = \epsilon_j^w | \epsilon^w = \epsilon_j^w] = 0.953$ for $j \in \{L, H\}$.

The dividend income risk plays a key role in the decision rules of the households and drives the *asset-wealth trade-off* discussed in Section 5.1. However, a proper estimation of this process is infeasible due to the lack of available data in most economies.[18] Due to the restrictions of the available data for Mexico, we take the following estimation strategy. We jointly estimate the three parameters that characterize the dividend income risk (d, ρ_d, σ_d) to match the leverage ratio distribution of households in 2005. Specifically, we focus on three distribution statistics: the measure of savers that have financial assets (negative leverage ratio), indebted households that have positive debts but are not close to their debt limit, and financially constrained households which have a leverage ratio above 0.168 (the 90th percentile). The calibrated parameters are $(d = 0.0445, \rho_d = 0.905, \sigma_d = 0.618)$ and similarly to the labor risk, the discrete dividend risk process is approximated using a symmetric 2-state

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[17]Bosch and Esteban-Pretel (2015) study the consequences on the labor market of implementing an unemployment benefit system in economies with large informal sectors and find that an unemployment benefit could increase the formality rate.

[18]One exemption is the work by Fagereng et al. (2020) which estimate the wealth risk using administrative data from Norway and find that there is high heterogeneity in the wealth returns and that these differences are highly persistent.

Markov chain using a simple persistence rule. Hence, the discretized risk takes the values $\epsilon^d \in \{\epsilon^d_L = 0.38, \epsilon^d_H = 1.62\}$ and the probability that the next period realization of the shock is the same as the current period is $Pr[\epsilon^{d'} = \epsilon^d_j | \epsilon^d = \epsilon^d_j] = 0.9525$ for $j \in \{L, H\}$. These estimates imply that the effective dividend yield ($\epsilon^d d$) the households will face can take the following two values: $\{1.7\%, 7.2\%\}$. The matched distribution is shown in Table 7. Lastly, the aggregate wage level, w, is set equal to $4d\bar{K}$ such that the average household has a total flow income that correspond to four-fifths labor income and one-fifth dividend income.

Table 7: Leverage Ratio Distribution of Households (%)

	Data	Stationary Model
Savers (lev. ratio ≤ 0)	14.2	14.3
Indebted not constrained (lev. ratio $\in (0, 0.168)$)	75.8	75.7
Financially constrained (lev. ratio $\geq 0.168]$)	10.0	10.0

Notes: The leverage ratio level considered to be the threshold between financially constrained an indebted unconstrained households is 0.168 and corresponds to the 90th percentile of its distribution in 2005. Source: MxFLS.

The calibrated parameters are ($d = 0.0445, \rho_d = 0.905, \sigma_d = 0.618$) and similarly to the labor risk, the discrete dividend risk process is approximated using a symmetric 2-state Markov chain using a simple persistence rule. Hence, the discretized risk takes the values $\epsilon^d \in \{\epsilon^d_L = 0.38, \epsilon^d_H = 1.62\}$ and the probability that the next period realization of the shock is the same as the current period is $Pr[\epsilon^{d'} = \epsilon^d_j | \epsilon^d = \epsilon^d_j] = 0.9525$ for $j \in \{L, H\}$. These estimates imply that the effective dividend yield ($\epsilon^d d$) the households will face can take the following two values: $\{1.7\%, 7.2\%\}$.

The last exogenous process that needs to be estimated corresponds to the international interest rate. This process was estimated using data from Meza (2018) and the parameters are ($R = 1.054, \rho_R = 0.905, \sigma_R = 0.019$). Similarly, the interest rate process is approximated using a symmetric 2-state Markov chain using a simple persistence rule. Hence, the discretized interest rate takes the values $R \in \{R_H = 1.073, \ R_L = 1.035\}$ and the probability that the next period realization of the interest rate is the same as the current period is $Pr[R' = R_j | R = R_j] = 0.9525$ for $j \in \{L, H\}$. These values are common in the literature of small open economies

and are close to the estimates obtained in studies of the Mexican economy (see Mendoza (2010), Bianchi (2016), among others.).

6.2   Stationary Model

In this subsection, we analyze the stationary equilibrium for an economy in which the interest rate is constant at its steady state value of 5.4% – i.e., a Bewley economy without aggregate risk. The stationary model does a good job capturing the wealth inequality, as seen in Table 8. This is the result of the *asset-wealth trade-off* described in Section 5.1.

Table 8: Non-targeted Inequality Measure

	Model	Data
Wealth Gini	0.592	0.733

Notes: Source: MxFLS.

Moreover, in Table 9 we show the average net wealth, assets, and debts by deciles relative to the median level of each variable for simulated data and observed data in 2005. As we can see in the top and medium rows, the net wealth and assets distributions generated by the model are very close to the ones obtained from the MxFLS in 2005 with the exception of the top deciles. Regarding the total debt, the only decile that is significantly different is the bottom decile. One possible reason for this difference is that we do not allow the households to default in the model and cannot hold more debt than the collateral limit. In contrast to the real data where households in the bottom decile have negative net wealth. However, for the rest of the deciles, the model does a good job of capturing the inequality in terms of the net wealth, total assets, and debt.

With respect to the aggregate equity premium, in Table 10 we show its level and decomposition. The model generates a high equity premium which is close to the data (first column). As expected, the risk component contributes the most to the equity premium, about 89%, while the other 11% corresponds to the constraint effect. Note that the calibration was done to capture the measure of constrained households in 2005 equal to 10% (see Table 7). Hence, even if only these households have an active

Table 9: Variables relative to the median, ordered by net wealth

	I	II	III	IV	V	VI	VII	VIII	IX	X
Net wealth relative to median										
Data	-0.1	0.1	0.3	0.6	1	1.6	2.2	3.2	4.9	22.3
Model	0.1	0.2	0.4	0.7	1	1.5	2.1	2.9	4.5	9.5
Assets relative to median										
Data	0.1	0.1	0.3	0.6	1	1.6	2.2	3.1	4.8	21.5
Model	0.1	0.2	0.3	0.6	1	1.5	2.2	3.4	5.9	18.3
Debt relative to median										
Data	2.9	0.4	0.5	0.9	1	1.8	0.8	1.5	2.6	5.2
Model	0	0.1	0.2	0.5	1	1.6	2.3	3.3	5	9.2

Notes: Deciles ordered by the net wealth. Source: MxFLS.

debt constraint, there is an important contribution to the aggregate equity premium.

Table 10: Decomposition of the Equity Premium (%)

	Model	Data
Equity Premium	5.2	6.5
Constraint Effect	0.6	-
Risk Effect	4.8	-
Trading Cost Effect	0.02	-
Short-Sales Effect	-0.02	-

Notes: Data from Damodaran (2013) corresponds to Mexico in 2005.

Finally, notice that the debt-deflation mechanism affects a household's consumption when two things happen. First, the household must be highly leveraged, so when the collateral constraint tightens, they are close to (or at) the binding region and they need to adjust their asset holdings; and second, the household must have a large debt-to-expenditure ratio so when they have to deleverage, there is a significant impact on their consumption. As a model validation exercise, Figure 4 shows how well the model replicates the distribution of households with respect to the joint leverage ratio and debt-to-expenditure ratio. In overall terms, the model does a good

job replicating the joint distribution, with a slight underestimation of the measure of households in the top quintile of leverage ratio and debt-to-expenditure ratio.

