Alternative Output, Input and Income Concepts for the Production Accounts

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Abstract

Gross National Product is a useful measure of the output of the production sector of an economy. However, it does not accurately measure the income generated by the production sector. A measure of income or net output is obtained by replacing gross output by the end of period capital stock less the beginning of the period capital stock and replacing the user cost of capital by waiting services. Essentially, depreciation is subtracted from gross output and smoothed capital gains (or losses) are added to nominal output. The switch from gross output to net output also requires a new measure of Total Factor Productivity for the production sector. The paper also looks at the problems that arise when aggregating over individual production units to arrive at national aggregates.

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Production, income, System of National Accounts, gross and net output, productivity, measurement of capital, accounting theory.

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1. Introduction

This paper takes a new look at the production accounts in the international System of National Accounts with emphasis on alternative measures of output and primary input, with implications for the resulting alternative measures of productivity. Perhaps more importantly, the paper considers alternative measures of *income* generated by the production sector of an economy.

As well as their central use in informing macroeconomic policy, national accounts data on inputs and outputs for countries are used extensively in the academic literature on productivity; see for example Solow (1957), Jorgenson and Griliches (1967), Diewert and Fox (1999) and Fernald and Inklaar (2020). They are also used in the literature on efficiency analysis; see for example Färe, Grosskopf, Norris and Zhang (1994) and Kumar and Russell (2002). Given their extensive use and broad acceptance as the authoritative source of information on economic performance, it is tempting to believe that all matters relating to national accounts have been settled by the international community. Yet the United Nations System of National Accounts (SNA), which provides guidance to countries on optimal practice, is periodically revised.[3] Hence, it is worthwhile considering whether the concepts currently employed are appropriate for all purposes to which they are put, or if there are alternatives.

Here we start from some basic definitions which lead us to propose alternatives measures of output, input and income. In doing so, we stay within the current production boundaries of the SNA 2008. That is, our paper is not a contribution to the growing literature on “Beyond GDP” concepts nor on “GDP and Beyond”, but rather stays focussed on alternatives within the existing SNA production boundary.[4]

[3] At the time of writing, the current version is the SNA 2008 (United Nations 2009), with the next revision due to be released in 2025.

[4] The “Beyond GDP” literature typically focusses on ending the use of GDP in policy making in favour of alternatives measures of progress. The “GDP and Beyond” literature focusses retaining GDP but with possible extensions to better capture things that are important yet are not currently (well) measured in the national accounts, such as household work, consumption of free digital goods, or the use of the environment as an input. See Coyle and Mitra-Kahn (2017), OECD (2018), Corrado, Fox, Goodridge, Haskel, Jona-Lasinio, Sichel and Westlake (2017) and Brynjolfsson, Collis, Diewert, Eggers and Fox (2019).

The model of production that we use in this paper is based on treating capital as both an input used and output produced by the production sector of an economy. This model of production was developed by the economist Hicks and the accountants Edwards and Bell as shown by the following two quotations:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period). This model of production could be viewed as an *Austrian model of production* in honour of the Austrian economist Böhm-Bawerk

(1891) who viewed production as an activity which used raw materials and labour to further process partly finished goods into finally demanded goods.[5]

The rest of the paper is organised as follows. The next section introduces production accounting using a simplified context of a single production unit. Section 3 considers alternative net output, input and income concepts for a production unit, while section 4 considers corresponding economy wide measures with multiple types of capital. Section 5 concludes.

2. Production Unit Accounting: The Hicks and Edwards and Bell framework

In order to simplify the notation, we consider a very simple model of production in this section for a *single production unit* that produces or uses only six types of goods and services during an accounting period t. A production unit could be a firm, a division of a firm or what national income accountants call an establishment. The establishment must be able to provide period by period accounting information about periodic revenues and costs as well as balance sheet information on the status of its asset holdings at the end of each accounting period.

