Labor Market Shocks and Monetary Policy*

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Abstract

We develop a heterogeneous-agent New Keynesian model featuring a frictional labor market with on-the-job search to quantitatively study the role of worker flows in inflation dynamics and monetary policy. Motivated by our empirical finding that the historical negative correlation between the unemployment rate and the employer-to-employer (EE) transition rate up to the Great Recession disappeared during the recovery, we use the model to quantify the effect of EE transitions on inflation in this period. We find that the four-quarter inflation rate would have been 0.6 percentage points higher between 2016 and 2019 if the EE rate increased commensurately with the decline in unemployment. We then decompose the channels through which a change in EE transitions affects inflation. We show that an increase in the EE rate leads to an increase in the real marginal cost, but the direct effect is partially mitigated by the equilibrium decline in market tightness through aggregate demand that exerts downward pressure on the marginal cost. Finally, we study the normative implications of job mobility for monetary policy responding to inflation and labor market variables according to a Taylor rule, and find that the welfare cost of ignoring the EE rate in setting the nominal interest rate is 0.2 percent in additional lifetime consumption.

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1 Introduction

The episode after the Great Recession in the U.S. led to discussions questioning the validity of the Phillips curve—the negative short-run relationship between the unemployment rate and the inflation rate. These discussions were motivated by the observation that there was no disinflation in the early years following the Great Recession (2009-2011) when unemployment remained significantly higher than its pre-recession level and there was no inflation in the years of recovery (2016-2019) when the unemployment rate fell 30 percent below its pre-Great Recession level. The ensuing body of work focused on several hypotheses such as the role of import competition and market concentration, a flattening aggregate supply curve, a declining natural rate of unemployment, and better anchored inflation expectations as potential explanations. Recent inflation readings in the aftermath of the COVID-19 recession—the highest in about four decades—are surprising through the lens of these explanations.

In this paper, we argue that some of this disconnect between inflation after the Great Recession and after the COVID-19 recession can be traced back to temporary frictions in the labor market, especially those pertaining to employment-to-employment (EE) transitions. Importantly, we demonstrate how job mobility dynamics have important welfare implications for the conduct of monetary policy. In doing so, we make three contributions. First, we show that the negative historical relationship between the unemployment and the EE rates significantly weakened after the Great Recession and eventually disappeared between 2016 and 2019. This co-movement then became significantly negative again in the aftermath of the COVID-19 recession. Second, we develop a Heterogeneous-agent New Keynesian (HANK) model that embeds a Diamond-Mortensen-Pissarides (DMP) model of the labor market featuring rich worker heterogeneity and on-the-job search (OJS). Using the model, we quantify the magnitude of the "missing inflation" between 2016 and 2019 that is accounted for by the weakening relationship between the unemployment rate and the EE rate. A key contribution we make is providing a decomposition of the channels through which EE transitions affect inflation. Finally, we study optimal monetary policy among a restricted class of policies in our environment, where we solve for the coefficients of a Taylor rule that maximizes welfare. This allows us to gauge the welfare costs of ignoring EE dynamics when setting the nominal interest rate.

We present three quantitative results. First, we find that the four-quarter inflation rate was 0.6 percentage points lower in the years of recovery from the Great Recession (2016-2019) because the EE rate remained around its trend while the unemployment rate declined significantly below its trend. Second, we decompose the overall effect of job mobility on inflation. We find that the total effect is a combination of various offsetting channels. Increased job mobility raises the frequency with which incumbent firms need to match outside offers or have their worker poached, thus reducing incentives to post vacancies. The reduction in expected profits entails a compensatory increase in prices to encourage vacancy creation, resulting in higher inflation. This direct effect, however, is offset by a general equilibrium response in labor market tightness. Higher job mobility implies that the unemployed now compete for jobs with a larger pool of employed job searchers. The increased unemployment risk dampens demand and creates more slack in the labor market. This slack in the labor market exerts downward pressure on inflation. Finally, we show that when allowed to respond to deviations of the EE rate from its trend, optimal monetary policy prescribes a sizeable and positive reaction of the nominal rate with respect to EE gaps. In addition, we show that ignoring EE dynamics has important implications on the design of optimal monetary policy. When the monetary authority is restricted to responding *only* to the inflation and unemployment gaps, optimal policy prescribes a weaker response to unemployment combined with a stronger response to inflation relative to the baseline Taylor rule. This is because the unemployment gap does not constitute a sufficient measure of labor market conditions during episodes when the strong negative relationship between the unemployment rate and the EE rate disappears, as observed after the Great Recession. However, this restricted optimal policy results in a welfare loss of 0.2 percent additional lifetime consumption relative to the optimal policy where the monetary authority can also respond to deviations of the EE rate.

Our starting point is to build a HANK model combined with a labor search model featuring on-the-job search. Individuals work, retire and die stochastically, and face idiosyncratic labor-market shocks. Markets are incomplete in that the labor market shocks are not fully insured: Individuals can smooth consumption through personal wealth, which they hold in terms of shares of a mutual fund. The government also provides unemployment benefits as social insurance. Individuals work in firms that produce labor services. Their productivity depends on their human capital and the match-specific productivity of their job. Their wage is an endogenous piece-rate of their output, which is determined through Bertrand competition based on their flow output. To find a job or to improve it (and therefore increase their wages), workers look for work in a labor market subject to search frictions. Workers accumulate human capital stochastically while employed and also engage in OJS, both of which allows them to obtain higher wages. In particular, contacting outside employers may potentially result in higher wages for the employed even when they stay with their firm because such contacts may lead to rebargaining. The rest of the model follows the New Keynesian tradition. Monopolistically competitive intermediate firms buy labor services from the service firms to produce their differentiated goods, which are then sold to final-good producers. The government issues nominal bonds and collects taxes on labor income and consumption to finance an exogenous stream of unproductive government expenditures, social security for retirees, and an unemployment insurance program.

We calibrate the steady state of the model to match several aspects of the U.S. economy over the period 2004–2006 before the onset of the Great Recession. In light of recent work highlighting the importance of the wealth distribution for monetary policy transmission, we calibrate our model to match the fraction of hand-to-mouth individuals. To discipline the relative importance of human capital formation and the job ladder for income dynamics, we target the wage growth of job switchers and the earnings loss associated with job loss during the first year of displacement. To discipline search frictions in the labor market, we match the unemployment rate, job separation rate, and EE rate. The New Keynesian block of the economy is calibrated to match the average level of markups, the slope of the Philips curve, and the responsiveness of the nominal rate to inflation and unemployment gaps.

Solving our model for the purposes of estimation as well as positive and normative analysis requires significant computational resources. To reduce the computational burden, we implement the sequence-space Jacobian method recently developed by Auclert, Bardóczy, Rognlie, and Straub (2021). To this end, we cast the model as a Directed Acyclic Graph (DAG) that represents equilibrium conditions as separate blocks that are interconnected via model variables. Along this DAG, we efficiently compute the partial Jacobians of each block's outputs with respect to its direct inputs in the sequence space, and use these partial Jacobians to compute the general equilibrium derivative of the outcome variables of interest with respect to exogenous variables. This step allows us to calculate impulse response functions (IRFs) and then simulate the economy hit by aggregate shocks. In the process, we extend the method in Auclert et al. (2021) and incorporate discretized endogenous worker distributions into the DAG, which are key objects in our model that features non-trivial asset and labor markets.

Our quantitative exercise is motivated by our empirical finding which shows that the strongly negative correlation between the unemployment rate and the EE rate up to the Great Recession disappeared between 2016 and 2019. In particular, the raw correlation between the monthly unemployment rate and the EE rate was significant and negative (-0.18) between 1995 and 2007 but it turned insignificant and positive (0.06) between 2016 and 2019. This was because, in the post-Great Recession episode, the unemployment rate declined by around 30 percent relative to its pre-Great Recession level, while the EE rate remained flat around its long-run trend. We use our model to quantify the effect of this weakening correlation on inflation. To do so, we compare outcomes of two economies that start at the steady state (which was attained in 2016) and exhibit the same declining path for the unemployment rate over four years, emulating the empirical pattern of unemployment. The two economies differ in that they are subject to a different combination of shocks. The first counterfactual economy undergoes a period of declining unemployment rate through a series of positive demand shocks (modeled as negative shocks to the discount factor), while the second post-Great Recession economy experiences the exact same declining path of unemployment rate but also a constant EE rate through a combination of positive demand shocks and negative job mobility shocks (modeled as negative shocks to OJS efficiency). Hence, the key difference between the two economies is the dynamics of the EE rate. In the first economy, positive demand shocks alone reduce the unemployment rate and increase vacancy posting by firms, resulting in an endogenous *increase* in the EE rate. In the second economy, the EE rate remains *flat* by construction, mimicking the empirical pattern observed during the post-Great Recession. We show that the presence of negative OJS efficiency shocks causes a drag on inflation, despite similar unemployment and output dynamics. In particular, the four-quarter inflation rate is around 0.6 pp lower in the second economy.

We then turn to studying the channels through which an OJS efficiency shock affects inflation. To do so, we analyze model outcomes to a positive unit shock to OJS efficiency and decompose the on-impact change in the real marginal cost of production (the real price of labor services sold to intermediate firms) and thus inflation. In doing so, we leverage the DAG representation of the model and rely on the sequence-space Jacobians we compute in the process of solving the model. In the labor market, a higher OJS efficiency reduces the service firm's expected value from a match. The worker's probability of contacting an outside firm increases, which leads to either rebargaining with the incumbent firm to extract a greater share of match surplus or a shorter match duration if the worker is poached. All else equal, this decline in the match value for service firms requires an increase in the price of their output (labor services) for the free-entry condition to hold. We quantify that this direct effect of OJS efficiency explains 306 percent of the total increase in the real marginal cost upon impact. Importantly, we show that this direct effect is partially mitigated by secondary effects through general equilibrium (GE) responses. In particular, higher job mobility leads to a decline in market tightness in equilibrium. This is mainly driven by a decline in aggregate consumption due to a higher unemployment rate which arises because unemployed job searchers are crowded out by their employed counterparts who now enjoy higher search efficiency. A decline in output reduces demand for labor services, which all else constant, implies a decline in market tightness for the labor market to clear. When the labor market is more slack, firms find it easier to fill vacancies. Therefore, the price of labor services, i.e., the marginal cost, declines to preserve the free-entry condition. We show that this GE response of market tightness accounts for -219 percent of the total increase in the marginal cost. Thus, counteracting labor market effects explain 87 ($306-219$) percent of the total increase in the marginal cost. The remaining 13 percent is accounted by the changes in the real rate due to the GE responses of inflation and unemployment. Specifically, in equilibrium, inflation and unemployment increase upon a rise in OJS efficiency. While higher inflation leads to higher nominal and real interest rates, higher unemployment implies the opposite, according to the Taylor rule and the Fisher equation. In turn, the real rate affects the continuation value of a service firm matched with a worker and thus the expected match value for an entrant. We find that this discount rate effect through inflation accounts for 17 percent of the total increase in the real marginal cost, while the remaining -4 percent is explained by the effect of unemployment.

Finally, we study the normative implications of job mobility for monetary policy. We first consider an augmented Taylor rule that responds to both deviations of unemployment and EE rates from their steady state values. Under this augmented Taylor rule, we jointly solve for the coefficients on the inflation gap, unemployment gap, and EE gap that maximize the ex-ante worker welfare under the veil of ignorance. We find that the optimal policy prescribes (i) a similar coefficient on inflation compared to the existing (calibrated) policy, (ii) a negative coefficient on unemployment gap that is roughly double that of the existing policy, and (iii) a sizable positive coefficient on the EE rate. In practice, this implies a more aggressive increase in the nominal rate during a recovery when the decline in the unemployment rate is accompanied by a rise in the EE rate (as observed after the COVID-19 recession) when compared to a recovery episode when the EE rate remains flat despite the decline in the unemployment rate (as observed after the Great Recession).

In order to compute the welfare implications of ignoring job mobility dynamics in the conduct of monetary policy, we jointly optimize over the coefficients on the inflation gap and unemployment gap, but shut down the response to EE dynamics. We find that the optimal policy in this case prescribes a weaker response to the unemployment gap but a stronger response to the inflation gap compared to the benchmark policy. The weaker response to the unemployment gap is due to the fact that the unemployment rate is not a sufficient measure of labor market slack especially during episodes when the correlation between the unemployment rate and EE rate becomes weaker. Overall, we compute that the welfare cost of ignoring the dynamics of EE rate is 0.2 percent in additional lifetime consumption.

Related literature. This paper contributes to several strands of the literature. A recent but expanding literature studies inflation dynamics after the Great Recession. Earlier studies focused on why there was no disinflation in the earlier years following the Great Recession (2009-2011) (Coibion and Gorodnichenko, 2015; Ball and Mazumder, 2011) despite high unemployment rates. The ensuing recovery phase and the "missing inflation" that would have been implied by low unemployment rates in the years after the Great Recession also motivated several other important studies. Hazell, Herreno, Nakamura, and Steinsson (2020) argue that well-anchored inflation expectations weaken the link between measures of labor market tightness and inflation and reduce the volatility of inflation. An alternative view is that structural shifts in the economy have caused the Phillips curve to flatten over time. Del Negro, Lenza, Primiceri, and Tambalotti (2020) find that the disconnect between the labor market and inflation is due primarily to the muted reaction of inflation to cost pressures and rule out stories centered around changes in the structure of the labor market or in how one should measure its tightness. Hooper, Mishkin, and Sufi (2020) estimate the slopes of the price and wage Phillips curves over time and reach a similar conclusion. These findings are consistent with those in Heise, Karahan, and Şahin (2020), who use disaggregated data to find a declining pass-through of wage pressures to inflation. Carvalho, Eusepi, Moench, and Preston (2017) estimate a decline in the natural rate of unemployment, and articulate this as a reason for why historically low unemployment rates do not have to translate to wage pressures. We view our work as complementary to these papers in that we focus on a specific labor market friction, the dynamics of the EE rate, quantify its independent effect on inflation, decompose channel through which it affects inflation, and study its normative implications without taking a stance on the slope of the price Phillips curve or inflation expectations.

Our work is most closely related to Moscarini and Postel-Vinay (2019), Faccini and Melosi (2021), and Alves (2019), who focus on the role of the job ladder in inflation dynamics. Seminal work by Moscarini and Postel-Vinay (2019) is the first in this literature to establish the distribution of workers across matches as an important determinant of wage pressures on inflation. Faccini and Melosi (2021) build on their work and highlight the role of variations on the OJS rate in explaining the missing inflation after the Great Recession. Relative to these papers, our model features imperfect insurance against labor market risk, and therefore changes in the job mobility are an important determinant of aggregate demand. Alves (2019) embeds the key insights in Moscarini and Postel-Vinay (2019) in a HANK model and obtains sizable demand side effects from changes in the job mobility. Our work differs from his in three important ways. First, our model features richer labor-market heterogeneity by allowing for differences in human capital as well as match productivity. Second, we not only quantify the total effect of job mobility on inflation but also decompose channels through which a change in job mobility affects inflation, using the DAG representation of the model and relying on the sequence-space Jacobian method. Finally, we study the normative implications of job mobility and calculate the welfare cost associated with ignoring EE dynamics when setting monetary policy.

On the modeling side, several papers bring together elements from search models together with those from New Keynesian models. Ravn and Sterk (2016) develop a tractable New Keynesian model with uninsurable risk and characterize the interactions between unemployment risk, aggregate demand and monetary policy. Gornemann, Kuester, and Nakajima (2021) develop a fully stochastic New Keynesian model with uninsurable idiosyncratic risk and search frictions. We add to this literature by allowing for on-the-job search and heterogeneity across jobs, which turn out to be important for aggregate dynamics.

wFinally, on the computational side, we build on the sequence-space Jacobian method of Auclert, Bardóczy, Rognlie, and Straub (2021). One challenge in adapting this method to our setting is that the endogenous distribution of workers across jobs and human capital levels matters for the free-entry condition. This is in contrast to settings where only scalars (such as aggregate capital and labor) enter equilibrium conditions. We show how their method can be generalized to a multi-stage model with search frictions, where one needs to keep track of worker distributions to ensure market clearing.

Outline. Section 2 presents our model combining the HANK framework with a search model of the labor market. Section 3 discusses how we discipline the model's parameters and Section

4 explains how we solve and simulate the model. Section 5 quantifies the role of job mobility in inflation and Section 6 studies the normative implications of job mobility for monetary policy. Section 7 concludes.

2 Model

We now describe our model combining a New Keynesian framework with heterogeneous agents and a frictional labor market, where both employed and unemployed workers search for jobs.

2.1 Environment

Time is discrete and runs forever. The economy is populated by a measure one of ex-ante identical individuals, firms in three vertically integrated sectors (producing labor services, intermediate goods, and final goods), a mutual fund, a fiscal and a monetary authority.