Figure 4: Joint leverage ratio and debt-to-expenditure ratio distribution

(a) LR q=I

(b) LR q=II

(c) LR q=III

(d) LR q=IV

(e) LR q=V
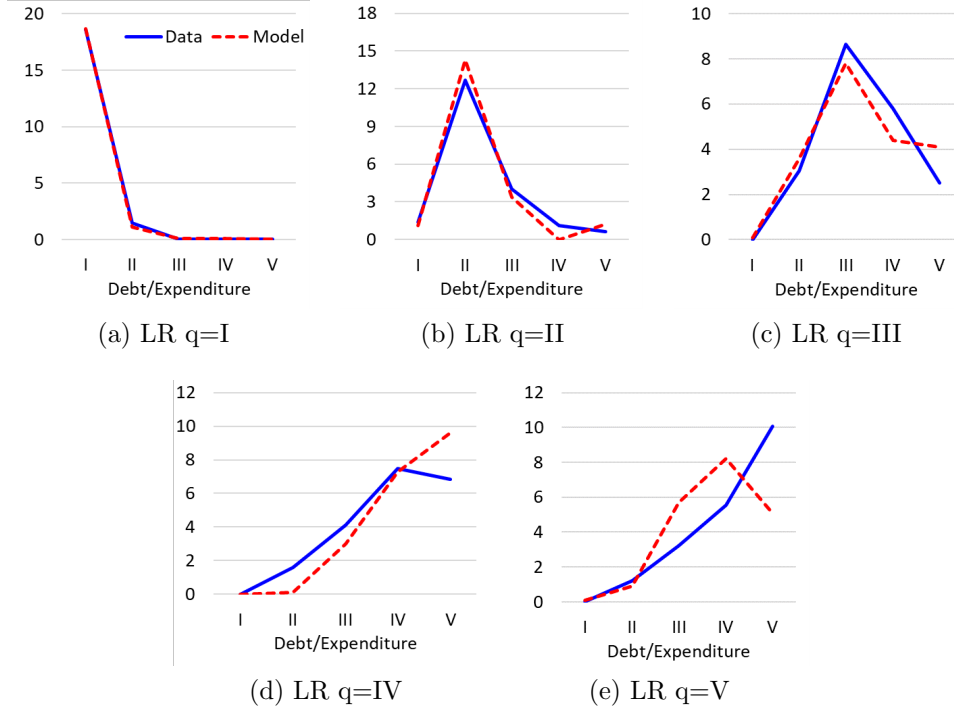
Notes: Solid blue lines correspond to the simulated distribution of the stationary model. Dashed red lines correspond to the distribution for Mexican households in 2005. Source: MxFLS.

6.3 Aggregate Risk Model

To solve the aggregate risk model, we adapt the *non-trivial* market clearing algorithm proposed by Krusell and Smith (1997) to a small open economy framework. Specifically, we use the current aggregate net foreign asset position, $B \equiv \int_0^1 b^i \, \mathrm{d}i$, and the current interest rate, $R - 1$, to forecast the next period's net foreign asset position, B'. Additionally, to forecast the domestic asset price, q, we also use last period's asset price, q_{-1}. This algorithm is computationally intensive since the current market clearing asset price depends on the whole distribution of asset holdings and not only on the aggregate holdings (which are constant). Hence, to obtain a simulated time

series, each period we use the aggregate law of motions to forecast the next period's aggregate net foreign asset position and the next period's asset's price. With these forecasts, we then solve a fixed-point problem for every period, which gives as solution the current equilibrium market clearing price.[19] The solution of the aggregate law of motions are:

$$B' = -0.020 + 0.798\ B + 0.129\ (R - 1), \quad R^2 = 0.99\ ,$$
$$q = 0.507 + 0.194\ B - 0.261\ (R - 1) + 0.083\ q_{-1}, \quad R^2 = 0.93\ . \tag{11}$$

6.3.1 Simulation and Event Study of Sudden Stops

Using the solution to the aggregate law of motions, we simulate a panel of 1,000 households for 6,000 periods and drop the first 1,000 periods. Table 11 columns (1) and (3) report long-run moments of the main macro aggregates from the benchmark model with heterogeneous-agents and a representative-agent version without idiosyncratic risk and a lower leverage limit, κ, that matches the same average leverage ratio of 0.122 obtained in the model with heterogeneity. Regarding the mean of the variables, the current account as a percentage of GDP is zero for both models. Average consumption is 4.5 percent higher, the net foreign asset position relative to GDP is 4.7 percentage points larger and the asset price is 26 percent higher in the heterogeneous-agents model. Since households do not need to self-insure against idiosyncratic shocks in the representative-agent model, there are less precautionary savings and lower demand pressure for the domestic asset. This equilibrium effect lowers the average asset price, tightening the aggregate financial conditions and lowering both average consumption and total debt.

Regarding the standard deviations, consumption volatility is about three times as volatile and asset price is about one half as volatile in the benchmark heterogeneous-agents economy compared to the representative-agent economy. This result comes from the larger consumption adjustments that high leveraged households have to do when they get hit by a negative shock.

To construct the event study of the simulated Sudden Stops, we average across all the identified crisis periods. Figure 5 shows the percent deviations from the steady

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[19]See Appendix B for a description of the solution algorithm.

Table 11: Business Cycle Statistics

	(1)	(2)	(3)
	Het. Agents	Het. Agents	Rep. Agent
	Benchmark	$(\sigma^d/2)$	Same Mean
	Eme. Eco.	Adv. Eco.	Lev. Ratio
Mean			
CA/GDP (%)	0.00	0.00	0.00
Consumption	0.23	0.22	0.22
NFA/GDP (%)	-29.12	-31.64	-24.43
Leverage ratio	0.122	0.157	0.122
Asset price	0.53	0.52	0.42
Standard deviation (%)			
CA/GDP	0.75	0.29	0.10
Consumption	1.35	0.97	0.43
NFA/GDP	4.44	1.02	0.09
Leverage ratio	1.95	0.67	0.00
Asset price	0.80	0.66	1.50

Notes: The representative-agent calibration has a lower leverage limit, κ, that matches the same average leverage ratio of the heterogeneous-agent model of 0.122.

state where the crisis period corresponds to $t = 0$. The average of the simulated crisis episodes in the heterogeneous-agents economy corresponds to the solid lines and the average of the data for Mexico around 1995 and 2009 Sudden Stops corresponds to the dashed line.

Figure 5.a shows that the Sudden Stops occur when there is an interest rate increase. This is expected since the interest rate is the only source of aggregate uncertainty in this economy. However, note that not all the interest rate increases cause a crisis. Specifically, the long-run probability of a Sudden Stop in the simulated benchmark economy is 2.16%. In 5.b we can see that a crisis episode is preceded by periods with current account below the long-run average. Then, when the crisis happens ($t = 0$) there is a sharp reversal in the current account which means that international capital stops flowing into the economy. Consistent with the data, the crisis is persistent and takes more than 3 years for the international capital to flow