Equation (1) below defines the production unit's *pure profits* in period t, Π^t, using the Hicks, Edwards and Bell approach to production theory:

$$\Pi^t \equiv P_Y^t\, Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t - P_L^t Q_L^t + P_K^t Q_K^t - (1 + r^t) P_K^{t-1} Q_K^{t-1}. \qquad (1)$$

The price and quantity variables appearing on the right hand side of (1) are defined as follows:

$P_Y^t \;\equiv$ (unit value) price of output Y during period t;

[5] This Austrian model of production was further developed by von Neumann (1937) and Malinvaud (1953) but these authors did not develop the user cost implications of the model. On the user cost implications of the Austrian model, see Hicks (1973; 27-35) and Diewert (1977; 108-111) (1980; 472-474).

Q_Y^t ≡ total quantity of output y produced during period t;

P_Z^t ≡ (unit value) price of intermediate input Z purchased during period t;

Q_Z^t ≡ total quantity purchased of intermediate input Z purchased during period t;

P_{IP}^t ≡ (unit value) price of one unit of an investment good *purchased* during period t;

Q_{IP}^t ≡ total number of units of the investment good *purchased* during period t;

P_L^t ≡ wage rate for one hour of labour used by the producer during period t;

Q_L^t ≡ total hours worked in period t by the type of labour under consideration;

P_K^t ≡ price of a unit of the capital stock held by the unit at the end of period t;

Q_K^t ≡ quantity of the capital stock held by the production unit at the end of period t;

P_K^{t-1} ≡ price of a unit of the capital stock held by the unit at the beginning of period t;

Q_K^{t-1} ≡ quantity of the capital stock held by the unit at the beginning of period t;

r^t ≡ period t cost of capital for the production unit.

Units of the total output Q_Y^t could be sold to domestic customers or could be exported. Later in the paper, this distinction will become important when we aggregate over producers but at present, we do not have to distinguish domestic sales from foreign sales. Similarly, units of the intermediate input and units of the investment good could be purchased from domestic suppliers or could be imported.

We note that prices and quantities of output, intermediate input, purchased investment goods and labour can in principle be observed by the accountant. However, the quantity and price of the production unit's beginning and end of period capita stocks, Q_K^{t-1}, Q_K^t, P_K^{t-1} and P_K^t, *typically cannot be observed* but must be estimated by the accountant. We will indicate how this can be done shortly. The production unit's period t *cost of capital* is denoted by r^t on the right hand side of (1). If the production unit purchased its beginning of period t capital stock and financed this purchase by issuing a one period bond at the interest rate r^{t^*} in the amount equal to $P_K^{t-1}Q_K^{t-1}$, then r^t in definition (1) would equal the observed bond interest rate r^{t^*}.[6] However, in general, since a firm's holdings of beginning

[6] See Diewert (2014) for a more complete accounting model that deals with the financing of the initial capital stock and other financial transactions using the Hicksian accounting framework.

of the period assets are financed by a mixture of debt and equity capital, a firm's weighted cost of capital must be estimated by the national income accountant since there is no unambiguous estimate for the equity portion of a firm's financial capital.

Standard firm accounting does not allow for a deduction for the cost of equity capital[7] but following Hicks' (1946) intertemporal theory of the firm, it is clear that future cash flows should be discounted by an appropriate interest rate or cost of capital in order to make future cash flows comparable to present cash. Accounting conventions suggest that current period flows should be cumulated over the accounting period and "realized" at the end of the accounting period.[8] Thus the discounted pure profits of the production unit for period t are equal to minus the beginning of the period cost of the capital stock, $-P_K^{t-1}Q_K^{t-1}$, plus the period t *discounted cash flow* of firm revenues minus firm expenditures on flow inputs and market purchases of investment goods, $(1+r^t)^{-1}(P_Y^t\,Q_Y^t - P_Z^tQ_Z^t - P_{IP}^tQ_{IP}^t - P_L^tQ_L^t)$, plus the discounted end of period value of the production unit's capital stock, $(1+r^t)^{-1}P_K^tQ_K^t$. But if we measure profits from the *perspective of the end of period t*, then the resulting "anti-discounted" profits are equal to $(1+r^t)P_K^{t-1}Q_K^{t-1}$ plus cash flow plus the value of the capital stock at the end of period t, which is equal to pure profits Π^t defined by (1).