Firms. Labor firms hire workers in a frictional labor market (to be described below) and produce labor services. These are sold in a competitive market to intermediate firms, who produce differentiated varieties of intermediate inputs using a linear technology with aggregate productivity z. As in the standard New Keynesian model, intermediate goods firms are monopolistically competitive and set prices subject to quadratic adjustment costs and a downward-sloping demand from final goods producers. Final goods firms produce the consumption good by combining the intermediate inputs using a constant elasticity of substitution (CES) technology.

Individuals. An individual's life consists of a working stage and a retirement stage. During their working lives, individuals are heterogeneous in their holdings of mutual fund shares $s \geq 0$, their employment status (employed E or unemployed U), their general human capital (skill) $h \in \mathcal{H} = \{\underline{h}, \ldots, \overline{h}\}$, and—among the employed—in their match-specific productivity $x \in \mathcal{X} \equiv \{\underline{x}, \ldots, \overline{x}\}$ and their piece-rate $\alpha \in (0, 1]$ governing the share of output that they receive as wages. Individuals are born with skill h drawn from distribution Γ^h. During their working lives, they experience stochastic appreciation or depreciation of skills depending on their employment status, as in Ljungqvist and Sargent (1998). In particular, an employed individual's skill increases by Δh percent with probability π^E, while an unemployed individual's skill depreciates by Δh percent with probability π^U in each period. Formally,

$$h' = \begin{cases} h \times (1 + \Delta h) & \text{with probability } \pi^E \\ h & \text{with probability } 1 - \pi^E \end{cases}$$

when employed and,

$$h' = \begin{cases} h \times (1 - \Delta h) & \text{with probability } \pi^U \\ h & \text{with probability } 1 - \pi^U \end{cases}$$

when unemployed. Individuals trade shares of the mutual fund and make consumption decisions (bought at price P_t) in the face of idiosyncratic income risk due to stochastic human capital evolution and frictions in the labor market. Each period, working-age individuals retire with probability ψ^R. Retirees finance consumption through their private savings and from pension income ϕ^R. They die with probability ψ^D, upon which they are replaced with unemployed individuals.[1]

Labor market. The labor market in the service sector is frictional and features random search. Unemployed and employed individuals search for jobs, and their probability of contacting a vacancy depends on their job search efficiency as well as the labor market tightness, θ_t. Upon meeting, the worker-firm pair draws a match-specific productivity x from distribution Γ^x, which remains constant throughout the match. The match operates a production technology given by $F(h, x) = hx$. The individual supplies labor inelastically and is paid real wages according to a predetermined rule $w(h, x, \alpha)$ every period until the termination of the match (described below). The match can dissolve because of an exogenous job separation which occurs at rate δ, retirement, or endogenous job-to-job transitions by the worker. Unemployed individuals receive unemployment insurance (UI) benefits from the government according to the function $UI(h) = \phi^U F(h, \underline{x})$ (denoted in consumption units), where we assume that UI payments are designed as a replacement rate ϕ^U of output that the worker would have received when working at a job with the lowest match productivity $\underline{x}$. On the other side of this labor market, service sector firms pay a per-period fixed cost κ to post vacancies and sell their output to intermediate firms at nominal price P_t^l ($p_t^l = P_t^l/P_t$ in units of the final good).[2]

Wage determination. In each period, the wage paid to an employed worker is an endogenous piece-rate α of the flow output from the worker-firm match. We follow the bargaining protocol in Graber and Lise (2015)—a simplified version of Bagger et al. (2014)—for the determination of α, where firms Bertrand compete based on current flow output (instead of present values).

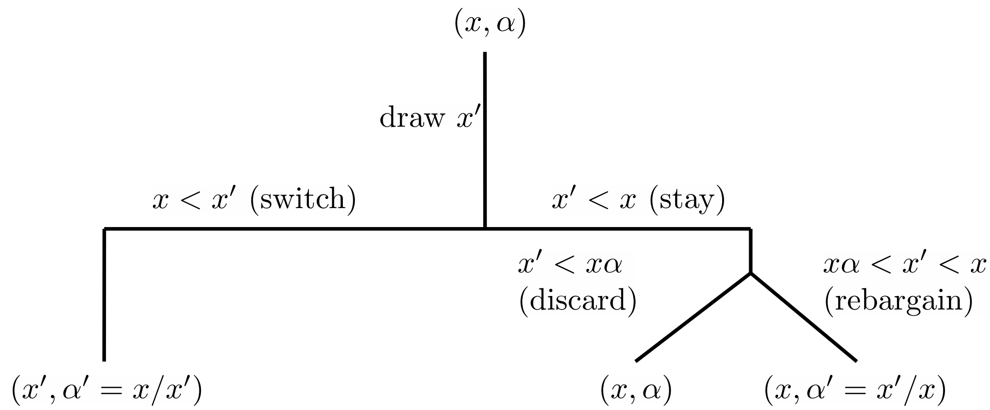
Consider a worker with human capital h employed in a match with productivity x and piece rate α, whose wage is given by $w(h, x, \alpha) = \alpha\phi^E F(h, x)$, where $\phi^E \in (0, 1)$ represents the maximum share of output that a worker with maximum piece rate $\alpha = 1$ can capture as wage.[3] Suppose this worker meets a new firm with a higher productivity $x' > x$, in which case she switches jobs. This is because the most the incumbent firm can offer to the worker is $w(h, x, 1) = \phi^E F(h, x)$. We assume that the new firm is willing to match this wage, i.e., $w(h, x', \alpha') = w(h, x, 1)$, which implies a new piece rate $\alpha' = x/x'$ for this worker. While this

[1]When an individual dies, she is replaced by an offspring who inherits her mutual fund holdings and enters working stage as unemployed with the lowest skill level $\underline{h}$.

[2]Unless otherwise stated, we use uppercase letters to denote nominal variables and lowercase letters for their real counterparts.

[3]This assumption guarantees that whenever $\phi^E < p^l$, the firm's flow profit is greater than zero. As a result, there are no firms with negative surplus.

Figure 1: Wage Determination



new piece rate is $\alpha' < 1$, the worker is better off in switching to the more productive firm with x' given that the piece rate can only become (weakly) larger in the future once a new contact is made with and outside firm with sufficiently high productivity, as discussed below.

Now suppose the same worker receives an offer with a lower productivity $x' < x$, resulting in the worker staying with the incumbent firm. This case induces two scenarios. First, the new productivity x' could be so low that even the maximum potential wage from the new job cannot match the worker's current wage, i.e., $w(h, x', 1) < w(h, x, \alpha)$, which happens when $x' < \alpha x$. In this case the worker simply discards the offer and continues with the same piece rate. Second, x' could be sufficiently high to serve as a credible threat for the worker to bid up her wage with the incumbent firm. This happens when $w(h, x, 1) > w(h, x', 1) > w(h, x, \alpha)$, i.e., $x > x' > \alpha x$, in which case the incumbent firm matches the maximum potential wage from the outside offer, $w(h, x, \alpha') = w(h, x', 1)$, implying an updated piece-rate $\alpha' = x'/x$. Figure 1 summarizes this bargaining protocol.

The piece rate for a worker out of unemployment follows the same logic. We assume that for a new match with productivity x', the piece rate is given by $\alpha' = \underline{x}/x'$. We also assume that all offers out of unemployment are accepted.[4]

Mutual fund. The mutual fund owns all the firms in the economy, as well as all nominal bonds B_t issued by the government, and sells shares in return. The fund pays a nominal dividend D_t per share and can be traded by individuals at price P_t^s.

Fiscal and monetary authorities. The government implements a linear consumption tax τ_c and a progressive income tax. For any gross income level ω, net income is given by $\tau_t \omega^{1-\Upsilon}$, where τ_t captures a potentially time-varying level of taxation and $\Upsilon \geq 0$ captures the rate of

[4] In equilibrium under our baseline calibration, we verify that all new matches out of unemployment indeed have positive surplus, even though there is an opportunity cost of accepting an offer as we ultimately estimate on-the-job search to be less efficient than searching while unemployed. This is because dynamic gains of being employed dominate the option value of waiting for another match with higher productivity.

progressivity built into the tax system, as in Benabou (2002) and Heathcote, Storesletten, and Violante (2014).[5] Together with these taxes, the government issues nominal bonds B_t to finance UI benefits, retirement pensions, and an exogenous stream of nominal expenditures G_t. The central bank sets the short-term nominal interest rate i_t using a reaction function responding to the inflation rate and the unemployment rate.

Timing of events. At the start of each period, (unanticipated) aggregate shocks realize, which we elaborate in subsequent sections. Then, the monetary authority decides on the nominal interest rate, the government sets taxes and government spending, and exogenous retirement, mortality and job destruction shocks realize. Next, worker skills evolve based on the beginning of period employment status and reborn workers replenish the dead starting as unemployed with the lowest skill level. Then, the job search stage opens. Firms post vacancies, and employed and unemployed workers search for jobs. Once new contacts are made, match productivities are observed, new matches are formed, and job-to-job transitions occur. Then in the production stage, each worker-firm pair produces labor services. Intermediate firms produce differentiated goods using these labor services and set their prices subject to nominal rigidities, and final goods are produced using the intermediate goods. Next, intermediate and service firms realize their profits, service firms pay wages to their workers, the mutual fund pays out dividends, and the government collects taxes, issues new bonds, pays out UI and retirement benefits, and spends an exogenous amount. In the final stage of the period, individuals decide on how much to consume and how many shares of the mutual fund to purchase.

2.2 Individuals

We turn to describing in detail the decision problem of individuals. They choose whether to accept a job offer (that are received while employed), how many shares of the mutual fund to buy, and how much to consume subject to a budget constraint and a short-selling constraint for the fund shares. We cast the problems recursively, where time subscripts encode all the relevant aggregate state variables. We now present the problem of unemployed, employed, and retired individuals in turn.

Unemployment. Let $V_t^U(s,h)$ denote the value of unemployed individuals with s shares of the mutual fund and skill h in period t. The problem of the unemployed worker is given by

$$\begin{aligned} V_t^U(s,h) = \max_{s' \geq 0,\, c} \; & u(c) + \beta(1-\psi^R)\mathbb{E}_{h'|h}\left[\Omega_{t+1}^U(s',h')\right] + \beta\psi^R V_{t+1}^R(s') \\ \text{s.t.} \quad & P_t c(1+\tau_c) + P_t^s s' = P_t \tau_t UI(h)^{1-\Upsilon} + (P_t^s + D_t)s, \end{aligned} \tag{1}$$

[5] Note that τ is inversely related to the tax rate. Under a linear schedule with $\Upsilon = 0$, the tax rate is $1-\tau$.

where we express the budget constraint in nominal terms. Here, $\Omega^U_{t+1}(s', h')$ is the value of job search for unemployed workers at the beginning of the next period that we describe below. Unemployed workers receive dividends D_t from the mutual fund in proportion to their share holdings s. They receive real after-tax UI benefits specified by $\tau_t UI(h)^{1-\Upsilon}$ and decide how much to consume and how many shares to buy for the next period subject to the budget constraint.

Employment. Let $V^E_t(s, h, x, \alpha)$ denote the value of employed individuals with s shares, skill h, match productivity x, and piece rate α. The employed individual's problem is given by

$$V^E_t(s, h, x, \alpha) = \max_{s' \geq 0,\, c} u(c) + \beta(1 - \psi^R)\mathbb{E}_{h'|h} \left\{(1 - \delta)\Omega^E_{t+1}(s', h', x, \alpha) + \delta\Omega^U_{t+1}(s', h')\right\} + \beta\psi^R V^R_{t+1}(s')$$
$$\text{s.t.} \quad P_t c(1 + \tau_c) + P^s_t s' = P_t\tau_t w(h, x, \alpha)^{1-\Upsilon} + (P^s_t + D_t)s. \tag{2}$$

Similar to the unemployed, employed individuals collect dividends D_t from the mutual fund, a real after-tax wage of $\tau_t w(h, x, \alpha)^{1-\Upsilon}$, and choose consumption and share holdings before entering the next period. At the beginning of the next period, the job might dissolve exogenously, in which case the worker becomes unemployed and searches for a new job. If not, the worker can engage in on-the-job search, whose value is given by $\Omega^E_{t+1}(s', h', x, \alpha)$.

Retirement. Finally, the value of retirement is given by

$$\begin{aligned} V^R_t(s) &= \max_{s' \geq 0,\, c} u(c) + \beta(1 - \psi^D)V^R_{t+1}(s') \\ \text{s.t.} \quad & P_t c(1 + \tau_c) + P^s_t s' = P_t\tau_t(\phi^R)^{1-\Upsilon} + (P^s_t + D_t)s. \end{aligned} \tag{3}$$

The retirees are only subject to mortality risk and make consumption-saving decisions given their real after-tax pension income $\tau_t(\phi^R)^{1-\Upsilon}$.

Job search problems. Employed and unemployed individuals search for jobs in a frictional labor market with tightness θ_t that we formally define below. Let $f(\theta_t)$ be the workers' aggregate job-finding rate per unit of search effort. The value of job search for an unemployed worker is

$$\Omega^U_t(s, h) = \zeta f(\theta_t)\mathbb{E}_x V^E_t(s, h, x, \underline{x}/x) + (1 - \zeta f(\theta_t)) V^U_t(s, h), \tag{4}$$

where ζ is the job search efficiency among unemployed workers. On-the-job search value is

$$\begin{aligned} \Omega^E_t(s, h, x, \alpha) = {} & \nu f(\theta_t)\mathbb{E}_{\widetilde{x}}\left[\max\left\{V^E_t(s, h, \widetilde{x}, x/\widetilde{x}), V^E_t(s, h, x, \max\{\alpha, \widetilde{x}/x\})\right\}\right] \\ & + (1 - \nu f(\theta_t)) V^E_t(s, h, x, \alpha), \end{aligned} \tag{5}$$

where ν is the search efficiency of the employed. Upon contact, the worker-firm pair draws match productivity x and the expectations are taken with respect to the sampling distribution Γ^x. The first term inside the expectation represents the worker's value when she switches to a new job with match productivity $\widetilde{x}$ and new piece rate $\alpha' = x/\widetilde{x}$. The second term represents the worker's value of staying with the incumbent firm, either with current piece rate α (if $\widetilde{x} < \alpha x$) or a higher piece rate $\widetilde{x}/x$ (if $\widetilde{x} > \alpha x$).

2.3 Production

The economy has three sectors that we now describe in more detail: final goods, intermediate goods, and labor services.

Final goods. The final-good producer purchases differentiated intermediate goods $y_t(j)$ at relative price $p_t(j) = P_t(j)/P_t$ and produces the final consumption good Y_t using the technology:

$$Y_t = \left(\int y_t(j)^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}, \tag{6}$$

where η is the elasticity of substitution between varieties, and solves the following profit maximization problem:

$$\max_{\{y_t(j)\}} \; Y_t - \int p_t(j) y_t(j) dj. \tag{7}$$

This problem determines the demand for each intermediate good, $y_t(j) = p_t(j)^{-\eta} Y_t$ as a function of the relative price of variety $p_t(j)$ and aggregate demand conditions Y_t, and implies an ideal price index satisfying $1 = \left(\int p_t(j)^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$ that the intermediate-goods firms take as given.

Intermediate goods. Intermediate firms produce $y_t(j)$ using a linear technology with labor services as the only input: $y_t(j) = z_t l_t(j)$, where z_t is the aggregate productivity. They set the price for their differentiated good taking into account the demand from the final-good producer and price adjustment costs à la Rotemberg (1982). Pricing frictions render the last period's *relative* price $p_{t-1}(j)$ a state variable for the intermediate goods producers. They solve the following profit maximization problem:

$$\Theta(p_{t-1}(j)) = \max_{p_t(j)} \; p_t(j) y_t(p_t(j)) - p_t^l \frac{y_t(p_t(j))}{z_t} - Q(p_{t-1}(j), p_t(j)) Y_t + \frac{1}{1 + r_{t+1}} \Theta(p_t(j)). \tag{8}$$

Price adjustment costs are given by

$$Q(p_{t-1}(j), p_t(j)) = \frac{\eta}{2\vartheta} \log \left(\frac{p_t(j)}{p_{t-1}(j)} (1 + \pi_t) - \pi^* \right)^2,$$

where π^* is the inflation target of the monetary authority. In Appendix A.1, we show that this profit maximization problem implies the following New Keynesian Phillips curve (NKPC):

$$\frac{\log(1 + \pi_t - \pi^*)(1 + \pi_t)}{1 + \pi_t - \pi^*} = \vartheta \left(\frac{p_t^l}{z_t} - \frac{\eta - 1}{\eta} \right) + \frac{1}{1 + r_{t+1}} \frac{\log(1 + \pi_{t+1} - \pi^*)(1 + \pi_{t+1})}{1 + \pi_{t+1} - \pi^*} \frac{Y_{t+1}}{Y_t}, \tag{9}$$

where $\pi_{t+1} = P_{t+1}/P_t - 1$ is the inflation rate between periods t and $t+1$, and $mc_t = p_t^l/z_t$ is the real marginal cost of production.