Figure 5: Event Study of a Sudden Stop
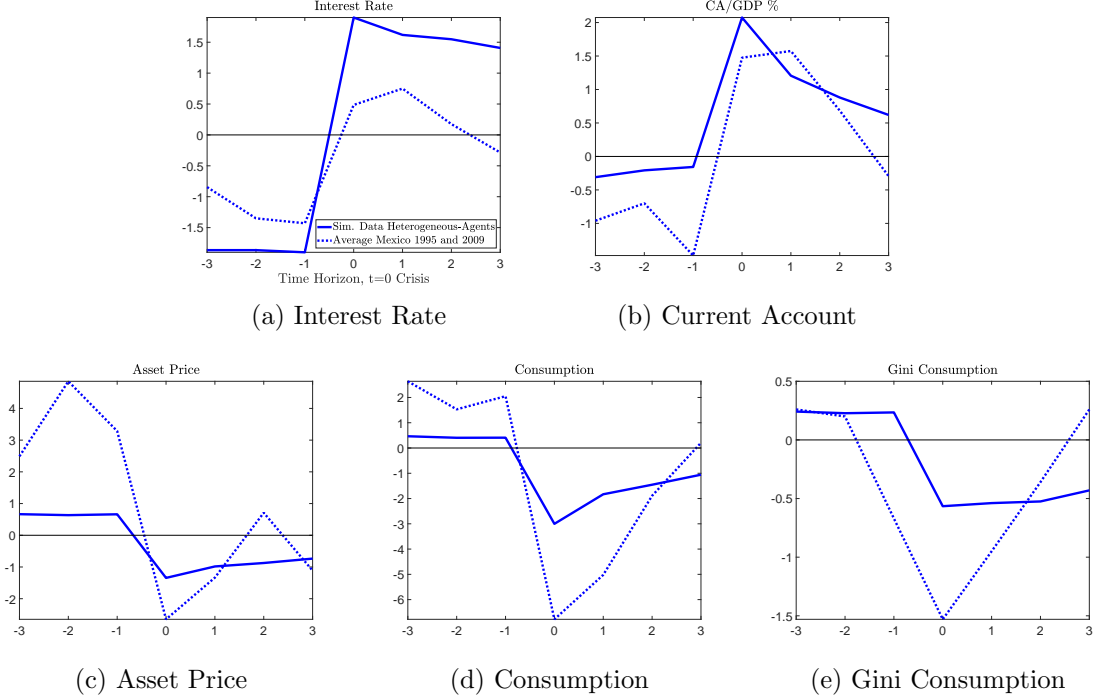
(a) Interest Rate

(b) Current Account

(c) Asset Price

(d) Consumption

(e) Gini Consumption

Notes: Solid lines correspond to the simulated data using the heterogeneous-agent model calibrated to the Mexican economy, dotted lines correspond to the average of the Mexican data around the 1995 and 2009 Sudden Stops. Panels a), b) and e) correspond to the level difference to the long-run mean. Panels c) and d) correspond to percentage point deviations from the long-run average.

back into the economy. Regarding the asset price drop, in 5.c we can see that the simulated price is 1.3% below the steady state which is below the asset price index for Mexico and in 5.d we can see that the model is able to generate a large and persistent aggregate consumption drop. Finally, 5.e shows that the model is able to capture a decline in consumption inequality during the crisis measured with the Gini coefficient, consistent with the data.

Regarding the differentiated individual effects during a Sudden Stop, in Tables 12 and 13 we show the dynamics of the asset holdings and consumption according to the leverage ratio and wealth of the households in a similar way as the results presented in Section 3.2. We can see that the model does a good job capturing the dampening effect coming from the wealthy unconstrained households that buy assets during a crisis and relieve the downward pressure on the price. In particular, these

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households increased by 6.4% their asset holdings during the crises. Moreover, in line with the empirical evidence on the amplifying effect, the financially constrained wealthy households are the ones that fire-sale the most their assets during the crisis and decreased their asset holdings by 13.6%. Although in the model, the households in decile IX of the leverage ratio do not sell their assets, we can see that they increase in a smaller amount than the households in deciles I-VIII. Hence, the model is able to capture both cross-sectional effects. In Table 13, we see that, in line with the empirical evidence, households with lower leverage ratios decrease the least their consumption. Hence, the model captures the heterogeneous consumption dynamics coming from the different leverage ratio levels and that crisis do no affect every household in the same way.

Table 12: Median % Asset Holdings Change in a Crisis

	Net Wealth	
Leverage Ratio	I-IX	X
	(Non-Wealthy)	(Wealthy)
I-VIII	-0.6	6.4
IX	2.2	3.4
X	1.8	-13.6

Notes: Ordered in the period of the crisis.

Table 13: Median % Consumption Change

Leverage Ratio	Pre-Crisis Period	Crisis Period
I	0.0	-1.9
II-IX	0.2	-2.7
X	0.5	-2.5

Notes: Ordered in the period previous to the crisis.

Lastly in Table 14, we show percent deviations from the steady state of the current account as a percentage of the GDP, consumption and the asset price for Mexico and different simulated economies. Column (1) and (2) show the observed deviations in 1995 and 2009 for Mexico, respectively. In column (3) we show the benchmark heterogeneous-agents model calibrated to an emerging economy (Mexico). We

can see that in the benchmark calibration, the asset price drop is smaller than the consumption drop, consistent with the data. Finally, in column (5) we show the representative-agent version of the model in which there is no idiosyncratic risk, and the leverage ratio limit, κ, is reduced to match the average leverage obtained in the heterogeneous-agents economy. Comparing columns (3) and (5) we can see that in the heterogeneous-agents economy the dampening effect dominates and asset prices drop less. However, there is a larger adjustment in aggregate consumption mainly driven by the most leveraged households (see Table 13).

Table 14: Comparison of Dynamics during Sudden Stops

|	(1)	(2)	(3)	(4)	(5)
	Mexico 1995	Mexico 2009	Het. Agents Benchmark Eme. Eco.	Het. Agents $(\sigma^d/2)$ Adv. Eco.	Rep. Agent Same Mean Lev. Ratio
CA / GDP (p.p.)	2.6	0.4	2.1	0.9	0.3
Consumption (%)	-8.3	-5.3	-3.0	-1.9	-1.3
Asset price (%)	-3.7	-1.8	-1.3	-0.9	-3.0

Notes: Sudden Stop episodes are defined as the periods where the current account as a percentage of GDP is 2 standard deviations above its mean.

6.3.2 Effect of a Lower Variance in the Dividend Risk

In this subsection, we compare the severity of Sudden Stops in economies with different degrees of inequality. Figure 6 shows descriptive evidence that crises are more severe in more unequal economies. The figure shows a scatter plot with the percentage change in consumption (panel (a)) and in GDP (panel (b)) during Sudden Stops for different economies (advanced in triangles and emerging in circles) against their income Gini index. This evidence suggests that emerging economies are more unequal and that there is a negative correlation between both variables.

To quantitatively asses the effects of lower income inequality, we calibrate the model to an advanced economy where the dividend risk is one-half of the benchmark emerging-markets model. The results summarized in Table 11 column (2) and Table 14 column (4), show that in the version of the model calibrated to an advanced economy, the average net foreign debt position is 8.5 percent larger, consumption

Figure 6: Severity of Sudden Stops and Inequality

(a) Change in consumption   (b) Change in GDP
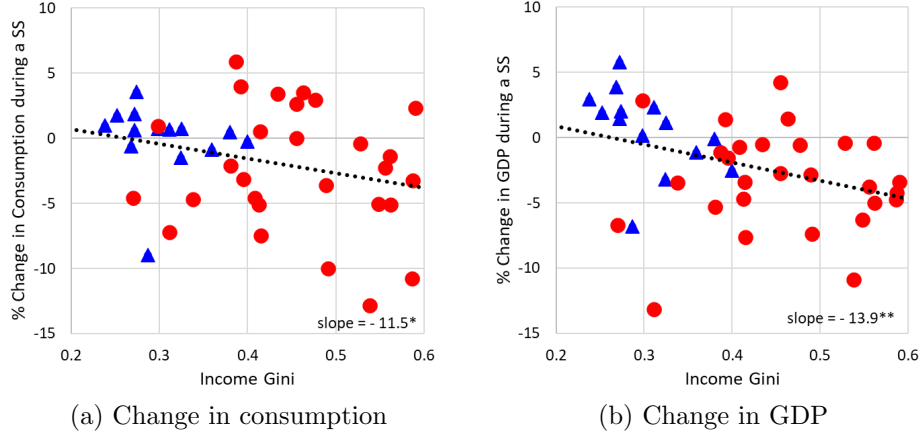
Notes: Triangle (circle) markers correspond to advanced (emerging) economies. Dates of Sudden Stop episodes come from Bianchi and Mendoza (2020). Gini index measures income inequality, larger numbers mean larger inequality (income instead of wealth is used due to the availability in a larger sample of countries). $^{***}p < 0.01,^{**} p < 0.05,^* p < 0.1$. Source: Own calculations with data from The World Bank.