At this point, we need to make some assumptions about investments, depreciation and capital stocks. The first point to note is that, in general, investment goods could be purchased or they could be manufactured by the production unit. Thus we have defined P_{IP}^t and Q_{IP}^t as the period t price and quantity of purchased investment goods. However, the production unit may also produce units of the investment good internally for its own use. Thus define $Q_{II}^t > 0$ as the amount of internally produced investment and P_{II}^t as the

[7] This accounting convention dates back to Garske and Fells (1893). For a discussion of this convention, see Anthony (1973). Diewert and Fox (1999) attributed some of the fall in the world wide fall in Total Factor Productivity during the 1970s to the problems associated with measuring income using historical cost accounting when inflation is high.

[8] "This [convention] accords with the assumption conventional in discrete compounding that flows occur at the end of each period." K.V. Peasnell (1981; 56).

imputed price for a unit of this internally produced investment.[9] Define period t *total investment* as the sum of purchased investment, Q_{IP}^{t}, plus internally produced investment, Q_{II}^{t}:

$$Q_I^t = Q_{IP}^t + Q_{II}^t \qquad (2)$$

Our next assumption relates period t total investment to the beginning and end of period t capital stocks held by the unit; i.e., we assume that the following equation holds:

$$Q_K^t = (1 - \delta^t)Q_K^{t-1} + Q_I^t \qquad (3)$$

where δ^t is the *period t geometric depreciation rate* that is applied to the production unit's beginning of the period capital stock Q_K^{t-1} in order to obtain the number of constant quality units of the initial capital stock at the end of period t that are equivalent to new units of the capital stock.[10]

The price of a new unit of the capital stock at the beginning of period t, P_K^{t-1}, should be equal to the price of a new investment good at the *beginning* of period t. Note that this beginning of the period price is not necessarily equal to the period t market price of the investment good, P_{IP}^t, since P_{IP}^t price represents the *average price* of the investment good over the entire duration of period t. Similarly, the price of a new unit of the capital stock at the end of period t, P_K^t, is not necessarily equal to P_{IP}^t. If inflation is low, then P_K^t could be approximated by P_{IP}^t. If general inflation is high during period t, then P_K^t could be approximated by $(½)P_{IP}^{t-1} + (½)P_{IP}^t$.[11] More generally, one could argue that in a situation

[9] If $Q_{II}^t = 0$, there is no need to impute P_{II}^t. If $Q_{II}^t > 0$, then define P_{II}^t as the average cost of producing the internally manufactured investment goods. Typically, Q_{II}^t will be a small amount of total investment. If firms make very large infrastructure investments such as building pipelines or new natural gas liquefaction plants, then internally produced investments become important.

[10] The geometric model of depreciation was used by Jorgenson and Griliches (1967) in their classic study of the Total Factor Productivity of the U.S. economy. For additional materials on the geometric model of depreciation, see Jorgenson (1989) (1996a) (1996b) and Schreyer (2001).

[11] Commercial accounting "solves" this capital stock valuation problem by using historical cost accounting which simply carries forward the initial purchase value of a capital stock and applies a suitable depreciation rate to this initial value without making any adjustment for price change. See Ijiri (1979) for a defense of historical cost accounting.

where asset prices are very volatile, instead of using the price of an investment good at the beginning and end of a period, one should use a longer run *smoothed investment price* for P_K^t that captures the *trend* in the price of a new unit of a particular capital stock component. Typically firms do not actually sell their capital stocks; they hold units of their capital stock until they are completely worn out. However, the owners of firms are interested in end of period values for the capital stocks held by the firm because there is always the option of selling these capital stocks. If asset prices are very volatile, using a smoothed estimate for the current values of capital stock components may give investors a more realistic picture of the current opportunity cost of holding the existing capital stocks in the production unit rather than using an estimated current value which is subject to large fluctuations.