Labor services. A continuum of service-sector firms post vacancies incurring a cost of κ per vacancy. Labor market tightness, θ_t, is defined as the ratio of vacancies v_t to the aggregate measure of job search effort by both unemployed and employed workers $S_t = \zeta \int d\mu_t^U(s,h) + \nu \int d\mu_t^E(s,h,x,\alpha)$, where μ^U and μ^E are distributions of unemployed and employed workers over their relevant states at the search stage within a period, respectively. Let $M(v,S)$ be a constant-returns-to-scale (CRS) matching function that determines the number of worker-firm matches as a function of vacancies and search effort. We can then define $q(v,S) = \frac{M(v,S)}{v} = M\left(1, \frac{1}{\theta}\right)$ to be the firm's contact rate and $f(v,S) = \frac{M(v,S)}{S} = M(\theta, 1)$ to be the worker's contact rate per unit search effort, where the CRS assumption implies that θ is sufficient to determine these rates.

We now turn to the problem of the service firms, which mirror those of the workers. Consider a firm that employs a worker with skill level h and piece rate α in a match with productivity x. The worker-firm pair produces labor services according to the production technology $F(h,x)$. The output is then sold to intermediate goods producers at real price p_t^l. Let $J_t(h,x,\alpha)$ denote the real value of this firm given by

$$J_t(h,x,\alpha) = p_t^l F(h,x) - w(h,x,\alpha) + \frac{1}{1+r_{t+1}}(1-\psi^R)(1-\delta) \\ \times \mathbb{E}_{h'|h}\Big\{ (1-\nu f(\theta_{t+1}))\, J_{t+1}(h',x,\alpha) + \nu f(\theta_{t+1}) \int_{\underline{x}}^{x} J\left(h', x, \max\{\alpha, \widetilde{x}/x\}\right) d\Gamma^x(\widetilde{x}) \Big\}, \tag{10}$$

where the match survives if the worker does not retire, does not exogenously separate into unemployment, and does not find a new job through on-the-job search. As discussed above, the worker accepts the new job offer if $\widetilde{x} > x$, in which case the firm's value is 0. If the new match quality $\widetilde{x}$ is below current x, then the firm keeps the worker either at a higher piece rate $\widetilde{x}/x$ (if $\widetilde{x} > \alpha x$) or at the current piece rate α (if $\widetilde{x} < \alpha x$). As firms are risk-neutral, they discount the future at the real interest rate r_{t+1} that we discuss below.

The real value of a firm posting a vacancy is

$$V_t = -\kappa + q(\theta_t)\frac{1}{S_t}\Big[\zeta \int_{s,h}\int_{\widetilde{x}} J_t\left(h, \widetilde{x}, \underline{x}/\widetilde{x}\right) d\Gamma^x(\widetilde{x})\, d\mu_t^U(s,h) \\ + \nu \int_{s,h,x,\alpha}\int_{x}^{\overline{x}} J_t\left(h,\widetilde{x}, x/\widetilde{x}\right) d\Gamma^x(\widetilde{x})\, d\mu_t^E(s,h,x,\alpha)\Big], \tag{11}$$

where the first term captures the value of filling a vacancy with workers originating from unemployment and the second term captures workers the firm can poach from other firms.

A free-entry condition implies that profits are just enough to cover the cost of filling a vacancy κ in expectation. Thus, we have $V_t = 0$, which together with Equation (11), pins down equilibrium market tightness θ.

Mutual fund. The mutual fund issues shares to raise funds and owns the intermediate and labor service firms, and all government bonds in the economy. The fund can issue shares at price P^s and short government bonds to earn a gross return of $1+i$. No arbitrage implies that the rate of return on shares must equal the rate of return on government bonds:

$$\frac{P^s_{t+1} + D_{t+1}}{P^s_t} = 1 + i_t. \tag{12}$$

The mutual fund is not allowed to retain any funds. All balances (positive or negative) are distributed to share owners in the form of dividends given by

$$D_t = B_{t-1} - \frac{B_t}{1+i_t} + P_t\Gamma^I_t + P_t\Gamma^S_t, \tag{13}$$

where the aggregate per-period real profits of intermediate and service firms are given by[6]

$$\Gamma^I_t = \left(1 - \frac{p^l_t}{z_t} - \frac{\eta}{2\vartheta}\log(1+\pi_t-\pi^*)^2\right) Y_t, \tag{14}$$

and

$$\Gamma^S_t = \int \left(p^l_t F(h,x) - w(h,x,\alpha)\right) d\lambda^E_t(s,h,x,\alpha). \tag{15}$$

Here, $\lambda^E_t(s,h,x,\alpha)$ is the distribution of employed workers at the consumption stage, i.e., at the end of the period. Equation (13) implies that the mutual fund collects payments for the existing debt obligations B_{t-1}, profits of intermediate firms Γ^I_t, profits of service firms Γ^S_t and finances all the new debt purchases B_t. The remaining balance accrues to the individuals as dividends in proportion to their shareholdings.

Fiscal authority. The fiscal authority taxes individuals and issues bonds to finance an exogenous stream of expenditures G_t as well as UI benefits and retirement pensions. The government budget constraint is given by

$$\begin{aligned} B_{t-1} + G_t + P_t\int UI(h)d\lambda^U_t(s,h) + P_t\int \phi^R d\lambda^R_t(s) = {} & \frac{B_t}{1+i_t} + P_t\tau_c\int c(s,h,x,\alpha)d\lambda_t(s,h,x,\alpha) \\ & + P_t\int \left(UI(h) - \tau_t UI(h)^{1-\Upsilon}\right) d\lambda^U_t(s,h) \\ & + P_t\int \left(w(h,x,\alpha) - \tau_t w(h,x,\alpha)^{1-\Upsilon}\right) d\lambda^E_t(s,h,x,\alpha) \\ & + P_t\int \left(\phi^R - \tau_t(\phi^R)^{1-\Upsilon}\right) d\lambda^R_t(s), \end{aligned} \tag{16}$$

[6] We assume that vacancy creation costs are psychic in that they do not consume real resources and hence do not show up in the profits of service-sector firms.

where the left hand side is total government expenses and the right hand side is the total government revenues generated from issuing bonds and consumption and income taxation, respectively. Here, $\lambda_t(s,h,x,\alpha)$, $\lambda_t^E(s,h,x,\alpha)$ and $\lambda_t^U(s,h)$, $\lambda_t^R(s)$ are the distributions of all, employed, unemployed, and retired individuals, respectively, with given (relevant) state variables as of the consumption stage, i.e., at the end of the period.

Monetary authority. A monetary authority controls the short-term nominal interest rate and we assume that this nominal rate i_t is set according to the following reaction function

$$i_t = i^* + \Phi_\pi\left(\pi_t - \pi^*\right) + \Phi_u\left(u_t - u^*\right). \tag{17}$$

Here, i^* denotes the steady-state nominal interest rate, Φ_π governs the responsiveness of the central bank to deviations from its inflation target, and Φ_u controls how much the central bank responds to deviations of the unemployment rate from its steady state value.

Finally, real interest rate, r_t, satisfies the Fisher equation

$$1 + i_t = (1 + \pi_{t+1})(1 + r_{t+1}). \tag{18}$$

Timing conventions for these variables are as follows: The nominal interest rate i_t is indexed to the period in which it is set, and is the interest rate that applies between periods t and $t+1$. The inflation rate is denoted by the period in which it is measured, i.e., π_{t+1} is the realized inflation between periods t and $t+1$. The real rate has the same timing convention as inflation: r_{t+1} is the ex-post realized real interest rate from t to $t+1$.

2.4 Equilibrium

In this section, we present the conditions that characterize the equilibrium of our model.

Market clearing requires that labor services demanded by intermediate firms Y_t/z_t is equal to the aggregate supply of labor services and mutual fund shares demanded by all individuals aggregate to one. Formally, these conditions are given by:

$$Y_t/z_t = \int F\left(h,x\right) d\lambda_t^E(s,h,x,\alpha), \tag{19}$$

$$1 = \int g_t^{Us}(s,h) d\lambda_t^U(s,h) + \int g_t^{Es}(s,h,x,\alpha) d\lambda_t^E(s,h,x,\alpha) + \int g_t^{Rs}(s) d\lambda_t^R(s), \tag{20}$$

where g_t^{es} denotes the saving decision of workers with employment status $e \in \{E, U, R\}$.

Definition of equilibrium. Given fiscal policy instruments that determine UI replacement rate ϕ^U, retirement transfers ϕ^R, tax parameters $\{\tau_c, \tau_t, \Upsilon\}$, and government spending G_t, monetary policy rule in Equation (17), and paths of exogenous shocks to discount factor β_t, on-the-job-search efficiency ν_t, and productivity z_t, an equilibrium of the model is a sequence of individual decision rules for consumption $g_t^{Ec}, g_t^{Uc}, g_t^{Rc}$ and mutual fund share demand $g_t^{Es}, g_t^{Us}, g_t^{Rs}$, intermediate and service firm profits Γ_t^I and Γ_t^S, dividends D_t, unit labor cost p_t^l, share price P_t^s, labor market tightness θ_t, interest rates r_t, i_t and bond holdings B_t, and worker distributions $\left\{\lambda_t^E, \lambda_t^U, \lambda_t^R\right\}$ such that

- Given the path of inflation π_t, the nominal and real interest rates satisfy the Taylor rule (17) and the Fisher equation (18).
- Intermediate and service firm profits satisfy Equations (14) and (15), respectively.
- Share prices satisfy Equation (12) and dividends are given by Equation (13).
- Bonds are such that the government budget constraint in Equation (16) holds every period.
- Individual decisions g_t^{Ec}, g_t^{Uc}, g_t^{Rc}, g_t^{Es}, g_t^{Us} and g_t^{Rs} are optimal.
- θ_t is such that value of posting a vacancy expressed in Equation (11) is zero.
- Unit labor costs p_t^l satisfy the Philips curve in Equation (9).
- The labor and shares markets clear as specified in Equations (19) and (20).[7]
- The worker distribution evolves according to the laws of motion in Appendix A.2.1.

The stationary equilibrium of the model is obtained by setting all exogenous shocks to zero. In steady state, we assume that tax parameter τ^* clears the government budget constraint, and that outstanding bonds and government expenditures are a fraction of output $B^* = x_B Y^*$ and $G^* = x_G Y^*$, respectively. We provide details on the computation of the economy's transitional dynamics in Section 4 and further computational details in Appendices A.2 and A.3.

3 Calibration

In this section we discuss how we discipline our model. We assume the economy is in steady state and calibrate the model to match several targets of the U.S. economy prior to the Great Recession, specifically, over the period 2004–2006. Our model period is a quarter. We first discuss the parameters that are set outside the model and then explain how we discipline the remaining ones using our model.

[7] We do not check for goods market clearing due to Walras's Law.

Table 1: Externally calibrated parameters

Parameter	Explanation	Value	Reason
σ	Curvature in utility function	2	Standard
ψ^R	Retirement probability	0.00625	40 years of work stage
ψ^D	Death probability	0.0125	20 years of retirement stage
Δh	Skill appreciation/depreciation amount	0.275	Set
π^E	Skill appreciation probability	0.018	Wage growth for job stayers
ξ	Matching function elasticity	1.6	Set
ζ	Search efficiency of the unemployed	1	Normalization
η	Elasticity of substitution	6	20 percent markup
ϑ	Price adjustment cost parameter	0.10	Slope of Phillips curve
x_G	Government spending/GDP ratio	0.19	Total net federal outlay/ GDP
x_B	Debt/GDP ratio	2.43	Total public debt/GDP
τ_c	Consumption tax rate	0.0312	Sales tax receipt/consumption exp.
Υ	Progressivity of income tax	0.151	Heathcote et al. (2014)
ρ_τ	Responsiveness of income tax parameter to debt level	0.10	Auclert et al. (2020)
π^*	Steady-state inflation rate	0.00496	2% annual inflation rate
Φ_π	Responsiveness of interest rate to deviations from inflation target	1.5	Taylor (1993) and Gali (2015)
Φ_u	Responsiveness of interest rate to deviations from unemployment target	-0.25	Taylor (1993) and Gali (2015)

Notes: This table summarizes externally calibrated parameters. See the main text for a detailed discussion.

Functional forms and externally calibrated parameters. Table 1 summarizes the externally calibrated parameters. The utility function over consumption is of the CRRA form with $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. As is standard in the literature, we set the risk aversion parameter to $\sigma = 2$. As for the life cycle, workers spend 40 years in the labor force and 20 years in retirement in expectation, which require setting $\psi^R = 0.625\%$ and $\psi^D = 1.25\%$ on a quarterly basis.

Turning to the evolution of worker productivity, we use five equally-spaced (in logs) grid points between the lowest value $\underline{h} = 1$ and the highest value $\overline{h} = 3$ for human capital. These choices imply that worker skills change by a proportion $\Delta h = (ln(3) - ln(1))/4 = 0.275$ between grid points when they appreciate while working or depreciate during unemployment. We discipline the probability of skill appreciation for the employed π^E by the annual wage growth of job stayers. Karahan, Ozkan, and Song (2022) document that this is around 2% for a large share of the U.S. population, which implies that expected quarterly wage growth of job stayers should be around 0.5%, which requires setting $\pi^E = 0.005/0.275 \approx 0.018$. We further assume that the match-specific productivity x is drawn from a log-normal distribution with standard deviation σ_x (to be discussed below). We discretize this process with 7 equally-spaced grid points (in logs) between the 1st and 99th percentiles of the log-normal distribution.

Following Menzio and Shi (2011) and Schaal (2017), we pick a CES matching function so that the worker and firm contact rates are given by $f(\theta) = \theta(1 + \theta^\xi)^{-1/\xi}$ and $q(\theta) = (1 + \theta^\xi)^{-1/\xi}$, respectively. Here, ξ controls the elasticity of contact rates with respect to market tightness, and we choose $\xi = 1.6$ following Schaal (2017). We also normalize the search efficiency of unemployed workers to $\zeta = 1$.

The elasticity of substitution across intermediate goods varieties η controls the markup of prices over the marginal cost—and therefore the profit share—at the steady state. We set this parameter to 6 so as to obtain a profit share of $\eta/(\eta - 1) = 20\%$ (Auclert, Bardóczy, Rognlie, and Straub, 2021; Faccini and Melosi, 2021). Without loss of generality, we normalize the productivity of the intermediate sector to $z = 1$ at the steady state. Finally, we set the price adjustment cost parameter ϑ to 0.1. As Equation (9) shows, this parameter directly dictates the slope of the Phillips curve. A slope of 0.1 is the same as in Kaplan, Moll, and Violante (2018) and falls within the range of estimates in the literature such as in Smets and Wouters (2007); Christiano, Eichenbaum, and Trabandt (2016); Auclert, Rognlie, and Straub (2020).

Given that Ricardian equivalence does not hold in our model, fiscal policy matters for how the economy responds to shocks. We assume that government transfers are a fixed share of output, $G_t/Y_t = x_G$. Over the period 2004-2006, the ratio of government spending to GDP was around 19 percent, so we set $x_G = 0.19$. We calibrate the model to have a realistic amount of government debt. In the data, the ratio of debt to annual GDP averages to 60.8% over the same period. The quarterly frequency in the model dictates us to set this ratio to $B_t/Y_t = x_B = 4 \times 0.608 = 2.43$. We set the consumption tax rate to $\tau_c = 3.02\%$, which we obtain as the ratio of state and local sales tax receipts to personal consumption expenditures in the data for 2006. There are two parameters related to labor income taxes, one governing the average level of taxes, τ and the other one governing its progressivity, Υ. We follow Heathcote, Storesletten, and Violante (2014), and set Υ exogenously to 0.151. We explain below how we calibrate τ jointly with other parameters to match a set of targets.

As we discussed above, the government uses debt to balance its budget. Along a transition path—off the steady state that we discuss below—the level of debt can go above or below its steady state level of $x_B Y$. In these cases, the fiscal authority follows an exogenous rule that adjusts the level parameter of income taxes τ to eventually bring the level of real debt back to its steady state value. This response function is given by

$$\tau_t = \tau^* - \rho_\tau \left(b_{t-1} - b^*\right)/Y^*. \tag{21}$$

Here, τ^* is the steady state value of τ, which is inversely related to the level of income taxes. The second term in Equation (21) controls how strongly fiscal policy reacts to deviations of debt-to-GDP from its steady state value. A higher value for ρ_τ indicates that taxes go up more when debt-to-GDP rises above the steady state. Following Auclert, Rognlie, and Straub (2020), we set $\rho_\tau = 0.1$.