drops 1.1 percentage points less and asset prices drop 0.4 percentage points less. Hence, the model predicts that in economies with less dividend return inequality, the economy supports larger debt positions and Sudden Stop crises are less severe, as observed in the data.
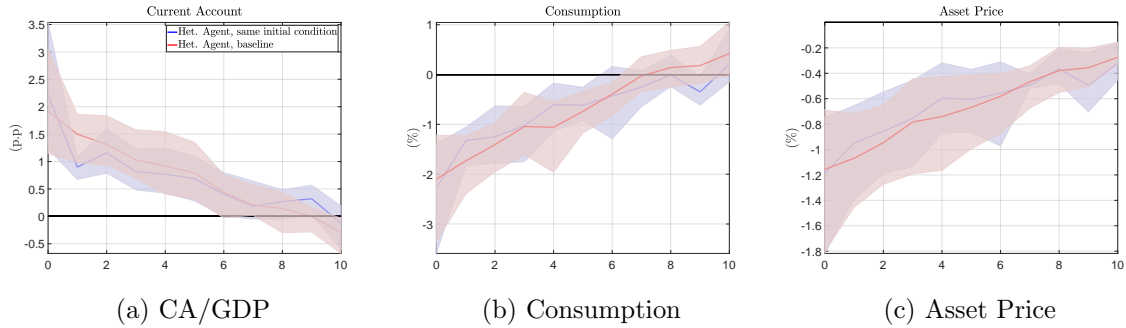
6.3.3   Impulse Response to an Interest Rate Shock

Lastly, this subsection looks at the impulse response functions after a two standard deviations interest rate shock. We compare the model with heterogeneity for different initial distributions and the representative-agent model. In the baseline model with heterogeneity, the responses are obtained by conditioning the economy to start at the stationary ergodic distribution when the aggregate interest rate is kept constant at its mean value. With respect to the same initial condition model, the responses are obtained by conditioning the economy to start with a perfect equality distribution where all the households hold the long-run average level of individual assets and the long-run average level of individual bonds. Finally, the representative-agent model results are obtained by conditioning the economy to start at the long-run mean bond

position. All three simulations also start at the long-run mean interest rate.

In line with the results from the previous subsection, Figure 8.a shows that the model with heterogeneity generates persistent current account reversals, which are two percentage points larger than the representative-agent model, which produces an isolating response in the current account around zero. In panels (b) and (c), we see that the response of the model with heterogeneity is about four times larger for consumption and about one-half for asset prices compared to the representative-agent economy, respectively. Lastly, in Figure 7, comparing both blue and red lines, we see that the effect of starting with a different initial distribution is minimal. However, in line with the results of the previous subsection, under a perfect equality initial condition, the responses are slightly smaller after impact. This set of results suggests that while inequality coming from the economy's fundamental processes matters for the severity and propagation of aggregate shocks, the effect coming from starting with different individual distributions is minimal.

Figure 7: Impulse Responses to an Interest Rate Shock:
Baseline and Perfect Equality Initial Condition Heterogeneous-Agent Economies

(a) CA/GDP   (b) Consumption   (c) Asset Price

Notes: Impulse response functions after a two standard deviations interest rate shock. In the baseline model with heterogeneity (red line), the responses are obtained by conditioning the economy to start at the stationary ergodic distribution when the aggregate interest rate is kept constant at its mean value. In the same initial condition model (blue line), the responses are obtained by conditioning the economy to start with a perfect equality distribution where all the agents have the long-run average level of individual assets and the long run average level of individual bonds. Both simulations also start at the long run mean interest rate. Bands represent 68% credible intervals, and solid lines are averages over ten simulations.

Figure 8: Impulse Responses to an Interest Rate Shock:
Baseline Heterogeneous and Representative-Agent Economies
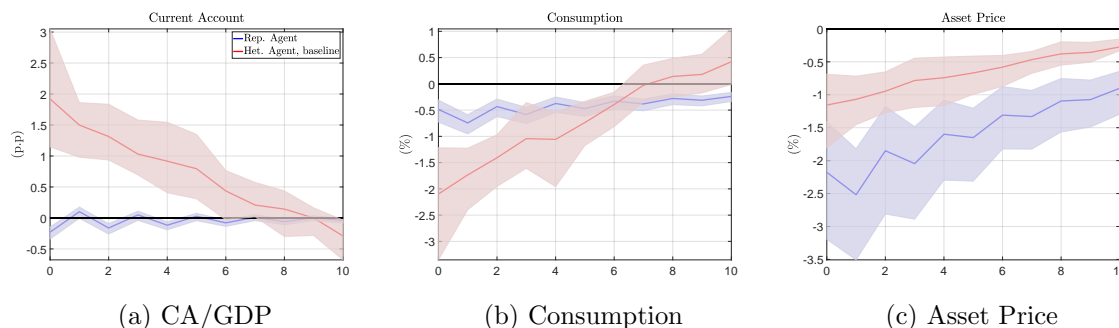
(a) CA/GDP (b) Consumption (c) Asset Price

Notes: Impulse response functions after a two standard deviations interest rate shock. In the baseline model with heterogeneity (red line), the responses are obtained by conditioning the economy to start at the stationary ergodic distribution when the aggregate interest rate is kept constant at its mean value. In the representative-agent model (blue line), results are obtained by conditioning the economy to start at the long-run mean bond position. Both simulations also start at the long-run mean interest rate. Bands represent 68% credible intervals, and solid lines are averages over ten simulations.

7 Conclusion

This paper studies the cross-sectional dimension of the debt-deflation mechanism that triggers endogenous financial crises of the Sudden Stop type. This dimension is relevant for the macroeconomy for two reasons. First, there is a dampening effect on the deflation of asset prices coming from the unconstrained wealthy households who buy depressed assets, relieving the downward pressure on asset prices. Second, there is an amplifying effect on the asset price deflation coming from the financially vulnerable households who fire-sale assets, generating a stronger downward pressure on asset prices. Because these two cross-sectional effects move asset prices in opposite directions, the cross-section and inequality role during crises is quantitatively ambiguous. Hence, this paper examines how the frequency and severity of Sudden Stops crises are affected by inequality in an economy.

Using panel data for Mexican households, we document micro-data evidence that supports both effects. Specifically, the 2009 crisis had different effects on the households depending on the composition of their balance sheets. The real estate holdings of low-leveraged wealthy households increased 59.4% during the crisis while wealthy

households with high-leverage fire-sold and decreased the most their assets during the crisis. Additionally, in terms of the consumption dynamics, households in the top decile of leverage decreased their expenditures by 6.2% while households in the bottom decile increased their expenditures by 5.4% during the crisis. These heterogeneous asset and consumption dynamics highlight the importance of the opposing forces that are missed when the financial crises are studied under a representative-agent framework. For this reason, we proposed a model to quantify a Sudden Stop's effect on asset prices and consumption, accounting for the household's heterogeneity in their balance sheet.