In any case, we assume that the national income accountant has estimates available for the beginning and end of period t prices of a new unit of the capital stock. These prices can be used to define the *period t asset inflation rat*e i^t using the following equation:

$$1 + i^t = P_K^t/P_K^{t-1}. \tag{4}$$

Thus $P_K^t = (1 + i^t)P_K^{t-1}$. Now use (4) to eliminate P_K^t and use (3) to eliminate Q_K^t from definition (1). This allows us to express period t pure profits Π^t for the production unit as follows:

$$\begin{aligned}\Pi^t &= P_Y^tQ_Y^t - P_Z^tQ_Z^t - P_{IP}^tQ_{IP}^t - P_L^tQ_L^t \\ &\quad +(1 + i^t)P_K^{t-1}[(1 - \delta^t)Q_K^{t-1} + Q_I^t] - (1 + r^t)P_K^{t-1}Q_K^{t-1} \\ &= P_Y^tQ_Y^t - P_Z^tQ_Z^t - P_{IP}^tQ_{IP}^t + P_K^tQ_I^t - P_L^tQ_L^t - U^tQ_K^{t-1}.\end{aligned} \tag{5}$$

The *period t user cost of capital U^t* which makes its appearance in the second line of (5) is defined as follows:[12]

[12] Babbage (1835; 287) described the user cost idea in words and Walras (1954; 268-269) developed an explicit user cost formula (in 1874) as did the industrial engineer Church (1901; 907-908). Alternative derivations of a user cost formula may be found in Jorgenson (1963) (1989) (1996b), Griliches (1963; 120), Christensen and Jorgenson (1969; 302), Diewert (1974; 504) and Diewert and Lawrence (2000; 276).

$$U^t = [(1 + r^t) - (1 + i^t)(1 - \delta^t)]P_K^{t-1} \qquad (6)$$
$$= [r^t - i^t + (1 + i^t)\delta^t]P_K^{t-1}.$$

Thus the user cost of capital consists of three terms: the interest rate term $r^t P_K^{t-1}$, less an asset price inflation term $-iP_K^{t-1}$, plus a depreciation term valued at the end of period price of a new asset, $(1 + i^t)\delta^t P_K^{t-1} = \delta^t P_K^t$.[13]

Note that the treatment of investment in expression (5) is not conventional: see the terms $-P_{IP}^t Q_{IP}^t + P_K^t Q_I^t$ which add to profits the value of *total investment* Q_I^t valued at the end of period price of a unit of capital, P_K^t, and subtract the value of *purchased investment* valued at market prices, $-P_{IP}^t Q_{IP}^t$. The remaining terms in (5) are conventional: $P_Y^t Q_Y^t - P_Z^t Q_Z^t$ is equal to revenues less payments for intermediate inputs, or value added, and $P_L^t\, Q_L^t + U^t Q_K^{t-1}$ is primary input cost made up of labour cost, $P_L^t\, Q_L^t$, and capital services cost, $U^t Q_K^{t-1}$.

It should be noted that a conventional economic treatment of firm accounting would not measure profits according to definition (1) or its special case (5) which was derived from (1) using assumptions (2)-(4). Conventional economic accounting would immediately capitalize all investments and define *conventional period t pure profits* of the production unit, Π^{t^*}, as follows:

$$\Pi^{t^*} \equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_L^t\, Q_L^t - U^t Q_K^{t-1}. \qquad (7)$$

However, Π^{t^*} defined by (7) will equal Π^t defined by (1) or (5) *if* the end of period t price of capital, P_K^t, is set equal to the period t average price of market purchased investments, P_{IP}^t, *and if* there are no internally produced investment goods so that total investment, Q_I^t, equals purchased investment, Q_{IP}^t.

[13] If the asset is a land or structure asset, then the use of this input may also be subject to a property tax. If the period t property tax rate τ^t is a percentage of the beginning of the period value of the asset, then the user cost becomes $[r^t - i^t + (1 + i^t)\delta^t + \tau^t]P_K^{t-1}$.

In the following section, we will look at alternative output and input measures that could be constructed using our Hicksian measurement framework.