Turning to monetary policy, the central bank targets an annual inflation rate of 2%. Quarterly calibration requires us to set $\pi^* = 1.02^{1/4} - 1 \approx 0.496\%$. In disciplining the Taylor rule, we follow

Taylor (1993) and Galí (2015). We set the coefficient on inflation to $\Phi_\pi = 1.5$ as in their work. One notable difference of our specification is that our Taylor rule reacts to the unemployment gap rather than to the output gap. Galí (2015) sets the the coefficient on the output gap in a quarterly model as 0.125. To map their coefficient on the output gap to the unemployment gap, we use Okun's law with a coefficient of -2, as in Okun (1962). This implies setting $\Phi_u = -0.25$.

Internal calibration. The remaining nine parameters are the discount factor β, vacancy creation cost κ, job separation probability δ, job search efficiency of the employed ν, skill depreciation probability when unemployed π^U, standard deviation parameter of match specific productivity distribution σ_x, maximum share of output potentially paid to worker as wages ϕ^E, UI replacement rate ϕ^U, and retirement benefit amount ϕ^R. These parameters are calibrated internally by matching a set of data moments that we now describe. Specifically, we use the simulated method of moments where we minimize the sum of squared percentage deviations of the model moments from their empirical counterparts. Table 2 summarizes the targeted moments and the calibrated parameter values. While all parameters are jointly calibrated, Table 2 presents each parameter next to the target its mostly informative about.

Given the recent work highlighting the role of the asset distribution in the transmission of monetary policy, we target the fraction of hand-to-mouth (HtM) households in the labor force to discipline discount factor β. We define HtM households as those with non-positive liquid wealth holdings. We use the 2004 panel of the Survey of Income and Program Participation (SIPP) and work with a sample of individuals aged 25–65, who do not own any business. 16 percent of our sample are HtM households according to our classification.

On the labor market side, we target a steady state unemployment rate of 5.1%, as well as worker flows. We obtain the targets for the flow rates from various sources. Using data from the Current Population Survey (CPS), we compute the average monthly employment-to-unemployment separation rate over the period 2004-2006. We convert this monthly job loss rate to a quarterly frequency and obtain our target of 3.8%. To compute the job-to-job transition rate, we make use of quarterly data from the Longitudinal Employer-Household Dynamics (LEHD). We find that the job-to-job transition rate (or EE rate), measured as the job switching rate of workers who do not have any intervening nonemployment spell, is around 2% over the same period. These moments are informative about the vacancy creation cost κ, job separation rate δ, and employed search efficiency ν, respectively.

The probability of skill depreciation when unemployed π^U is informative about the magnitude of earnings loss upon job displacement. Getting this moment right is not only important to discipline skill depreciation but also to get at the cost of job loss and the welfare effects of stimulating the economy. A large literature has estimated the magnitude of earnings losses upon job displacement using a variety of datasets and approaches (see, for example, Jacobson, LaLonde, and Sullivan, 1993; Stevens, 1997; Davis and von Wachter, 2011; Jarosch, 2021; Birinci,

Table 2: Internally calibrated parameters

Parameter	Explanation	Value	Target	Data	Model
β	Discount factor	0.981	Fraction of individuals with non-positive liquid wealth	0.16	0.11
κ	Vacancy creation cost	0.670	Unemployment rate	0.051	0.052
δ	Job separation probability	0.091	EU rate	0.038	0.033
ν	Search efficiency of employed	0.108	EE rate	0.02	0.02
π^U	Skill depreciation probability	0.022	Earnings drop upon job loss	-0.35	-0.36
σ_x	Standard deviation parameter of match productivity distribution	0.063	Wage growth of job switchers	0.09	0.09
ϕ^E	Maximum share of output as wages	0.823	Labor share	0.67	0.74
ϕ^U	UI replacement rate	0.385	UI replacement rate	0.40	0.44
ϕ^R	Retirement benefit amount	0.473	Retirement income/labor income	0.34	0.41

Notes: This table summarizes internally calibrated parameters. See the main text for a detailed discussion.

2021, among others). Across these studies, the median estimate of the earnings loss in the year of job displacement is about 35%. To facilitate comparison with the literature, we generate a simulated panel of households in the model, aggregate quarterly simulations to an annual frequency, and estimate a distributed-lag regression on these model-generated data, analogously with empirical studies.

Another important aspect of the model is what happens to wages when workers change employers. This feature disciplines how important job-to-job transitions are for aggregate demand in the economy. Using the LEHD, we calculate the change in earnings for continuously employed workers upon a job change, which we find to be around 9%. This moment is informative about the dispersion parameter for match productivity σ_x, which governs the increase in wages upon a job-to-job switch in the model. Given σ_x, we pick the mean parameter of the match productivity distribution $\mu_x = -\sigma_x^2/2$ so that the mean of the distribution is normalized to one. Finally, we choose the maximum share of output that is paid to workers as wages ϕ^E to target an average labor share of 0.67.

Turning to the generosity of government programs, we calibrate the UI replacement rate ϕ^U to match an average replacement rate of 40%. To discipline pension benefits during retirement ϕ^R, we calculate the average retirement income to labor income ratio in the SIPP. Specifically, we add up Social Security Income and pension incomes from federal, state, and local governments for the sample of retirees and compute a per-person retirement income as an average of this measure in our sample. We then divide it by the average labor income among nonretirees to obtain a ratio of 0.34 in the data.

4 Solving for transitional dynamics

In this section, we discuss our methodology to solve the model's transitional dynamics upon a shock with further details relegated to Appendix A.3.

We assume that the economy is in steady state at time $t = 0$ and people expect it to remain that way. Entering period $t = 1$, they observe an unexpected and transitory shock to the economy (e.g. productivity, discount rate, and labor market shocks). Because the shock is transitory, the economy returns to the same real allocations but potentially with different nominal price levels. We conjecture the transition is completed by period $t = T$ for some large enough T.

We use the sequence space Jacobian method developed by Auclert, Bardóczy, Rognlie, and Straub (2021), which allows us to efficiently solve for the impulse responses to shocks. To apply this method, we first recast key model equations in terms of real variables and relative prices so that the terminal value of all variables following a shock attain their initial steady state values.[8] We then cast the model as a directed acyclical graph (DAG), presented in Figure A.1, which expresses the model as various nodes and how they relate to one another. The nodes in the DAG can be classified into three groups: the initial node that contains potential exogenous shocks to the economy as well endogenous variables to be solved for, the intermediate (green) nodes that represent blocks that contain the model's various components (such as the conduct of monetary policy via the Taylor rule, fiscal policy via the tax rule, or the heterogeneous agent household problem), and the terminal nodes that represent equilibrium conditions. Importantly, the DAG relates each node by specifying variables which are used as inputs to and generated as outputs from these nodes. At each node, we calculate partial Jacobians of each output with respect to each input. We then forward accumulate these partial Jacobians along a topological sort of the DAG and use the implicit function theorem to obtain the general equilibrium Jacobians of the model. These general equilibrium Jacobians can in turn can be used to compute the response of any endogenous variable to any exogenous shock. Furthermore, using the equivalence of the impulse response function and the moving average representation of the process generating that variable, we simulate the time path of aggregate variables as well as a large panel of individuals to obtain a rich set of aggregate and cross-sectional moments of the model under aggregate shocks.

Relative to a standard shooting algorithm to obtain general equilibrium impulse responses to a shock, the method of Auclert, Bardóczy, Rognlie, and Straub (2021) provides major computational efficiency improvements along two dimensions. The first improvement allows for the computation of policy function responses to any shock that may hit at any period by a single backward value function iteration. The second improvement introduces a fake-news algorithm to offer an efficient method of forward iteration of equilibrium distributions in a model with rich heterogeneity. We closely follow Auclert et al. (2021) to implement both of these improvements when solving for transitional dynamics in our model.

Importantly, we generalize the sequence-space Jacobian method and allow for model blocks to interact not only via aggregate variables but also through the discretized distribution of indi-

[8]Note that we assume a trend inflation of 2% per year, and hence the nominal variables in the initial and terminal steady states are not necessarily the same.

viduals across state variables. This modification is crucial for our application because outcomes of the heterogeneous-agent (HA) block in the DAG include various distributions of employed and unemployed individuals, which are required as inputs for the labor-service firm and other equilibrium conditions. First, the distribution of employed individuals across human capital and match productivity levels and the distribution of unemployed individuals across human capital at the job search stage within a period, i.e., $\mu^E(h,x)$ and $\mu^U(h)$, respectively, affect the expected value from a match $\mathbb{E}J$ for firms deciding on vacancy creation. This is because (1) human capital affects the magnitude of output in a match and (2) employed workers' match productivity with their current employer affects their job acceptance decision and the piece rate that the poaching firm would offer to the worker (and thus their wage level) upon a new match. Second, the distribution of employed workers across human capital, match productivity, and piece rate levels at the consumption/production stage in a period, $\lambda^E(h,x,\alpha)$, affect service-firm profits Γ^S by determining the output and wage levels in a match, which in turn affect dividend payments as all profits are collected by the mutual fund and are distributed back to households in proportion to their share holdings.

To summarize, we generalize the sequence-space Jacobian method and incorporate discretized distributions across state variables as inputs and outputs along the DAG to solve our model which combines a New Keynesian framework with heterogeneous agents and a frictional labor market featuring on-the-job search.

5 Positive implications of job mobility on inflation

In this section, we use our calibrated model to understand how macroeconomic outcomes respond to changes in worker mobility between employers, which we parsimoniously capture by shocks to the OJS efficiency parameter ν. To motivate our exercise, we start by presenting empirical evidence showing a significant weakening of the negative correlation between the employment-to-employment (EE) and unemployment rates during the recovery following the Great Recession. We then use our model to simulate this time period with a declining unemployment rate but constant EE rate and compare this model economy with a counterfactual economy where unemployment follows the exact same dynamics but the EE rate is left untargeted. The former economy represents a breakdown in the historical relationship between the EE rate and the unemployment rate while the later preserves their co-movement. Importantly, we show that a negative OJS efficiency shock that keeps EE transitions suppressed causes a drag on inflation without a change in aggregate output, providing an explanation to the "missing inflation" puzzle debated in this time period. Finally, we use the model's DAG representation to quantify the contribution of the different channels through which OJS shocks affect inflation dynamics.

5.1 The missing correlation between EE and unemployment rates post-Great Recession

We start by documenting the historical relationship between EE and unemployment rates. To do so, we use monthly data on the EE rate constructed by Fujita, Moscarini, and Postel-Vinay (2020) using the Current Population Survey (CPS) and the unemployment rate from the U.S. Bureau of Labor Statistics (BLS).

Panel (a) of Figure 2 presents a scatter plot of monthly EE rate and unemployment rate across different episodes: prior to the Great Recession (1995-2007), during the Great Recession and the subsequent recovery (2008-2015), post-Great Recession (2016-2019), and the Covid-19 period (2020-2022). We document that the raw correlation between the two series is usually significant and negative, except for the post-Great Recession episode when this correlation turned insignificant and positive. Presenting a more continuous view of time, Panel (b) plots the rolling correlation between the cyclical components of the log unemployment and EE rates using a five-year window where both time series are detrended using the HP filter with a smoothing parameter of 10^5. We show that, historically, there exists a strong negative correlation between the cyclical components of both series. This historical relationship weakened significantly between 2016 and 2019. These patterns can be further seen in Appendix Figure B.1 where between 2016 and 2019, the unemployment rate declined around 30 percent below its trend while the EE rate remained unchanged. This is a clear departure from the historical negative co-movement between the two variables.

Our subsequent exercises are motivated by this missing correlation between EE rate and unemployment rate. This episode presents a useful case study that can be used to differentiate real and nominal macroeconomic outcomes between an economy where this correlation has disappeared and in another where the correlation is preserved.

5.2 Inflation dynamics during an episode of declining unemployment

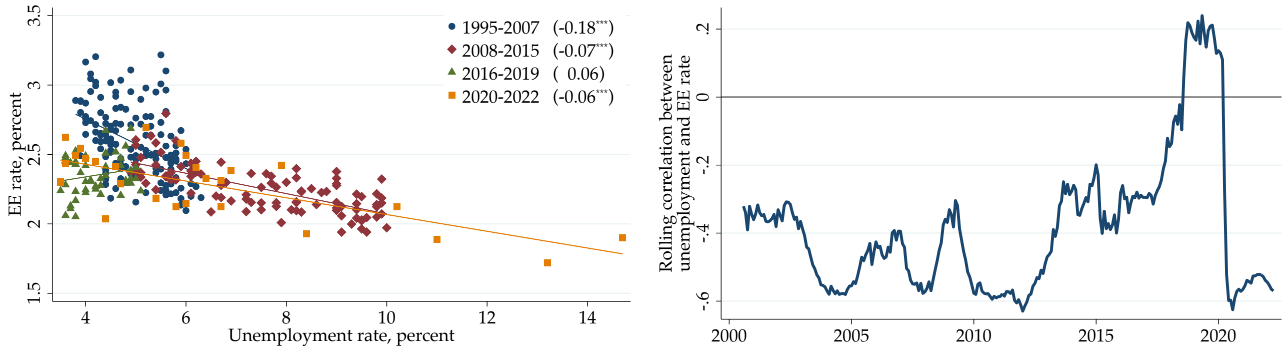
We interpret the period between 2016 and 2019 as a time when the economy was hit by a series of positive demand and negative OJS efficiency shocks, which prevented the EE rate from increasing despite a declining unemployment rate in the context of a tightening labor market. We consider two scenarios starting from the steady state described in Section 3, to study the implications of such shocks on the economy.

First, we mimic an economy undergoing a period of declining unemployment rate through a series of positive demand shocks. We estimate the path of these shocks such that the unemployment rate declines by 15% relative to its steady state of 5.2%.[9] Given that the unemployment

[9]This is consistent with the decline in unemployment rate attributable to an increase in the job-finding rate observed between 2016 and 2019. In particular, holding the separation rate fixed at its January 2016 level, the rise in the job-finding rate between 2016 and 2019 alone leads to a 15% decline in the unemployment rate.

Figure 2: Correlation between EE rate and unemployment rate over time

(a) Unemployment vs EE rate (b) Rolling correlation between unemployment and EE



Notes: Panel (a) presents a scatter plot of monthly EE rate and unemployment rate across different episodes: prior to the Great Recession (1995–2007), during the Great Recession and the subsequent recovery (2008–2015), post-Great Recession (2016–2019), and the Covid-19 episode (2020–2022). Values in parenthesis report the coefficient from regressing the EE rate on the unemployment rate and *** denotes significance at the 1 percent level. Panel (b) presents the rolling correlation between the cyclical components of the logs of unemployment and EE rates using a five-year window. Both time series are detrended using the HP filter with a smoothing parameter of 10^5. Source: BLS and Fujita, Moscarini, and Postel-Vinay (2020).

rate steadily declined from its trend over 16 quarters in the data, we assume that the decline in unemployment rate is linear and is completed within $\overline{T} = 16$ quarters from the onset of the first shock. Upon reaching its trough, the unemployment rate is assumed to revert back to its steady state in accordance with the following law of motion for $t > \overline{T}$:

$$u_t = (1 - \rho_u)u^* + \rho_u u_{t-1},$$

where $\rho_u = 0.85$ governs the speed of mean reversion in the unemployment rate.

We model demand shocks as innovations to the discount factor β following an AR(1) process:

$$\beta_t = (1 - \rho_\beta)\beta^* + \rho_\beta \beta_{t-1} + \varepsilon_{\beta,t}. \tag{22}$$

Here, we estimate the time path of *unexpected* demand innovations $\varepsilon_{\beta,t}$ to exactly match the path of the unemployment rate we posit above.[10]

The second scenario is a modification of the first one, where we consider an economy subject to both positive demand shocks and negative OJS efficiency shocks. This case generates the exact same path of unemployment rate as above and a constant EE rate as in the post-Great

[10] In Section 6.1, we jointly estimate the parameters of the AR(1) processes governing discount factor β, productivity z, and OJS efficiency ν to match empirical moments of the unemployment rate, average labor productivity, and the EE rate. Foreshadowing our estimation exercise, in Equations (22) and (23), we use the estimated persistence parameters ρ_β and ρ_ν for these AR(1) processes.

Recession period. To operationalize this exercise, we assume that the OJS efficiency parameter ν also follows an AR(1) process given by:

$$\nu_t = (1 - \rho_\nu)\nu^* + \rho_\nu \nu_{t-1} + \varepsilon_{\nu,t}. \tag{23}$$

Similar to the first exercise, we jointly estimate a series for $\varepsilon_{\beta,t}$ and $\varepsilon_{\nu,t}$ such that the unemployment rate exactly mimics the path described in the first scenario and, in addition, the EE rate remains unchanged.