Using the proposed asset-pricing Bewley model of a small open economy, we find that in a version of the model calibrated to an emerging economy (Mexico), the model can explain Sudden Stops' key stylized facts and generate persistent current account crises. Regarding the cross-sectional forces, the dampening effect dominates and asset prices drop less during Sudden Stop episodes in heterogeneous-agents economies. In contrast to the representative-agent framework, the model produces an empirically plausible leverage ratio distribution and generates persistent current account reversals with larger drops in consumption driven by the most leveraged households, consistent with the data. Moreover, calibrating the model to an advanced economy where the dividend risk is one-half of the benchmark emerging-markets model, the average net foreign debt position is 8.5 percent larger, consumption drops 1.1 percentage points less, and asset prices drop 0.4 percentage points less. Additionally, an impulse response analysis, which compares the effects of a interest rate shock, suggests that while inequality coming from the economy's fundamental processes matters for the severity and propagation of aggregate shocks, the effect coming from starting with different individual distributions is minimal. Hence, the model predicts that larger debt positions are supported in economies with less dividend return inequality, and Sudden Stop crises are less severe, as observed in the data.

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Online Appendix to "Inequality and Asset Prices during Sudden Stops"

Sergio Villalvazo

This Online Appendix consists of the following sections:

A The 2009 Mexican Sudden Stop at the Aggregate Level

A Sudden Stop is a fast and large outflow of international capital (Calvo, Izquierdo, and Talvi (2006)). Hence these types of episodes are characterized by large Current Account (CA) movements.[20] In this appendix, we use aggregate data to show the Sudden Stop that the Mexican economy experienced in 2009.

In Figure A-1 we can see that the current account deficit reversed around 1.5 percentage points of GDP. Also, GDP and consumption declined, there was a drop in the consumer confidence and a decline in consumption credit while firm and housing credit was not affected.

On the prices side, in Figure A-2 we see that there was a large decline in the stock market, house prices decelerated and remained constant for about 4 years since the crisis burst, the J.P. Morgan EMBI+ spread that measures the Mexican sovereign bonds risk increased about 2 percentage points and there has a large depreciation of the Mexican peso against the dollar.
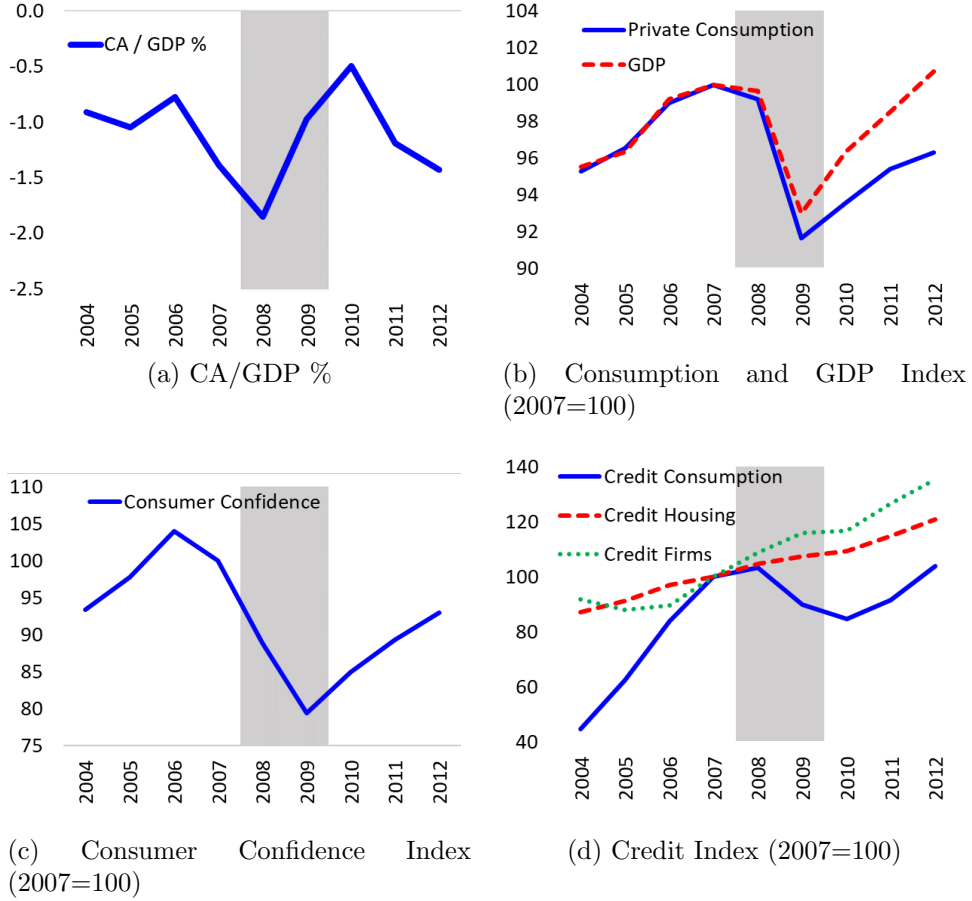
The aggregate dynamics shown in this Appendix are not particular to Mexico. See Bianchi and Mendoza (2020) for a recent survey of Sudden Stop episodes both among advanced and emerging economies.

B Solution Algorithm

In this appendix we describe the solution method. Building from Krusell and Smith (1997), we adapt their *non-trivial* market clearing algorithm to a small open economy framework. In particular, instead of solving problem 5, we solve:

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[20]Some Sudden Stop episodes have even registered CA reversals. Meaning that the economy transits from having a negative CA (foreign capital entering the economy) to positive CA surpluses (capital leaving the economy).
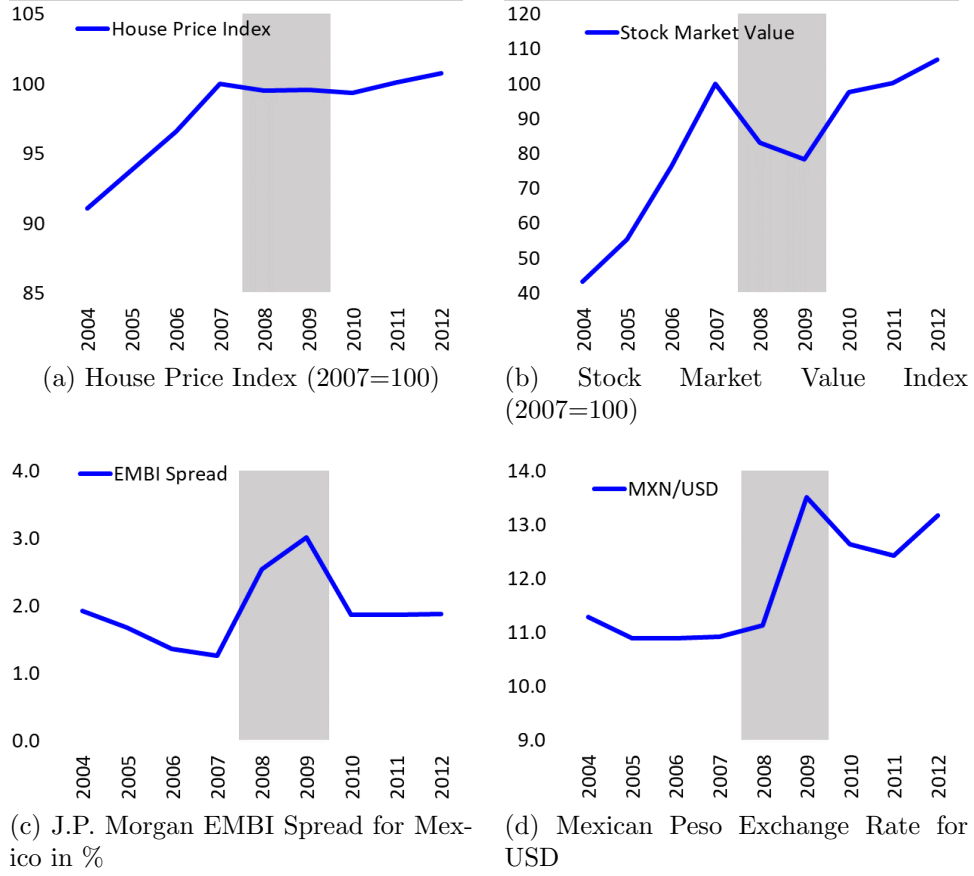
Figure A-1: Quantities and Consumption determinants