We note that in the first exercise where we assume no OJS efficiency shocks, the EE rate rises endogenously as unemployment declines. In this sense, the model is able to generate the co-movement of unemployment and job-to-job transitions using only demand shocks. The second exercise breaks this negative co-movement between unemployment and EE rates by introducing negative shocks to ν such that the EE rate remains unchanged.

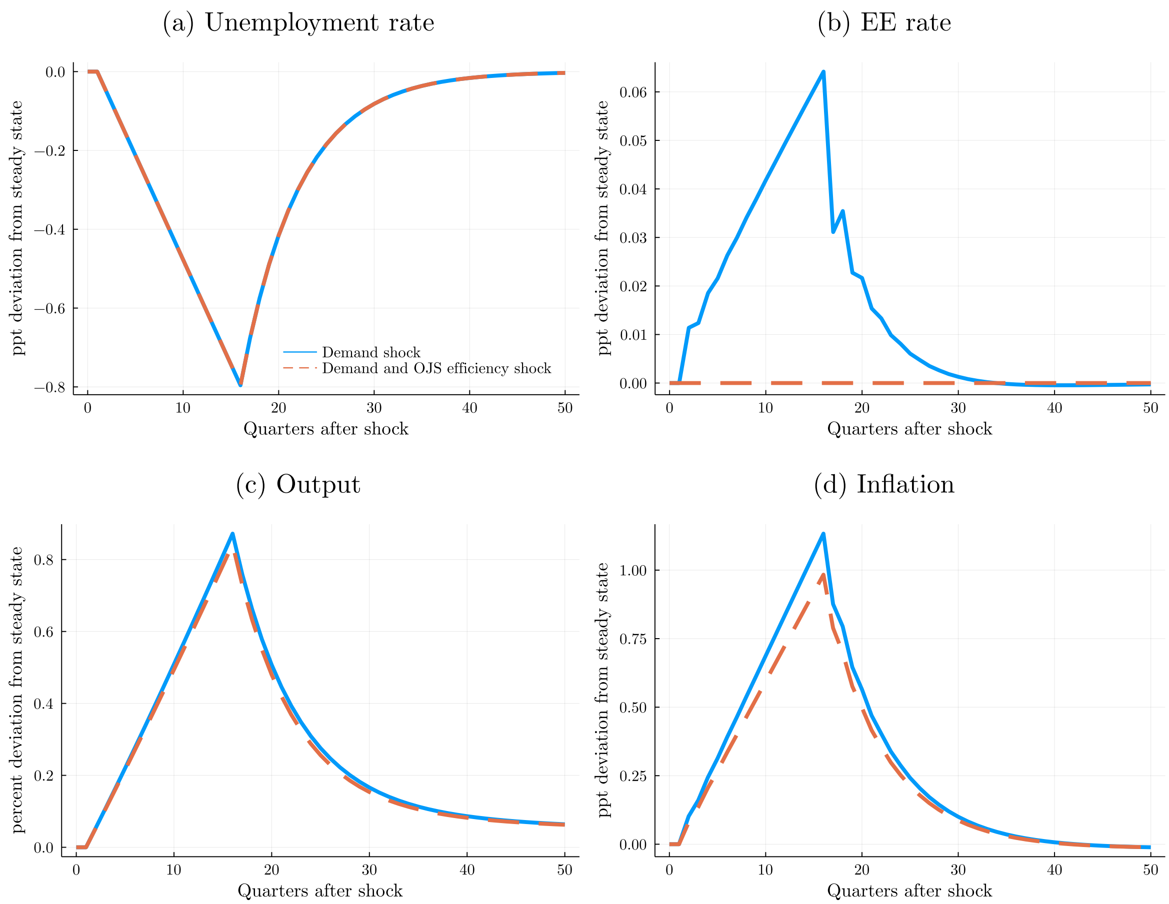
Figure 3 presents the results. Panel (a) shows that positive demand shocks alone and the combination of positive demand and negative OJS efficiency shocks yield identical paths for the unemployment rate, as targeted. Panel (b) illustrates the key difference between the two economies. In the first case, positive demand shocks alone reduce the unemployment rate and increase vacancy posting by firms, resulting in an endogenous increase in the EE rate from its steady state level of 2% to a peak of 2.06%. In the second scenario with both positive demand shocks and negative OJS efficiency shocks, by construction, the EE rate remains flat, mimicking empirical patterns observed during the post-Great Recession episode illustrated in Figure 2 and Figure B.1. The two economies have nearly identical output dynamics as depicted in Panel (c). This is because the average labor productivity (ALP) increases slightly and by almost the same amount in the two economies, as shown in Appendix Figure B.2.[11] In addition, given identical dynamics for the employment rates (as we target the same unemployment path), changes in output are almost identical between the two economies.

We note that if we were to solely focus on the dynamics of unemployment and output, and ignored job mobility dynamics, we would infer that the two economies were hit by identical shocks. However, Panel (d) shows that the presence of negative OJS efficiency shocks causes a drag on inflation in the second economy, despite similar output (and unemployment) dynamics. In particular, we find that the four-quarter inflation rate is around 0.6 percentage points smaller in the second economy.[12]

[11] The increase in ALP is slightly higher in the economy with an increase in EE transitions since more worker-firm contacts through OJS materialize into productivity-improving employment switches.

[12] Specifically, we calculate the four-quarter inflation rate in each economy by compounding quarterly inflation over time. We then report the maximum difference between these inflation rates.

Figure 3: Effects of negative OJS efficiency shocks on output and inflation



Notes: This figure presents the dynamics of unemployment rate, EE rate, output, and inflation in an economy subject to (1) only a series of positive demand shocks (solid-blue lines) and (2) series of positive demand shocks and negative OJS efficiency shocks (dashed-orange lines). The shocks in the two economies are estimated to generate the same path of unemployment. The EE rate is untargeted in the first economy whereas the OJS efficiency shocks are such that the EE rate remains unchanged in the second economy.

5.3 Decomposing effects of OJS efficiency shocks on inflation

The preceding exercises demonstrated that job mobility can have important effects on inflation dynamics. In the following discussion, we dissect and quantify the channels through which a positive shock to OJS efficiency translates to higher inflation. To this end, we analyze model outcomes in response to a unit shock to ν and provide a decomposition of the response of inflation on-impact by leveraging the DAG representation of the model and relying on the sequence-space Jacobians we compute in the process of solving the model.

The NKPC in Equation (9) reveals that—to a first-order approximation—a change in inflation is driven by a change in the real price of labor services p^l, which determines the real marginal cost of production for intermediate firms. Therefore, we focus on the dynamics of p^l in studying inflation.

In our decomposition exercise, the DAG setup proves useful as it allows us to keep track of key model variables that affect p^l through various model blocks and equilibrium conditions. For instance, p^l has a direct effect on service firms, given that this is the price of their output. Accordingly, the only outcome of the service firm block—the expected value of forming a match $\mathbb{E}J$—affects the free-entry condition captured by the H6 block in the DAG.[13] Since the free-entry condition expresses a relationship between the expected value of a match (which in itself is a function of other model variables and parameters) and p^l, we focus on this equilibrium condition to decompose the effects of a shock to ν on p^l (and thus inflation).[14]

To clarify how we operationalize the DAG and its associated input-output structure for this decomposition exercise, some discussion is warranted. We start from the total Jacobians of each block's outputs with respect to their inputs already computed in the solution of the model, as described in Section 4. We then use the implicit function theorem (IFT) at each block to compute the derivative of the output of interest with respect to all the endogenous and exogenous model variables listed in the initial node in the DAG. In the specific case of utilizing the free-entry condition captured in H6 for decomposing the changes in the marginal cost of production, we obtain the derivative of p^l with respect to all model variables by applying the IFT to H6. We then multiply the total derivative of p^l with respect to these variables with the general equilibrium IRF of these variables with respect to ν, which were also computed while solving the model. As a result, we obtain the response of each component that makes up p^l with respect to the shock of interest ν. Specifically, we end up with the effect of ν on p^l through the shock's direct effect as well as its effect on market tightness θ, inflation π, and unemployment u. Given the effect of ν on p^l through θ is large, we follow similar steps to above to further decompose θ to its subcomponents based on the labor-market clearing condition captured in the $H2$ block, which yields a break down of the response of p^l through changes in θ into the direct effect of ν on θ and the indirect effect of ν through output Y.

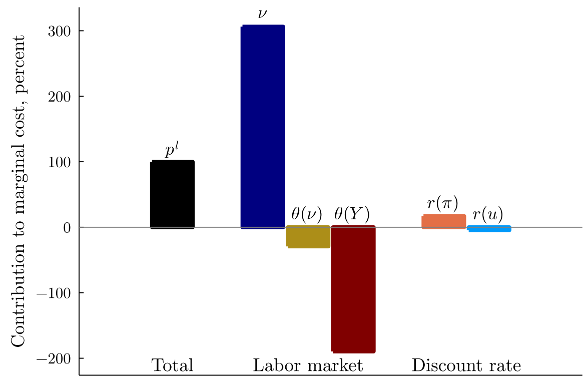
Figure 4 plots the share each of these components contribute to the overall impact response of p^l to a shock to ν. We group the overall effect into two broad categories, namely the labor market and discount rate effects, which we now discuss.

Labor market effects. A shock to the OJS efficiency parameter ν directly affects the firm's expected match value $\mathbb{E}J$. A higher ν reduces the firm's match value J because the worker's probability of contacting an outside firm increases. This could either result in more offers that translate to rebargaining wages with the incumbent firm such that the worker's piece rate α

[13]The expected value from a match is given by $\mathbb{E}J = \frac{1}{S_t}[\zeta \int_{s,h} \int_{\widetilde{x}} J_t\left(h, \widetilde{x}, \underline{x}/\widetilde{x}\right) d\Gamma^x\left(\widetilde{x}\right) d\mu_t^U\left(s,h\right) + \nu \int_{s,h,x,\alpha} \int_x^{\overline{x}} J_t\left(h, \widetilde{x}, x/\widetilde{x}\right) d\Gamma^x\left(\widetilde{x}\right) d\mu_t^E\left(s,h,x,\alpha\right)]$.

[14]We note that this decomposition is not unique. We choose the blocks that we consider to be most closely related to the variable of interest.

Figure 4: Decomposition of channels that OJS efficiency shock affects the marginal cost



Notes: This figure presents the share of marginal cost, i.e., the price of labor services p^l, explained by its components in response to an increase in the OJS efficiency parameter ν. In particular, the fraction of the total change in p^l is accounted for by labor market effects and discount rate effects. ν refers to the direct effect of OJS efficiency on p^l; $\theta(\nu)$ refers to the effect of ν on market tightness θ through its effect on the total supply of labor services $\mathcal{L}$; $\theta(Y)$ denotes the effect of ν on θ through its GE effect on output Y; $r(\pi)$ denotes the effect of ν on real rate r through inflation π; and $r(u)$ refers to the effect of ν on real rate r through unemployment u.

increases or to shorter match duration if the worker is poached by the outside firm. Thus, an increase in ν leads to a decline in $\mathbb{E}J$.[15] All else equal, the decline in $\mathbb{E}J$ necessitates an increase in p^l for the free-entry condition in H6 to hold. Quantitatively, we find that the direct effect of ν on p^l, labeled as ν in Figure 4, explains 306% of the total (100%) increase in p^l upon impact.

In addition to the direct effect, there are secondary effects of an increase in ν through general equilibrium (GE) responses. Focusing on the GE responses in the labor market, we find that an increase in ν leads to a decline in the market tightness θ as demonstrated in Appendix Figure B.3 panel (b). How does this decline in θ affect p^l? According to the DAG, θ enters the free-entry condition (H6) through its effects on the service firm.[16] For an unmatched service firm, a lower θ increases the probability of filling a vacancy $q(\theta)$. In addition, when the firm matches with a worker, a lower θ reduces the worker's probability of contacting other firms in the future, implying less frequent wage rebargaining and longer match durations. Thus, the matched firm's value J and $\mathbb{E}J$ increase. All else constant, a higher vacancy filling rate and expected match value require a decline in p^l for the free-entry condition to hold. Quantitatively, we find that this GE effect of ν on p^l through θ (separately labeled as $\theta(\nu)$ and $\theta(Y)$, which we discuss below)

[15] Note that a higher ν implies a higher weight for employed job-searchers in the aggregate measure of job searchers S. This compositional change also affects $\mathbb{E}J$. However, we find that this effect is quantitatively small due to counteracting effects of the changes in the composition of job-searchers on $\mathbb{E}J$. On the one hand, a higher likelihood of meeting an employed worker increases $\mathbb{E}J$ given that the employed typically have higher skill levels than the average unemployed. On the other hand, this higher likelihood reduces $\mathbb{E}J$ because employed individuals may reject job offers or receive higher piece rates than the new hires who were previously unemployed.

[16] We note that market tightness θ also affects the distribution of employed workers over time in the heterogeneous agent (HA) block. However, this change does not affect the distribution of the employed at the search stage μ^E in the first period of a positive ν shock. Therefore, it does not have an effect on p^l upon impact.

accounts for -219% of the increase in p^l.

As the indirect effect of ν on p^l through θ is quantitatively large and acts as a mitigating factor to the direct effect, we further decompose the response of θ to ν. To do so, we focus on the market clearing condition for labor services given by Equation (19), captured in the H2 block in the DAG.

Based on the DAG, the direct effect of ν on θ is through the aggregate supply of labor services, $\mathcal{L} = \int F(h,x)\, d\lambda_t^E(s,h,x,\alpha)$ via the HA block. This is because ν affects the distribution of employed workers at the consumption/production stage $\lambda_t^E(\cdot)$, which is the only determinant of $\mathcal{L}$ given the production function. An increase in ν leads to an improvement in the match productivity distribution among the employed, which increases $\mathcal{L}$. All else constant, for the labor market clearing condition (H2) to hold, this increase in $\mathcal{L}$ should be counteracted by a decline in θ.

In addition to the effect of ν on θ through $\mathcal{L}$, there is a separate effect of ν on θ through the GE effect of ν on output Y. Appendix Figure B.3 shows that output declines (panel (e)) in response to an increase in ν. This is driven by a decline in aggregate consumption (panel (f)) due to a higher unemployment rate (panel (d)). The unemployment rate rises because unemployed job searchers are crowded out by the employed whose contact rate with firms rise due to a higher ν. This lower output reduces demand for labor services, $L = Y/z$. All else constant, for the labor market to clear (H2), the decline in L requires a commensurate decline in θ.

Having decomposed the effect of ν on θ directly and through Y, we can now quantify how much each of these components eventually contribute to p^l by scaling their individual effects by the overall effect of θ on ν. We find that the direct effect, $\theta(\nu)$, explains -29% of the total increase in p^l, while the GE effect, $\theta(Y)$, accounts for -190% of the total increase in p^l. We conclude that the GE effect of ν on θ through output Y is the main driver behind the large effect of θ on p^l that mitigates a much larger increase on p^l when ν increases.

Overall, an increase in ν results in lower expected match values since firms face more frequent wage re-bargaining and shorter match durations. This entails a compensatory increase in price p_l to maintain the free-entry condition. However, higher ν also reduces θ because of lower demand arising from the crowding-out of unemployed job-seekers among new hires and because of increased labor supply arising from higher productivity. When the labor market is less tight, firms find it easier to fill vacancies and face less pressure from quits and outside offers. This raises the expected value of a match, thus necessitating an offsetting decline in p_l to reduce firm entry.

Discount rate effects. We now turn to the indirect effects of ν on p^l via unemployment and inflation, which enter the reaction function of the monetary authority and hence determine the nominal and real interest rates, which we label as discount rate effects. In equilibrium, in response to an increase in ν, inflation and unemployment increase (Appendix Figure B.3 Panels

(c) and (d)). We again use the DAG to quantify how much these GE responses affect p^l. An increase in inflation upon a rise in ν induces a more than one-for-one increase in the nominal rate i—as the monetary policy follows the Taylor principle ($\Phi_\pi = 1.5 > 1$)—and hence the real rate r, which is captured by the monetary policy (MP) block in the DAG. An increase in the real rate r reduces the continuation value of a service firm that is matched with a worker, i.e., the third term in the right-hand side of Equation (10), which in turn reduces the expected value from a match $\mathbb{E}J$. All else constant, this decline in $\mathbb{E}J$ requires an increase in p^l for the free-entry condition (H6) to keep holding. Quantitatively, we find that the effect of ν on the real rate through inflation, $r(\pi)$, accounts for 17% of the total increase in p^l. We note that this effect is small when compared to contributions of the direct of ν or the GE effects of θ to the increase in p_l.