(a) CA/GDP %

(b) Consumption and GDP Index (2007=100)

(c) Consumer Confidence Index (2007=100)

(d) Credit Index (2007=100)

Notes: The grey area corresponds to the crisis. Source: INEGI, World Bank, Banxico.

$$\tilde{v}(b, a, \epsilon^w, \epsilon^d, B, \epsilon^R, q) = \max_{\{c, b', a' \geq 0\}} \; u(c) + \beta \mathbb{E}[v(b', a', \epsilon^{w'}, \epsilon^{d'}, B', \epsilon^{R'})] \quad s.t.$$

$$c + R(\epsilon^R)^{-1} b' + q(a' + \Phi(a', a)) = \epsilon^w w + a(q + \epsilon^d d) + b,$$

$$R(\epsilon^R)^{-1} b' \geq -\kappa q a',$$

$$\Phi(a', a) = \frac{\phi}{2}(a' - a)^2$$

$$B' = \gamma_B^0 + \gamma_B^1 B + \gamma_B^2 (R - 1)$$

$$q = \gamma_q^0 + \gamma_q^1 B + \gamma_q^2 (R - 1) + \gamma_q^3 q_{-1} \tag{A.1}$$

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Figure A-2: Asset Prices

(a) House Price Index (2007=100)

(b) Stock Market Value Index (2007=100)

(c) J.P. Morgan EMBI Spread for Mexico in %

(d) Mexican Peso Exchange Rate for USD

Notes: The grey area corresponds to the crisis. Source: Sociedad Hipotecaria Federal, Moodys Analitics, INEGI, World Bank.

Where we replaced the full household distribution Ω with the aggregate bond position $B = \int b \, d\Omega$, and market clearing in the asset holdings is achieved using a fixed-point iteration on q such that $\bar{K} = \int a'(\cdot) \, d\Omega$. Then, the solution algorithm follows the simulation method described in Krusell and Smith (1997).

C Non-linearities in the Stationary Model

To understand better the mechanism and the *asset-wealth trade-off*, Figures A-3-A-6 show the policy functions and the non-linearities generated in the model. In the

upper row of Figure A-3 the solid lines correspond to the bond policy for the high (low) dividend shock in blue (red) and the average labor income shock as a function of the current asset holdings for three different values of the current bond holding $b^{\#}$. Additionally, the dashed lines represent the corresponding debt limits, and the black dashed lines correspond to the bottom 1% and top 99% percentiles of bond and asset holdings obtained from the model's simulated cross-section. The figure, shows that for low dividend shocks (red lines) a household lowers their bond holdings (or gets more debt) as they increase their asset holdings. This effect is stronger for constrained households, as shown in panels (b) and (c). As described in Section 5.1, the *asset-wealth trade-off* generates the convex form of the bond policy for high dividend shocks (blue lines). For asset-poor households, as they increase their assets, they also lower their bond holdings (or get more debt if the holdings are negative) and there is a certain level for which the dividend risk exposure overcomes the benefit from more debt capacity that makes the households increase their bond holdings. Regarding the bottom row of the figure, we can see the asset policy function that is highly linear and behaves as expected: for high-dividend shocks the households accumulate more assets, and for low-dividend shocks the households decumulate assets.

Moreover, in Figure A-4 we show similar bond and asset policies but now as a function of the current bond holdings. In the upper row, we can see the standard bond policies under a binding debt limit. Panel a) shows the policy for a high-asset holder. Here we can see that the debt limit is not binding for the states within the 1st and 99th percentiles. However, as we move to lower asset holdings, in panels b) and c), we can see that the LtV becomes binding when households accumulate enough debt. With respect to the cross-sectional fire-sales in the model, in the bottom row of the figure we can see that households accumulate less assets as they increase their debt holdings. However, this relation is highly strengthened (households incur in fire-sales) when the debt limit becomes binding. This can be seen using panels b) and e) and also panels c) and f). There are strong declines in the asset holdings (panels e) and f)) in the states where the bond holdings reach the debt limit (panels b) and c)).