Similar reasoning implies that an increase in unemployment upon a rise in ν induces a decline in the nominal rate i—as the monetary policy tries to close the output/unemployment gap—and the real rate r through the MP block in the DAG. A decline in the discount rate for firms implies an increase in $\mathbb{E}J$ and thus a commensurate decline in p^l for the free-entry condition to be satisfied. We show that this effect is much smaller in magnitude; in particular, the effect of ν on real rate through unemployment, $r(u)$, explains -4% of the total increase in p^l.

Taking stock. We find that while an increase in ν increases p^l through its direct effect, GE effects on market tightness θ, especially through the response of output Y, partially mitigate the increase in p^l. Overall, the labor market effects account for 87% of the total increase in p^l upon a rise in ν. The remaining 13% is accounted for by the changes in real rate due to the GE effects of ν on inflation and unemployment.

6 Optimal monetary policy with labor market dynamics

Thus far, we have established that changes in job mobility can have important effects on the relationship between unemployment and inflation as well as on labor market outcomes. A natural question to ask is whether job mobility matters for the conduct of monetary policy. We now turn to a normative analysis and study the implications of ignoring job mobility dynamics when setting monetary policy, especially in periods when the unemployment rate and EE rate do not strongly correlate, i.e., when the unemployment rate alone is a poor indicator for the state of the labor market.

In this section, we solve for the optimal monetary policy under the standard Taylor rule and a Taylor rule that also responds to deviations of EE rate from its steady state. We then compare outcomes under the two optimal policies, which allows us to uncover the welfare consequences of ignoring job mobility dynamics when setting monetary policy.

Table 3: Estimation of shocks

	Data			Model		
	UR_t	EE_t	ALP_t	UR_t	EE_t	ALP_t
σ_X	0.210	0.049	0.014	0.200	0.076	0.007
corr (Y_t, X_t)	-0.778	0.394	0.324	-0.764	0.341	0.299

Notes: This table compares model outcomes with their empirical counterparts using the estimated AR(1) processes for the discount rate β, aggregate labor productivity z, and OJS efficiency ν. All time series are logged and HP filtered with a smoothing parameter of 10^5 both in the data and the model. UR, EE, ALP, and Y denote the unemployment rate, EE rate, average labor productivity, and output respectively.

6.1 Estimation of shocks

A prerequisite for studying optimal policy is to determine the nature of the economy in which monetary policy will be conducted. We assume that the economy starts from steady state and is subject to demand, supply, and labor market shocks, which are modeled as innovations to the discount rate β, aggregate labor productivity z, and OJS efficiency ν. To do so, we consider AR(1) processes for β, z, and ν given by:

$$\beta_t = (1-\rho_\beta)\beta^* + \rho_\beta\beta_{t-1} + \sigma_\beta\epsilon_{\beta,t} \quad z_t = (1-\rho_z)z^* + \rho_z z_{t-1} + \sigma_z\epsilon_{z,t} \quad \nu_t = (1-\rho_\nu)\nu^* + \rho_\nu\nu_{t-1} + \sigma_\nu\epsilon_{\nu,t},$$

where ρ_x denotes the persistence of the AR(1) process, $\epsilon_x \sim N(0,1)$ is i.i.d. and $\sigma_x > 0$ denotes the standard deviation of innovations for $x \in \{\beta, z, \nu\}$.

We estimate the parameters of these processes by matching moments between the model and the data. In particular, we jointly estimate the persistence and standard deviations of innovations to β, z, and ν by targeting the correlations of the unemployment rate, ALP, and EE rate with output as well as the standard deviations of these variables.[17] We find that $\rho_\beta = 0.906$, $\rho_z = 0.440$, $\rho_\nu = 0.5$ and $\sigma_\beta = 0.006$, $\sigma_z = 0.006$, and $\sigma_\nu = 0.005$. Table 3 compares the resulting model moments their data counterparts. The model moments approach the data moments reasonably well.

To understand the contribution of each shock to the cyclical movements of our target outcomes, we provide a variance decomposition of these moments. Table 4 presents the fraction of variance of ALP, EE rate, and unemployment rate explained by shocks to only aggregate productivity z, shocks to aggregate productivity z and OJS efficiency ν, and all shocks. Shocks to z

[17] We obtain monthly data on the unemployment rate from the BLS which we convert to a quarterly frequency by taking averages; quarterly data on ALP—measured as output per employed in the nonfarm business sector—from the BLS; quarterly data on real GDP from U.S. Bureau of Economic Analysis; and monthly data on the EE rate from Fujita, Moscarini, and Postel-Vinay (2020), which we convert to a quarterly frequency by compounding $EE_t^{\text{qrt}} = 1 - (1 - EE_t)^3$. All data cover the period between 1995:Q3 and 2019:Q4. Both in the model and the data, we take logs and detrend the time series using the HP filter with a smoothing parameter of 10^5 and calculate correlations and standard deviations of the cyclical components. To calculate model moments, we simulate aggregate time series many times and take averages of moments across these simulations.

Table 4: Variance decomposition of moments

	Fraction of variance explained by shocks to		
	z	z and ν	z, ν and β
ALP	0.985	0.987	1
EE rate	0.163	0.529	1
Unemployment rate	0.093	0.097	1

Notes: This table presents a variance decomposition of average labor productivity, EE rate, and unemployment rate. The columns represent the fraction of each moment's variance explained by shocks to only aggregate productivity z, shocks to aggregate productivity z and OJS efficiency ν shocks, and all shocks.

explain almost all the fluctuations in ALP, while shocks to z account for 16.3% of fluctuations in the EE rate and 9.3% of fluctuations in the unemployment rate. We also find that variations in both ν and β are important factors in fluctuations in the EE rate: the former accounts for 36.6% of the variance of EE rate while the latter explains 47.1% of the variance of EE rate. Finally, shocks to β explain the majority of fluctuations in the unemployment rate.

6.2 Optimal monetary policy

Computing the optimal policy. Having estimated the shock processes, we study optimal monetary policy among a restricted class of monetary policy rules. In particular, we solve for the coefficients of the Taylor rule that maximizes welfare in two settings. First is one where the central bank augments the traditional Taylor such that it responds to the EE gap in addition to the inflation and unemployment gap. The second is one where the central bank responds only to inflation and unemployment gap and ignores EE dynamics.

The central bank chooses the coefficients of the Taylor rule to maximize the ex-ante lifetime utility of an individual who—under the veil of ignorance—is born into an economy that is at its steady state at period t_0 and experiences a path of aggregate shocks—governed by the processes estimated in the previous section—until period T sufficiently far away from t_0. Let $i \in \{1, ..., \mathcal{N}\}$ index individuals in the stationary distribution of the economy at period t_0. We define ω to be the percent additional lifetime consumption that must be endowed at all dates and states to all individuals $i \in \{1, ..., \mathcal{N}\}$ under the existing monetary policy—the Taylor rule in Equation (17) with calibrated coefficients in Section 3—that renders the individual indifferent between the benchmark monetary policy and an alternative policy. More concretely, to compute ω for monetary policy Θ, we simulate the economy subject to aggregate shocks N times. For a given simulation $n \in \{1, ..., N\}$ and policy Θ, we compute the present discounted value of utility at t_0 under the realized shocks and consumption paths for each individual i as follows:

$$V_i^n(\omega, \Theta) = \sum_{t=t_0}^{\min\{T,T_i\}} \beta_t^{t-t_0} u\left((1+\omega)c_{it}\right),$$

where time t runs until the end of simulation horizon T or the period of death T_i, whichever happens earlier.

We then approximate the ex-ante lifetime utility of an individual starting from t_0 as follows:

$$\mathbb{E}V(\omega, \Theta) = \frac{1}{N}\sum_{n=1}^{N}\frac{1}{\mathcal{N}}\sum_{i=1}^{\mathcal{N}} V_i^n(\omega, \Theta) = (1+\omega)^{1-\sigma}\,\mathbb{E}V(0, \Theta),$$

where the outer average is taken over N simulations subject to different sequences of aggregate shocks and the inner average is taken over the cross-section of $\mathcal{N}$ individuals subject to the same path of aggregate shocks but to different idiosyncratic shocks.

Let Θ^e denote the existing monetary policy. Then, the certainty-equivalent consumption ω for alternative policy Θ^a satisfies the following indifference condition:

$$\mathbb{E}V(\omega, \Theta^e) = \mathbb{E}V(0, \Theta^a).$$

Due to our choice of a CRRA utility function, we can solve for ω as follows:

$$\omega = \left(\frac{\mathbb{E}V(0, \Theta^a)}{\mathbb{E}V(0, \Theta^e)}\right)^{\frac{1}{1-\sigma}} - 1.$$

The alternative policy with the highest ω is the optimal policy within a class of Taylor rules.

Optimal monetary policy. We now consider an augmented Taylor rule that also responds to deviations of EE rate from its steady state value:

$$i_t = i^* + \Phi_\pi(\pi_t - \pi^*) + \Phi_u(u_t - u^*) + \Phi_{EE}(EE_t - EE^*), \tag{24}$$

where Φ_{EE} governs the responsiveness of the central bank to the EE rate. We find that the welfare maximizing augmented Taylor rule prescribes $\Phi_\pi = 1.47$, $\Phi_u = -0.52$, and $\Phi_{EE} = 0.33$. The optimal policy yields welfare gains equivalent to around $\omega = 0.5$ percent of additional lifetime consumption relative to the benchmark policy with $\Phi_\pi = 1.50$, $\Phi_u = -0.25$, and $\Phi_{EE} = 0$. Compared to the existing policy, the optimal policy under the augmented Taylor rule prescribes a similar coefficient on inflation, double the coefficient on the unemployment rate, and a sizeable coefficient on the EE rate.

To put into context the difference between the optimal policy and the existing policy, note that the unemployment rate declined by around 25 percent below its trend both between 2016 and 2019 (post-Great Recession; see Figure B.1) and between August 2021 and April 2022 (after the Covid recession). The difference between the two recovery episodes is that while the EE rate remained flat around its trend during the former episode, it increased by around 10 percent during the latter one. Suppose that inflation dynamics were similar between the two episodes so as to focus solely on the implications that differences in job mobility dynamics would have on monetary policy. Given that the benchmark policy does not respond to the deviations in EE rate, it would prescribe the same interest rate response across the two episodes. However, the optimal policy would prescribe a more aggressive response in increasing the nominal rate during the latter post-COVID episode which featured increasing EE rates.

Implications of ignoring job mobility dynamics when setting monetary policy. Next, to understand the welfare implications of ignoring job mobility dynamics in the conduct of monetary policy, we jointly optimize over coefficients of inflation and unemployment gaps, i.e., Φ_π and Φ_u, under the restriction that $\Phi_{EE} = 0$, i.e., we optimize over the coefficients of the standard Taylor rule given in Equation (17). In this case, we find that the optimal policy prescribes $\Phi_\pi = 1.68$ and $\Phi_u = -0.17$, which yields welfare gains equivalent to around $\omega = 0.3$ percent of additional lifetime consumption compared with the benchmark policy. Thus, we conclude that the welfare cost of ignoring the dynamics of EE rate is $0.5 - 0.3 = 0.2$ percent additional lifetime consumption relative to the augmented Taylor rule.

When the central bank ignores job mobility dynamics, the optimal policy prescribes a weaker response to the unemployment gap but a stronger response to the inflation gap compared to the existing policy. The weaker response to the unemployment gap is due to the unemployment rate not being a sufficient measure of labor market slack, especially during episodes when the unemployment rate and EE rate are decoupled. However, to compensate for periods of high inflation due to a less aggressive unemployment gap response, the restricted optimal policy reacts more aggressively to the inflation gap.

7 Conclusions

In this paper, we combine a HANK and a DMP model featuring on-the-job search and rich labor-market heterogeneity. In the model, employed and unemployed individuals with different human capital levels search for jobs in a frictional labor market and contact vacancies. Upon contact, they draw a match-specific productivity and their wages depend on their human capital, match-specific productivity, and a piece-rate of the output they capture as compensation, which is endogenously determined through Bertrand competition. The rest of the model follows the New Keynesian tradition. Service firms sell labor services to monopolistically competitive intermediate firms that produce differentiated goods, which are then sold to final good producers.

We quantitatively study the impact of the weakening correlation between the unemployment rate and EE rate after the Great Recession on inflation. We compare two economies that have the same path of declining unemployment rates driven by positive demand shocks but different paths of EE rates: The first economy experiences an increase in the EE rate, while the second economy observes a flat EE rate caused by additional negative OJS efficiency shocks, mimicking the period between 2016 and 2019 in the U.S. We find that the four-quarter inflation rate is around 0.6 percentage points smaller in the second economy. We then decompose channels through which an OJS efficiency shock affects inflation. We show that while the direct effect of an increase in OJS efficiency on match value leads to a significant increase in the real marginal cost pushing up inflation, this effect is partially mitigated by the equilibrium decline of market tightness through changes in aggregate demand and labor supply. Overall, these counteracting labor market effects explain 87% of the total increase in the real marginal cost upon impact and the remaining 13% is accounted for by changes in the real interest rate due to the GE effects of OJS efficiency on inflation and unemployment. Finally, we study the normative implications of job mobility dynamics for monetary policy. We find that the welfare cost of ignoring the dynamics of EE rate is 0.2 percent additional lifetime consumption.

Our model features a rich set of fiscal policy instruments such as a consumption tax, progressive labor income tax, unemployment and retirement benefits, and government debt. Therefore, it provides a framework to quantitatively study fiscal and monetary policy interactions, accounting for rich labor market dynamics. In addition, it is straightforward to introduce other exogenous shocks (such as shocks to monetary policy, markups, and other labor market parameters) into our model. Given our solution method, it is feasible to estimate a rich set of exogenous shocks jointly to evaluate the model's performance in matching time-series and cross-sectional empirical moments. We leave these considerations for future research.

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Online Appendix

A Model

A.1 Solving the intermediate firm's problem

The problem of the intermediate firm can be solved analytically. The solution is used to obtain an expression for profits in steady state—used to calculate dividends—and also to derive the New Keynesian Phillips curve. The pricing problem of an intermediate firm j whose last period *relative* price is $p_{t-1}(j)$ is given by

$$\Theta\left(p_{t-1}\left(j\right)\right) = \max_{p_t(j)} p_t\left(j\right) y_t\left(p_t\left(j\right)\right) - p_t^l \frac{y_t\left(p_t\left(j\right)\right)}{z_t} - \frac{\eta}{2\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right)^2 Y_t + \frac{1}{1+r_{t+1}} \Theta\left(p_t\left(j\right)\right).$$

Substituting in the demand for each variety, $y_t(j) = p_t(j)^{-\eta} Y_t$, the problem can be written as

$$\Theta\left(p_{t-1}\left(j\right)\right) = \max_{p_t(j)} p_t\left(j\right)^{1-\eta} Y_t - p_t^l p_t\left(j\right)^{-\eta} \frac{Y_t}{z_t} - \frac{\eta}{2\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right)^2 Y_t + \frac{1}{1+r_{t+1}} \Theta\left(p_t\left(j\right)\right).$$

The first-order condition with respect to relative price $p_t(j)$ is given by

$$\begin{aligned} 0 &= \left(1-\eta\right) p_t\left(j\right)^{-\eta} Y_t + \eta p_t^l p_t\left(j\right)^{-\eta-1} \frac{Y_t}{z_t} \\ &- \frac{\eta}{\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right) \frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right) - \pi^*} \frac{1+\pi_t}{p_{t-1}\left(j\right)} Y_t + \frac{1}{1+r_{t+1}} \Theta'\left(p_t\left(j\right)\right), \end{aligned}$$

and the envelope condition is

$$\Theta'\left(p_{t-1}\left(j\right)\right) = \frac{\eta}{\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right) \frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right) - \pi^*} \frac{p_t\left(j\right)\left(1+\pi_t\right)}{p_{t-1}\left(j\right)^2} Y_t.$$

Iterating the envelope condition forward by one period yields

$$\Theta'\left(p_t\left(j\right)\right) = \frac{\eta}{\vartheta} \log\left(\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)}\left(1+\pi_{t+1}\right) - \pi^*\right) \frac{1}{\frac{p_{t+1}(j)}{p_t(j)}\left(1+\pi_{t+1}\right) - \pi^*} \frac{p_{t+1}\left(j\right)\left(1+\pi_{t+1}\right)}{p_t\left(j\right)^2} Y_{t+1}.$$

Consolidating the envelope and the first-order conditions, we obtain:

$$\begin{aligned} 0 = \left(1-\eta\right) p_t\left(j\right)^{-\eta} Y_t + \eta p_t^l p_t\left(j\right)^{-\eta-1} \frac{Y_t}{z_t} - \frac{\eta}{\vartheta} \log\left(\frac{p_t\left(j\right)}{p_{t-1}\left(j\right)}\left(1+\pi_t\right) - \pi^*\right) \frac{1}{\frac{p_t(j)}{p_{t-1}(j)}\left(1+\pi_t\right) - \pi^*} \frac{1+\pi_t}{p_{t-1}\left(j\right)} Y_t \\ + \frac{1}{1+r_{t+1}} \underbrace{\frac{\eta}{\vartheta} \log\left(\frac{p_{t+1}\left(j\right)}{p_t\left(j\right)}\left(1+\pi_{t+1}\right) - \pi^*\right) \frac{1}{\frac{p_{t+1}(j)}{p_t(j)}\left(1+\pi_{t+1}\right) - \pi^*} \frac{p_{t+1}\left(j\right)\left(1+\pi_{t+1}\right)}{p_t\left(j\right)^2} Y_{t+1}}_{\Theta'(p_t(j))}. \end{aligned}$$

All firms set the same price due to symmetry, $p_t(j) = 1 \; \forall \, t, \; j$, and the equation simplifies to

$$0 = (1-\eta) Y_t + \eta p_t^l \frac{Y_t}{z_t} - \frac{\eta}{\vartheta} \frac{\log(1+\pi_t - \pi^*)(1+\pi_t)}{1+\pi_t-\pi^*} Y_t + \frac{1}{1+r_{t+1}} \frac{\eta}{\vartheta} \frac{\log(1+\pi_{t+1}-\pi^*)(1+\pi_{t+1})}{1+\pi_{t+1}-\pi^*} Y_{t+1}.$$

Rearranging terms and using the definition of π_t, we obtain the Phillips curve in Equation (9):

$$\frac{\log(1+\pi_t-\pi^*)(1+\pi_t)}{1+\pi_t-\pi^*} = \vartheta \left(\frac{p_t^l}{z_t} - \frac{\eta-1}{\eta} \right) + \frac{1}{1+r_{t+1}} \frac{\log(1+\pi_{t+1}-\pi^*)(1+\pi_{t+1})}{1+\pi_{t+1}-\pi^*} \frac{Y_{t+1}}{Y_t}.$$

A.2 Solving for a steady state equilibrium

A.2.1 Laws of motion

In this section we present the laws of motion that characterize the worker distribution measured at the consumption stage within a period. We denote by λ_t the distribution of agents across individual states (i.e., share holdings s, human capital h, match productivity x, and piece rate α) at time t. As the population is normalized to one and the dead are replenished with unemployed workers, we have

$$\sum_{s,h,x,\alpha} \lambda_t^E(s,h,x,\alpha) + \sum_{s,h} \lambda_t^U(s,h) + \sum_s \lambda_t^R(s) = 1,$$

where $\lambda_t^E(\cdot)$, $\lambda_t^U(\cdot)$ and $\lambda_t^R(\cdot)$ denote the mass of employed, unemployed and retired workers by individual state variables, respectively, and we omit states which are not relevant for the agents. Also for reference below, let $\boldsymbol{S}_t^E(s';h,x,\alpha) = \{s \in \mathbf{S} : \; g_t^{Es}(s,h,x,\alpha) = s'\}$, $\boldsymbol{S}_t^U(s';h) = \{s \in \mathbf{S} : \; g_t^{Us}(s,h) = s'\}$, and $\boldsymbol{S}_t^R(s') = \{s \in \mathbf{S} : \; g_t^{Rs}(s) = s'\}$ denote the set of period t share holdings s that map into a given level of share holdings s' in $t+1$ by employment status.

We now turn to explicitly writing down the system of equations that determine worker flows. To reduce notational clutter, we define $f_{t+1} = f(\theta_{t+1})$ and suppress some of the function arguments.

Flows into employment. Conditional on not retiring, flows into employment include the following mutually exclusive events.

- Employed worker stays with the same employer, skill appreciates or skill does not appreciate.
  - The worker's piece rate can either (i) remain the same ($\alpha' = \alpha$) either because no meeting occurs or an offer is not met with a counteroffer or (ii) rise to due rebargaining induced by an external offer. When considering inflows into specific match productivity x' and piece rate α', it must be that the poaching firm's match productivity is $\tilde{x} = x'\alpha'$ in the latter case. Further, it must be that the poaching firm's match productivity is higher than the current output share: $x\alpha < x'\alpha' = \tilde{x}$.

- Employed worker accepts new offer, skill appreciates or skill does not appreciate.
  - The worker's piece rate changes (declines) due to a job-to-job transition. When considering inflows into specific match productivity x' and piece rate α', it must be that $\alpha' = \frac{x}{x'}$, where x is the productivity of the previous match. This implies that previous match productivity must have been $x = \alpha' x'$.
- Employed worker loses job but finds new one, skill appreciates or skill does not appreciate.
  - When considering inflows into specific match productivity x' and piece rate α' from unemployment, it must be that $\alpha' = \frac{x}{x'}$. Here, it does not matter what the previous job's x or α was.
- Unemployed worker accepts new offer, skill depreciates or skill does not depreciate.
  - The evolution of piece rate is similar to above.

We then have the following law of motion for the distribution of employed workers:

$$\lambda_{t+1}^{E}\left(s',h',x',\alpha'\right) = \left(1-\psi_R\right)\times$$

$$\left(\sum_{s\in\boldsymbol{S}_t^E} \underbrace{\lambda_t^E\left(s,h'-1,x',\alpha'\right)}_{\substack{\text{no outside offer/discard offer;}\\ \alpha \text{ remains the same}}} \pi^E\left(1-\delta\left(h'-1,x'\right)\right)\left[\left(1-\nu\left(h',x'\right)f_{t+1}\right)+\nu\left(\cdot\right)f_{t+1}\underbrace{\sum_{\tilde{x}<x'\alpha'}}_{\text{discard offers}}\Gamma^x\left(\tilde{x}\right)\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right] \right.$$

$$+ \underbrace{\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h'-1,x',\alpha\right)}_{\text{received offer from firm with productivity } \alpha' x'} \pi^E\left(1-\delta\left(h'-1,x'\right)\right)\left[\left(1-\nu\left(h',x'\right)f_{t+1}\right)+\nu\left(\cdot\right)f_{t+1}\Gamma^x\left(x'\alpha'\right)\underbrace{\mathbf{1}_{x'\alpha'>x'\alpha}}_{\text{rebargain}}\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right]$$

$$+\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x',\alpha'\right)\left(1-\pi^E\right)\left(1-\delta\left(h',x'\right)\right)\left[\left(1-\nu\left(h',x'\right)f_{t+1}\right)+\nu\left(h',x'\right)f_{t+1}\sum_{\tilde{x}<x'\alpha'}\Gamma^x\left(\tilde{x}\right)\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right]$$

$$+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x',\alpha\right)\left(1-\pi^E\right)\left(1-\delta\left(h',x'\right)\right)\left[\left(1-\nu\left(h',x'\right)f_{t+1}\right)+\nu\left(h',x'\right)f_{t+1}\Gamma^x\left(x'\alpha'\right)\mathbf{1}_{x'\alpha'>x'\alpha}\left(1-g_{t+1}^{Ea}\left(\cdot\right)\right)\right]$$

$$+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h'-1,\underbrace{\alpha'x'}_{x},\alpha\right)\pi^E\left[\left(1-\delta\left(h'-1,\alpha'x'\right)\right)\nu\left(h',\alpha'x'\right)g_{t+1}^{Ea}\left(\cdot\right)\right]f_{t+1}\Gamma^x\left(x'\right)$$

$$+\sum_{\alpha}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x',\alpha\right)\left(1-\pi^E\right)\left[\left(1-\delta\left(h',\alpha'x'\right)\right)\nu\left(h',\alpha'x'\right)g_{t+1}^{Ea}\left(\cdot\right)\right]f_{t+1}\Gamma^x\left(x'\right)$$

$$+\sum_{\alpha}\sum_{x}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h'-1,x,\alpha\right)\pi^E\left[\delta\left(h'-1,x\right)\zeta\left(h'\right)g_{t+1}^{Ua}\left(\cdot\right)\right]f_{t+1}\Gamma^x\left(x'\right)\underbrace{\mathbf{1}_{\alpha'=\frac{x}{x'}}}_{\alpha\text{' must be }\frac{x}{x'}}$$

$$+\sum_{\alpha}\sum_{x}\sum_{s\in\boldsymbol{S}_t^E}\lambda_t^E\left(s,h',x,\alpha\right)\left(1-\pi^E\right)\left[\delta\left(h',x\right)\zeta\left(h'\right)g_{t+1}^{Ua}\left(\cdot\right)\right]f_{t+1}\Gamma^x\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}$$

$$+\sum_{s\in\boldsymbol{S}_t^U}\lambda_t^U\left(s,h'+1\right)\pi^U\zeta\left(h'\right)f\left(\theta_{t+1}\right)\Gamma^x\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}g_{t+1}^{Ua}\left(\cdot\right)$$

$$\left.+\sum_{s\in\boldsymbol{S}_t^U}\lambda_t^U\left(s,h'\right)\left(1-\pi^U\right)\zeta\left(h'\right)f\left(\theta_{t+1}\right)\Gamma^x\left(x'\right)\mathbf{1}_{\alpha'=\frac{x}{x'}}g_{t+1}^{Ua}\left(\cdot\right)\right). \tag{A.1}$$

Flows into unemployment. Conditional on not retiring, flows into unemployment include the following transitions.

- Employed worker loses job and does not find job or refuses offer, skill appreciates
- Employed worker loses job and does not find job or refuses offer, skill does not appreciate
- Unemployed worker does not find job or refuses offer, skill depreciates
- Unemployed worker does not find job or refuses offer, skill does not depreciate
- Dead retiree is reborn, inherits shares but draws new human capital

Hence, we have the following law of motion for the distribution of unemployed workers

$$
\begin{aligned}
&\lambda^U_{t+1}(s',h') = (1-\psi_R)\times \\
&\Bigg(\sum_{\alpha}\sum_{x}\sum_{s\in \boldsymbol{S}^E_t} \lambda^E_t(s,h'-1,x,\alpha)\pi^E\delta(h'-1,x)\left[1-\zeta(h')f_{t+1}+\zeta(h')f_{t+1}\sum_{\tilde{x}}\Gamma^x(\tilde{x})\left(1-g^{Ua}_{t+1}(\cdot)\right)\right] \\
&+\sum_{\alpha}\sum_{x}\sum_{s\in \boldsymbol{S}^E_t} \lambda^E_t(s,h',x,\alpha)(1-\pi^E)\delta(h',x)\left[1-\zeta(h')f_{t+1}+\zeta(h')f_{t+1}\sum_{\tilde{x}}\Gamma^x(\tilde{x})\left(1-g^{Ua}_{t+1}(\cdot)\right)\right] \\
&+\pi^U\left[1-\zeta(h')f_{t+1}+\zeta(h')f_{t+1}\sum_{\tilde{x}\in\tilde{X}}\Gamma^x(\tilde{x})\left(1-g^{Ua}_{t+1}(\cdot)\right)\right]\sum_{s\in \boldsymbol{S}^U_t}\lambda^U_t(s,h'+1) \\
&+(1-\pi^U)\left[1-\zeta(h')f(\theta_{t+1})+\zeta(h')f(\theta_{t+1})\sum_{\tilde{x}}\Gamma^x(\tilde{x})\left(1-g^{Ua}_{t+1}(\cdot)\right)\right]\sum_{s\in \boldsymbol{S}^U_t}\lambda^U_t(s,h')\Bigg) \\
&+\psi_D\Gamma^h(h')\sum_{s\in \boldsymbol{S}^R_t}\lambda^R_t(s). \qquad \text{(A.2)}
\end{aligned}
$$

Flows into retirement. Flows into retirement include the following set of transitions.

- Employed worker retires
- Unemployed worker retires
- Retired worker does not die

These inflows imply, we have the following law of motion for the distribution of retirees:

$$\lambda^R_{t+1}(s') = \psi_R \sum_{s\in \boldsymbol{S}^E_t,h,x,\alpha} \lambda^E_t(s,h,x,\alpha) + \psi_R \sum_{s\in \boldsymbol{S}^U_t,h} \lambda^U_t(s,h) + (1-\psi_D)\sum_{s\in \boldsymbol{S}^R_t}\lambda^R_t(s). \qquad \text{(A.3)}$$

A.2.2 Casting the model in relative prices and real variables

Nominal frictions are not relevant in the steady state, where prices rise by the rate of long-run inflation π^* and hence firms do not incur an adjustment cost while increasing their prices by that amount. For the same reason, the price level is indeterminate. Therefore, we solve for relative prices (relative to the price of output) and allocations. We start by deriving the equations governing these relative prices, real dividends, and real profits of intermediate firms in steady state.

- Evaluating the NKPC at the steady state, we obtain the real marginal cost $mc = p^l/z$:

$$mc = \frac{\eta-1}{\eta}.$$

- The price of labor services is then given by

$$p^l = mc \times z = \frac{\eta - 1}{\eta} z. \tag{A.4}$$

- Per-period real profits of the intermediate firms are given by

$$\Gamma^I = (1 - mc)Y = \frac{Y}{\eta}. \tag{A.5}$$

- Real dividends in the steady state are given by

$$\begin{aligned} d &= x_B Y - \frac{x_B Y(1+\pi^*)}{(1+i)} + \Gamma^I + \Gamma^S \\ &= x_B Y \frac{r}{1+r} + \Gamma^I + \Gamma^S. \end{aligned} \tag{A.6}$$

- Dividing the no arbitrage condition by aggregate price level P, we solve for share price

$$\begin{aligned} \frac{(p^s + d)(1+\pi^*)}{p^s} &= 1 + i \\ (1+r)p^s &= p^s + d \\ p^s &= \frac{d}{r}. \end{aligned} \tag{A.7}$$

- Finally, we rewrite the government budget constraint in real terms as follows. Let $b_t = B_t / P_{t+1}$. Then, dividing both sides by P_t, multiplying the first term on the right hand side by $\frac{P_{t+1}}{P_{t+1}}$, and recognizing that $1 + i_t = (1 + r_{t+1})(1 + \pi_{t+1})$, we get

$$\begin{aligned} b_{t-1} + g_t + \int UI\left(h\right) d\lambda_t^U\left(s,h\right) + \int \phi^R d\lambda_t^R(s) = \frac{b_t}{1+r_{t+1}} &+ \tau_c \int c\left(s,h,x,\alpha\right) d\lambda_t\left(s,h,x,\alpha\right) \\ &+ \int \left(UI\left(h\right) - \tau_t \left(UI\left(h\right)\right)^{1-\Upsilon}\right) d\lambda_t^U\left(s,h\right) \\ &+ \int \left(w(h,x,\alpha) - \tau_t w(h,x,\alpha)^{1-\Upsilon}\right) d\lambda_t^E\left(s,h,x,\alpha\right) \\ &+ \int \left(\phi^R - \tau_t \left(\phi^R\right)^{1-\Upsilon}\right) \lambda_t^R(s). \end{aligned}$$

Here, the lower case variables b_{t-1} and g_t represent the real values of government debt and government spending, respectively. It is useful to define the real net revenue of government (tax proceeds minus outlays for pensions and unemployment insurance), $\mathcal{R}_t$, as

$$\begin{aligned}\mathcal{R}_t = &-\int UI\left(h\right)d\lambda_t^U\left(s,h\right) - \int \phi^R \lambda_t^R\left(s\right) \\ &+ \tau_c \int c\left(s,h,x,\alpha\right)d\lambda_t\left(s,h,x,\alpha\right) + \int \left(UI\left(h\right) - \tau\left(UI\left(h\right)\right)^{1-\Upsilon}\right)d\lambda_t^U\left(s,h\right) \\ &+ \int \left(w(h,x,\alpha) - \tau w(h,x,\alpha)^{1-\Upsilon}\right)d\lambda_t^E\left(s,h,x,\alpha\right) + \int \left(\phi^R - \tau\left(\phi^R\right)^{1-\Upsilon}\right)\lambda_t^R\left(s\right).\end{aligned} \tag{A.8}$$

With these definitions, the government budget can be expressed in real terms as

$$\begin{aligned} b_{t-1} + g_t &= \frac{b_t}{1+r_{t+1}} + \mathcal{R}_t \\ \Rightarrow 0 &= (1+r_{t+1})(b_{t-1} + g_t - \mathcal{R}_t) - b_t. \end{aligned} \tag{A.9}$$

A.2.3 Solution algorithm for the steady state equilibrium

We solve for the steady state using the following algorithm by bisecting over a nominal interest rate i that clears the share market given by Equation (20).

1. For a given nominal interest rate i, given π^*, obtain r from the Fisher equation (18).

2. Outer loop: Guess a tax parameter τ, level of output Y, and service firm profits Γ^S.

   - Calculate the real bond holdings $b = x_B Y$, real government expenditures $g = x_G Y$, and real intermediate firm profits Γ^I using Equation (A.5).
   - Calculate real dividends d using Equation (A.6).
   - Calculate real share price p^s using Equation (A.7).