Additionally, in Figure A-5 we show the difference in the bond policy function for a high and a low dividend shock in panel (a) and labor income shock in panel (b). We can see a positive and increasing difference in the next period bond holdings
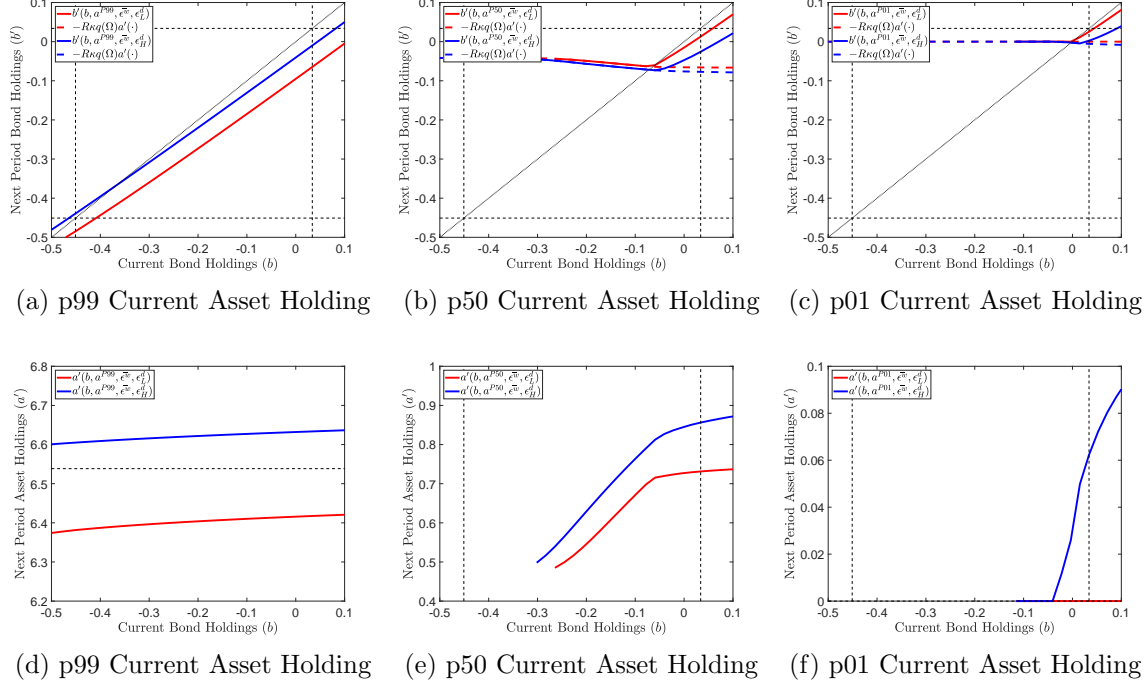
Figure A-3: Stationary bond and asset policies as a function of current asset holdings
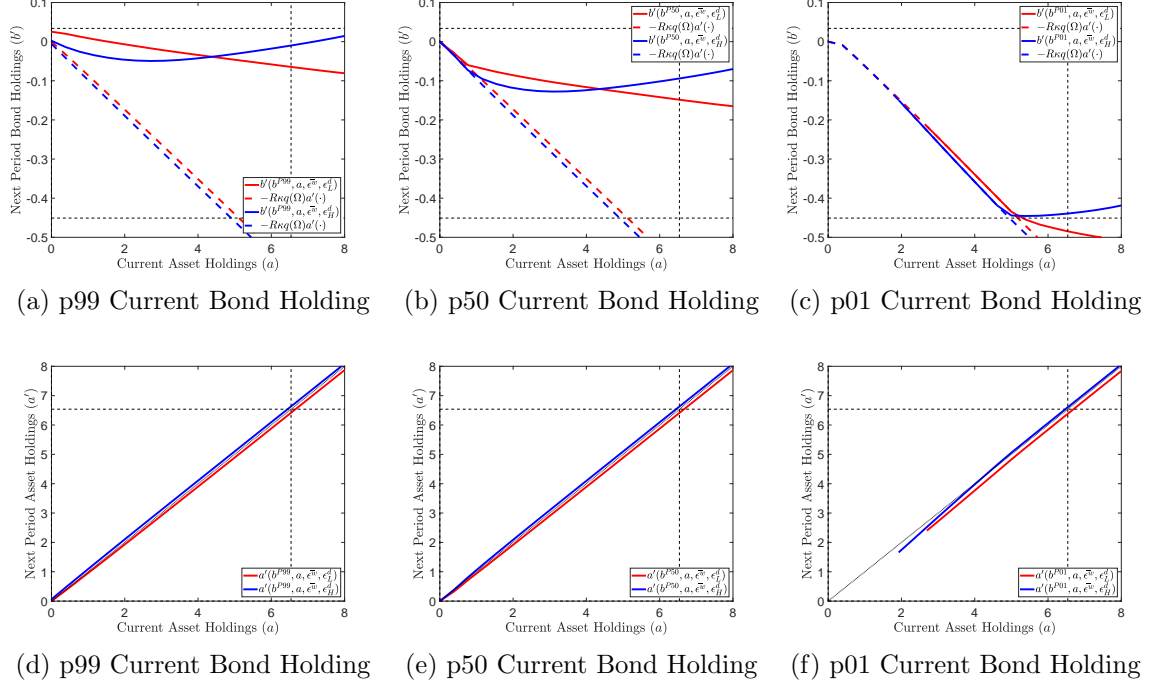
(a) p99 Current Bond Holding   (b) p50 Current Bond Holding   (c) p01 Current Bond Holding

(d) p99 Current Bond Holding   (e) p50 Current Bond Holding   (f) p01 Current Bond Holding

Notes: For a current bond holding $b^{\#}$ and mean labor shock $\bar{\epsilon}^w$, the upper (lower) row correspond to the bond (asset) policies, the solid blue (red) line corresponds to the policy function with the high (low) dividend shock and the dashed blue (red) line corresponds to the debt limit with the high (low) dividend shock. Black dashed lines correspond to the bottom 1% and top 99% percentiles of bond and asset holdings obtained from the model's simulated cross-section. Black dotted lines correspond to the 45-degree line. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

between the high and low dividend productivities as we move to higher current asset holdings (Figure A-5.a). This means that when the idiosyncratic dividend realization is high, the household optimally chooses larger bond holdings for the next period. Moreover, this difference is kept almost constant (only increases close to the debt limit) across the current bond holdings. In contrast, in Figure A-5.b we can see that the difference in the bond policy function between the high and low idiosyncratic labor productivity realization is positive but close to zero and constant throughout all the feasible state-space. Similarly, in Figure A-6 we show the difference in the asset policy function for a high and a low dividend shock in panel (a) and labor income shock in panel (b). We can see a positive and increasing difference in the

Figure A-4: Stationary bond and asset policies as a function of current bond holdings
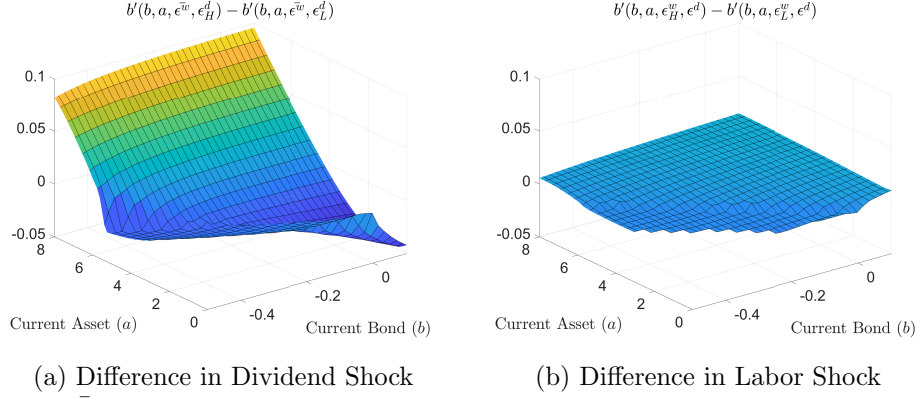

(a) p99 Current Asset Holding   (b) p50 Current Asset Holding   (c) p01 Current Asset Holding

(d) p99 Current Asset Holding   (e) p50 Current Asset Holding   (f) p01 Current Asset Holding

Notes: For a current bond holding $b^{\#}$ and mean labor shock $\bar{\epsilon}^w$, the upper (lower) row correspond to the bond (asset) policies, the solid blue (red) line corresponds to the policy function with the high (low) dividend shock and the dashed blue (red) line corresponds to the debt limit with the high (low) dividend shock. Black dashed lines correspond to the bottom 1% and top 99% percentiles of bond and asset holdings obtained from the model's simulated cross-section. Black dotted lines correspond to the 45-degree line. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

next period asset holdings between the high and low dividend productivities as we move to higher current asset holdings (Figure A-6.a). However, for high enough asset values, this positive difference becomes relatively constant. Moreover, this difference is kept almost constant (only increases close to the debt limit) across the current bond holdings. Finally, similarly to the bond policy function, in Figure A-6.b we can see that the next period asset holdings difference between the high and low idiosyncratic labor productivity realization is positive but close to zero and constant throughout all the feasible state-space.
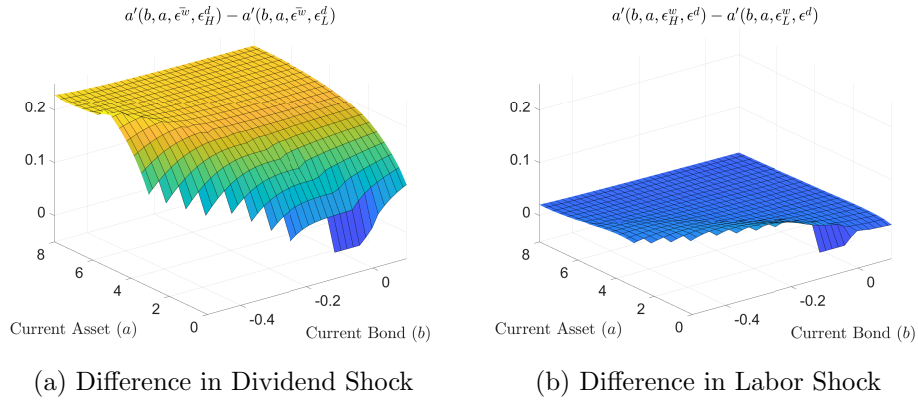
In summary, we used the stationary model to show the cross-sectional behavior of households. We can see that households with high-dividend shocks will accumulate