3. Inner loop: Guess a market tightness θ.

   - Calculate worker contact rate $f(\theta)$.
   - Solve the workers' problems given by Equations (1), (2), (3).
   - Compute the stationary worker distributions over state variables μ^E, μ^U, μ^R, λ, λ^E, λ^U, and λ^R.
   - Solve the matched firm problem in the labor services sector given by Equation (10).
   - Given the solution to the firm problem and worker distributions, calculate the implied market tightness $\widetilde{\theta}$ consistent with the free-entry condition $V = 0$, where V is satisfies Equation (11).
   - Iterate over the inner loop until $\widetilde{\theta}$ agrees with the guessed market tightness θ.

4. Using the worker distributions, calculate the implied output $\widetilde{Y}$ using market clearing for labor services in Equation (19) and real service firm profits $\widetilde{\Gamma}^S$ in Equation (15).

5. Calculate the implied tax parameter $\widetilde{\tau}$ that clears the government budget constraint, which can be obtained from Equations (A.8) and (A.9) as:

$$\widetilde{\tau} = \frac{-\frac{r}{1+r} x_B Y - x_g Y + \tau_c \int c d\lambda + \int w d\lambda^E + \int UI d\lambda^U + \int \phi^R d\lambda^R}{\int w^{1-\Upsilon} d\lambda^E + \int UI^{1-\Upsilon} d\lambda^U + \int \left(\phi^R\right)^{1-\Upsilon} d\lambda^R}.$$

6. Iterate over the outer loop until $\widetilde{\tau}$, $\widetilde{Y}$, and $\widetilde{\Gamma}^S$ agree with guesses for τ, Y, and Γ^S.

A.3 Solving for the transition path using DAGs

In Section 4, we discuss how we employ and expand the sequence-space Jacobian method detailed in Auclert, Bardóczy, Rognlie, and Straub (2021) to solve for the transitional dynamics in our model. In the following discussion, we provide additional details on this procedure.

To solve the model using sequence-space Jacobians, we first cast the model as a DAG, depicted in Figure A.1.[18] The leftmost red node contains exogenous variables which represent shocks the economy might be subject to as well as endogenous variables (unknowns) whose dynamics we are interested in. The intermediate (green) nodes represent various model components and importantly, demonstrate how each component relates with one another via their respective input and output variables. The intermediate nodes can be categorized into simple blocks and the heterogeneous agent block. An example of the former would be model components that relate various aggregate variables such as the fiscal policy rule (Equation 21), the Taylor rule (Equation 17) or the expression for dividends and no-arbitrage that relate to the mutual fund (Equations 12 and 13). The latter is the most complex model component wherein heterogeneous agents solve for decision rules that govern their consumption-saving choices and labor market outcomes, which play an important role in the dynamics of aggregates and distributions in the economy. Finally, the rightmost red node represents the target sequences that must equal zero in equilibrium (market clearing and consistency conditions).[19] This final node might take inputs directly from the initial node with exogenous and endogenous variables, as well as outputs from the intermediate nodes.

Formally, let $\upsilon = \left(\{\pi_t, Y_t, p^l_t, b_t, u_t, \theta_t, \Gamma^S_t, e2e_t\}_{t=0}^{T-1}\right)$ represent the path of unknown endogenous variables and $\Theta = \left(\{z, \beta, \nu\}_{t=0}^{T-1}\right)$ represent the path of exogenous variables.[20] The system

[18]For visual clarity, we consolidate the terminal "target" blocks that capture various equilibrium conditions into a single node. One should think of the last node as consisting of eight different ones representing each of the equilibrium conditions separately with inputs from the relevant intermediate blocks.

[19]Note that the number of unknown variables specified in the leftmost node must be equal to the number of target conditions in the rightmost node.

[20]Namely, the endogenous variables in the model are inflation, real output, real price of labor services, real debt,

of equations, labeled as "targets" in the rightmost node, that govern the transition path is:[21]

$$H(\upsilon;\Theta) = \begin{pmatrix} \log\left(1+\pi_t-\pi^*\right)\frac{1+\pi_t}{1+\pi_t-\pi^*} - \vartheta\left(\frac{p^l_t}{z_t}-\frac{\eta-1}{\eta}\right) \\ +\frac{1}{1+r_{t+1}}\log\left(1+\pi_{t+1}-\pi^*\right)\frac{1+\pi_{t+1}}{1+\pi_{t+1}-\pi^*}\frac{Y_{t+1}}{Y_t} \\ \mathcal{L}_t - L_t \\ \mathcal{S}_t - 1 \\ (1+r_{t+1})(b_{t-1}+g_t-\mathcal{R}_t)-b_t \\ \mathcal{U}_t - u_t \\ \theta_t - q^{-1}\left(\kappa/\mathbb{E}J_t\right) \\ \mathit{\Gamma}^S_t - \Gamma^S_t \\ \mathcal{E}2\mathcal{E}_t - e2e_t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{A.10}$$

The main purpose of setting up the model as a DAG is for us to be able to systematically solve for Jacobians that summarize the partial equilibrium responses of each node's output (including targets in the rightmost node) with respect to each direct input to that node. We are then able to forward accumulate—that is apply the chain rule in a systematic fashion—the partial Jacobians along the DAG to obtain the total Jacobians of any output (again including targets) with respect to changes in any exogenous variable or unknown endogenous variable. Simply put, a total Jacobian combines the direct and indirect responses of an output with respect to an input. For example, the response of the value of posting a vacancy $\mathbb{E}J$ (service firm block output) is affected directly by the real rate r through discounting in the firm's match value but also indirectly through how the real rate affects share prices and dividends, which ultimately affect household decisions and thus the distribution of workers that vacancies contact with.

Having obtained the total Jacobians of targets $H(\upsilon;\Theta)$ with respect endogenous unknowns υ and with respect to exogenous variables Θ, we can apply the implicit function theorem to compute the response of any endogenous unknown $d\upsilon$ to a change in the exogenous variables $d\Theta$. Formally, let $H_\upsilon = \partial H/\partial\upsilon$ and $H_\Theta = \partial H/\partial\Theta$ be the total Jacobians of targets with respect to endogenous unknowns and exogenous variables, then, the impulse responses of unknowns is given by:

$$d\upsilon = \underbrace{-H_\upsilon^{-1}H_\Theta}_{G_\upsilon} d\Theta,$$

where G_υ denotes the GE Jacobians of the endogenous variables.

Equipped with the partial Jacobians of the intermediate variables and GE Jacobians of the

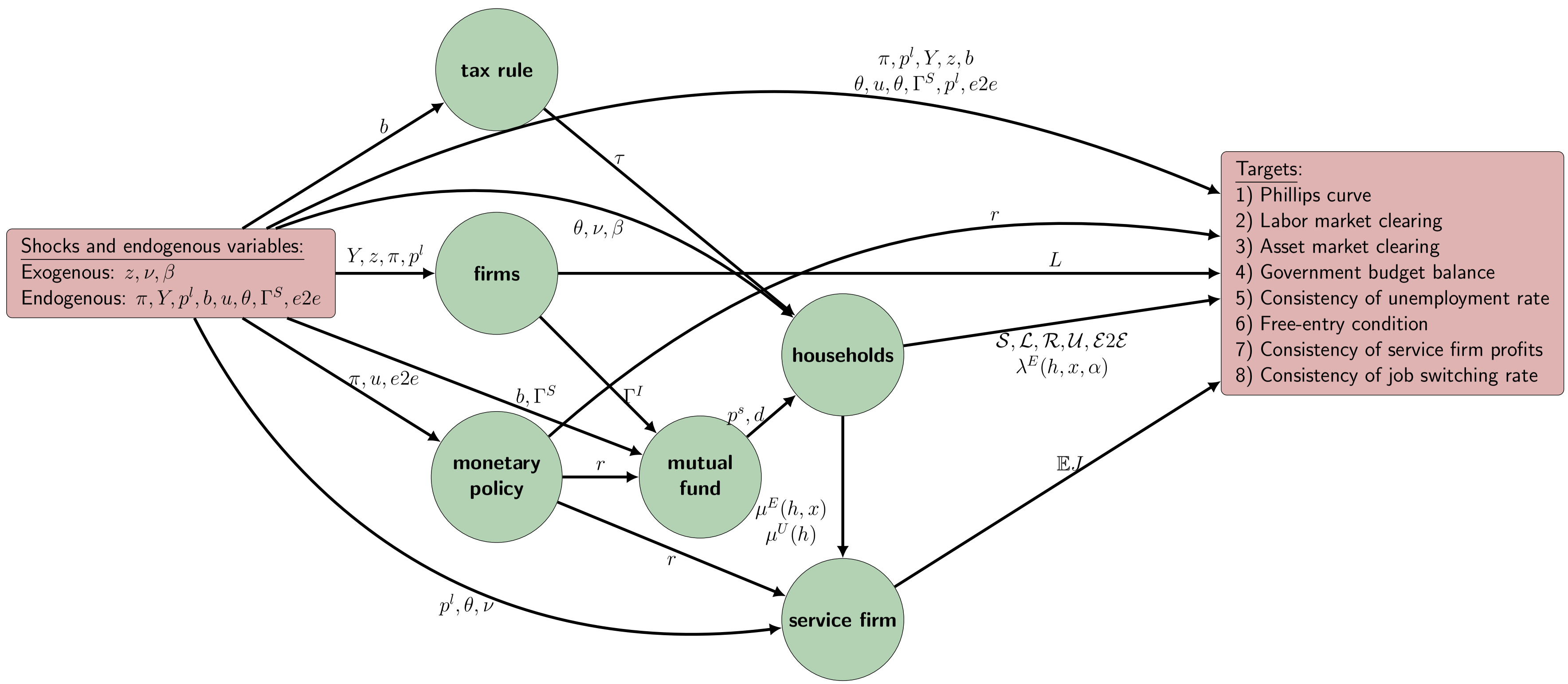
unemployment rate, labor market tightness, real profits of the labor-service sector and the mass of employer-to-employer transitions. The exogenous variables are the total factor productivity in intermediate goods production, the discount factor and the on-the-job search efficiency.

[21] These equations in order capture the New-Keynesian Phillips curve, market clearing for labor services, market clearing for mutual fund shares, government budget balance, consistency of the unemployment rate, the free-entry condition, consistency of labor-service profits and consistency of employer-to-employer transitions.

unknown variables, we compute the GE Jacobians of the intermediate variables too, which allow us to compute their IRFs with respect to exogenous variables as well.

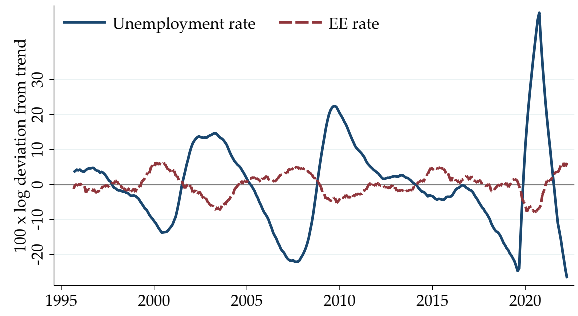
Finally, we use the equivalence of the impulse response function with the moving-average process representation of a time series. This allows us to flexibly simulate a time-path of aggregate variables and—given the path of these aggregate variables and policy responses in response to aggregate shocks—also simulate a large panel of workers. We in turn use this micro worker panel in to study a wide range of cross-sectional outcomes and to evaluate the welfare effects of monetary policy.

Figure A.1: DAG representation of the HANK model with a frictional labor market and on the job search



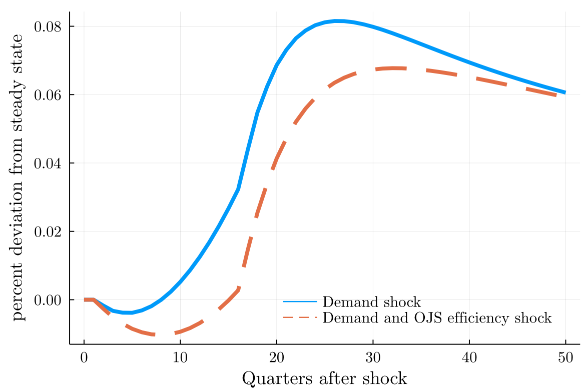
B Additional results

Figure B.1: Cyclical components of unemployment and EE rates



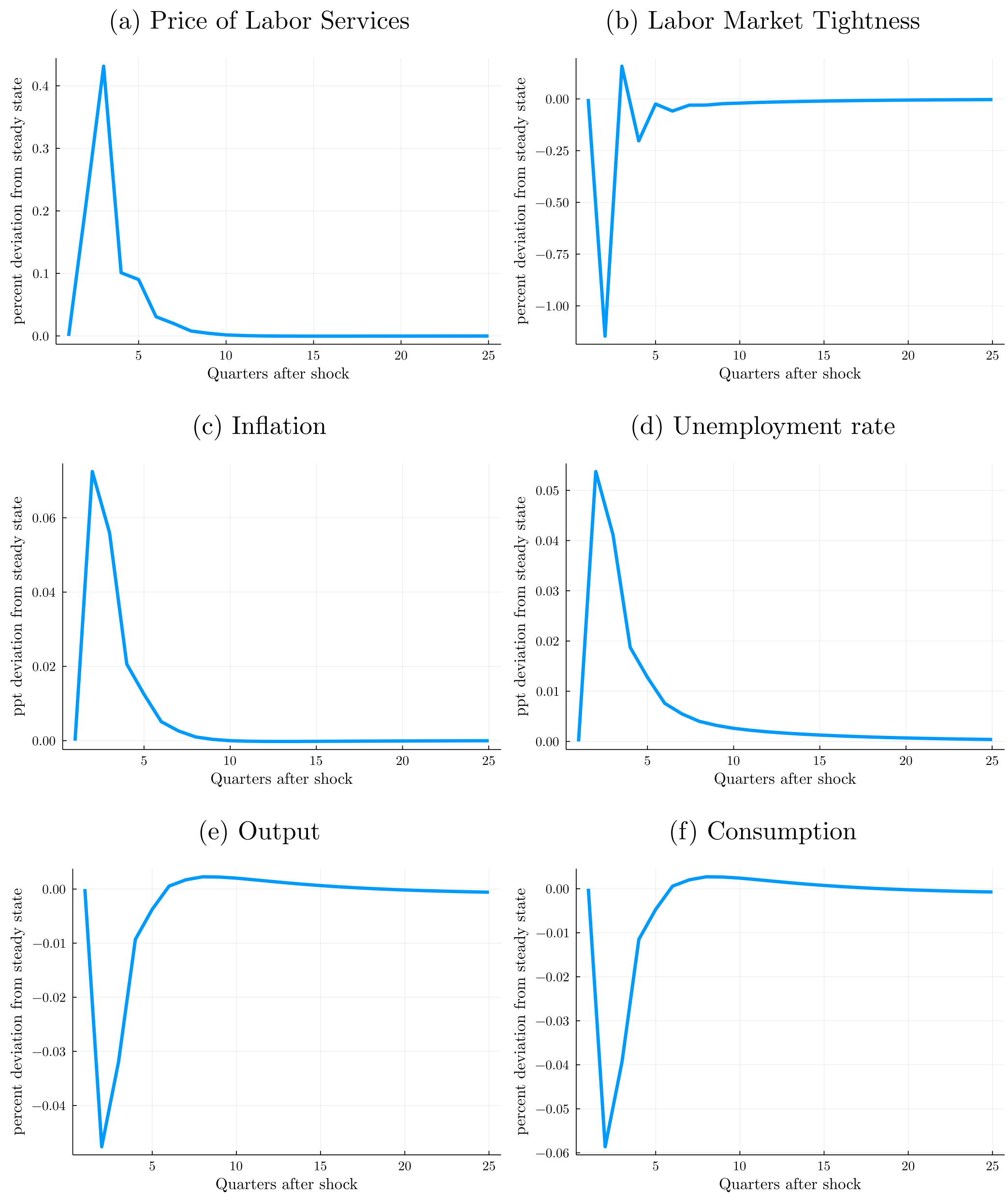
Notes: This figure plots the time series of the cyclical components of log unemployment rate and EE rate. Both time series are detrended using the HP filter with a smoothing parameter of 10^5.
Source: BLS and Fujita, Moscarini, and Postel-Vinay (2020).

Figure B.2: Effects of negative OJS efficiency shocks on average labor productivity



Notes: This figure presents dynamics of average labor productivity (ALP) in an economy with (1) only a series of positive demand shocks (solid-blue lines) and (2) series of positive demand shocks and negative OJS efficiency shocks (dashed-orange lines). These shocks in the two economies are calibrated such that they lead to the same path of unemployment rate. The additional negative OJS shocks in the second economy are estimated to keep the EE rate unchanged.

Figure B.3: Impulse responses to an OJS efficiency shock



Notes: This figure presents the impulse responses of model outcomes to a positive unit shock to the OJS efficiency parameter ν.