Figure A-5: Effect of Non-insurable Individual Shocks in the Bond Policy

$$b'(b,a,\bar{\epsilon}^w,\epsilon_H^d) - b'(b,a,\bar{\epsilon}^w,\epsilon_L^d)$$ $$b'(b,a,\epsilon_H^w,\epsilon^d) - b'(b,a,\epsilon_L^w,\epsilon^d)$$

(a) Difference in Dividend Shock   (b) Difference in Labor Shock

Notes: $\bar{\epsilon}^w$ and $\bar{\epsilon}^d$ correspond to the mean shock values. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

Figure A-6: Effect of Non-insurable Individual Shocks in the Asset Policy

$$a'(b,a,\bar{\epsilon}^w,\epsilon_H^d) - a'(b,a,\bar{\epsilon}^w,\epsilon_L^d)$$ $$a'(b,a,\epsilon_H^w,\epsilon^d) - a'(b,a,\epsilon_L^w,\epsilon^d)$$

(a) Difference in Dividend Shock   (b) Difference in Labor Shock

Notes: $\bar{\epsilon}^w$ and $\bar{\epsilon}^d$ correspond to the mean shock values. The missing values across the state space correspond to the infeasible individual states that would imply a negative consumption.

more assets and, while they are still asset-poor, they decumulate bonds. Once they become asset-rich, because of the *asset-wealth trade-off*, they start accumulating more bonds (Figure A-3). This behavior generates wealthy unconstrained households that drive the dampening cross-sectional effect. Moreover, we also show that households decumulate assets as they increase their debts, and that this relation strengthens (households incur in fire-sales) when the debt limit is reached, driving the strength of the amplifying effect (Figure A-4). Note that the representative-agent model would miss both effects. First, since there are no individual shocks, every household will

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behave in the same way. Hence, they either want to sell or buy more assets. Second, in that model, the average debt constraint multiplier will be the same as the individual debt multiplier, while in the heterogeneous-agents model, although fewer households could be constrained (calibrated to be only 10%), they could have a larger multiplier given the individual states. Finally, we used the stationary solution for simplicity and to avoid the extra aggregate states that would be needed in the aggregate risk model.

# D Event Study of an Economy with Lower Variance in the Dividend Risk

In Figures A-7 and A-8 we show the event study analysis for the same history of individual and aggregate shocks for the two calibrations: the baseline emerging economy in solid lines and the advanced economy with the same calibration but with half variance in the dividend risk in dashed lines.

Figure A-7: Event Study of a Sudden Stop in Simulated Economies
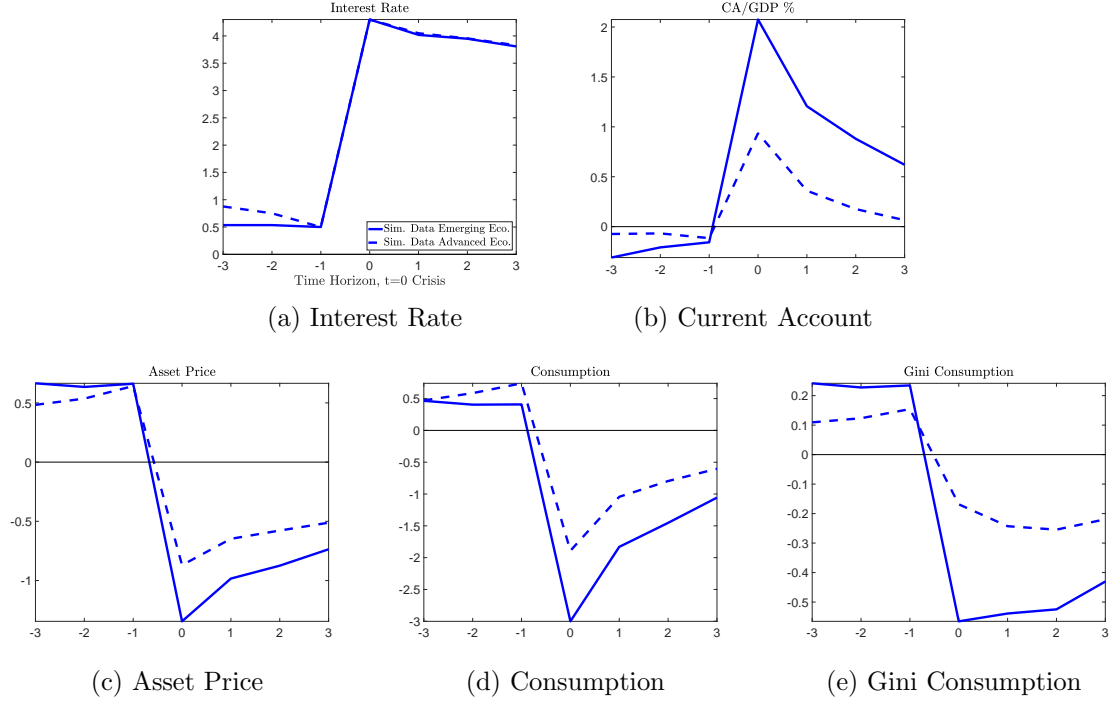
(a) Interest Rate

(b) Current Account

(c) Asset Price   (d) Consumption   (e) Gini Consumption

Notes: Solid lines correspond to the simulated data using the heterogeneous-agent model calibrated to an emerging economy (Mexico) and dashed lines to the heterogeneous-agent model calibrated to an advanced economy which has one half the variance in the dividend risk. Panels a), b) and e) correspond to the level difference to the long-run mean. Panels c) and d) correspond to percentage point deviations from the long-run average.

Figure A-8: Net Foreign Asset Position Event Study of a Sudden Stop in Simulated Economies

(a) Emering Economy   (b) Advanced Economy
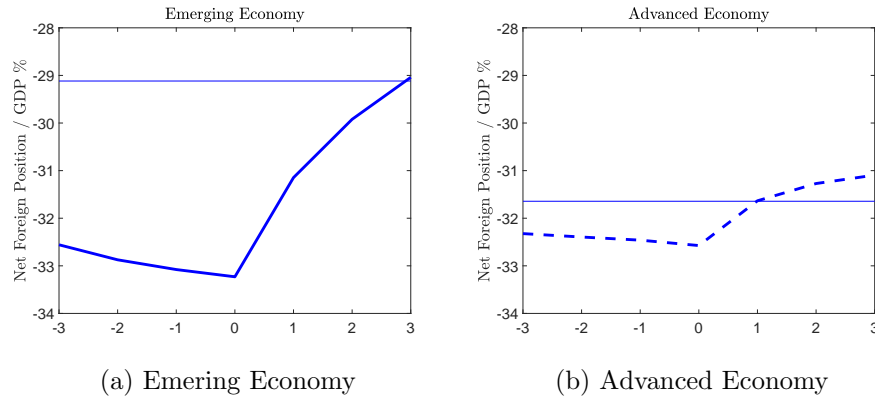
Notes: Solid blue horizontal lines correspond to the long-run averages.

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