3. Alternative Domestic Net Output, Input and Income Concepts

Period t Gross Domestic Input or Income generated by the production unit, GDI^t, can be defined as the value of labour services $P_L^t\, Q_L^t$ plus the value of capital services $U^t Q_K^{t-1}$ plus the value of pure profits Π^t:

$$GDI^t \equiv P_L^t\, Q_L^t + U^t Q_K^{t-1} + \Pi^t \tag{8}$$

To get the measure of production unit output that corresponds to the income measure defined by (8), replace Π^t in (8) by the right hand side of (5). Period t *Gross Domestic Output*, GDO^t, is then defined as follows:

$$\begin{aligned} GDO^t &\equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t + P_K^t Q_I^t \\ &= CVA^t + P_K^t Q_I^t \\ &= GDI^t \end{aligned} \tag{9}$$

where period t *Comprehensive Value Added* of the production unit, CVA^t, is defined as *Regular Value Added*, $VA^t \equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t$, less market expenditures on the investment good, $P_{IP}^t Q_{IP}^t$.[14] Thus period t CVA^t is defined as:

$$\begin{aligned} CVA^t &\equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t \\ &= VA^t - P_{IP}^t Q_{IP}^t \end{aligned} \tag{10}$$

Suppose the following conditions hold:

[14] The production unit could be producing units of the capital stock and this production would be included in the definition of a firm's regular value added. However, purchases of units of the capital stock are not included in regular value added because the cost of purchased investment goods is capitalized and depreciated over time using normal accounting procedures. Comprehensive Value Added allows revenues from sales of the investment good and costs from purchases of the investment good to enter the net output aggregate.

$$P_K^t = P_{IP}^t\,; Q_I^t = Q_{IP}^t \tag{11}$$

Then it can be seen that our measure of gross output, GDO^t, is equal to Regular Value added, VA^t.

The problem with the gross income measure, GDI^t defined by (8) is that it includes the value of depreciation as a component of income. But depreciation is not a component of income that can be spent on the purchase of consumer goods and services. Thus the depreciation component of user cost should be removed as a source of income and transferred to the net output accounts; i.e., depreciation should be treated as deduction from production unit revenues and be treated as a type of intertemporal intermediate input.[15] The period t value of depreciation (valued at end of period prices of the capital stock) is $P_K^t \delta^t Q_K^{t-1} = (1+i^t)\delta^t P_K^{t-1} Q_K^{t-1}$. Subtract this term from period t Gross Domestic Income to define the period t *Net Domestic Income*, NDI^t, generated by the production unit:

$$\begin{aligned} NDI^t &\equiv GDI^t - (1+i^t)\delta^t P_K^{t-1} Q_K^{t-1} \\ &= P_L^t Q_L^t + [r^t - i^t] P_K^{t-1} Q_K^{t-1} + \Pi^t \quad \text{using (8) and (6).} \end{aligned} \tag{12}$$

In order to obtain the output measure NDO^t that matches up with the net income measure NDI^t defined by (12), substitute the right hand side of (5) to eliminate $\Pi^t$from the second line in (12). We obtain the following expression for the *Net Domestic Output* NDO^t produced by the production unit during period t:

$$\begin{aligned} NDO^t &\equiv Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t + P_K^t [Q_I^t - \delta^t Q_K^{t-1}] \\ &= CVA^t + P_K^t [Q_I^t - \delta^t Q_K^{t-1}] \quad \text{using definition (10)} \\ &= CVA^t + P_K^t [Q_K^t - Q_K^{t-1}] \quad \text{using (3)} \\ &= NDI^t. \end{aligned} \tag{13}$$

[15] See Hicks (1946; 174) (1973; 155) and Samuelson (1961) on alternative definitions of income and on the treatment of depreciation.

The second line of (13) tells us that period t Net Domestic Output is equal to the production unit's Comprehensive Value Added, CVA^t, plus the production unit's period t gross investment, Q_I^t, less period t depreciation of the starting capital stock, $\delta^t Q_K^{t-1}$, valued at the end of period capital stock price, P_K^t. Note that $Q_I^t - \delta^t Q_K^{t-1} = Q_K^t - Q_K^{t-1}$ is period t net investment.

The measure of net output defined by (13) looks reasonable enough. It adds the value of net investment (valued at the end of period price for units of the capital stock) to a comprehensive measure of value added produced by the production unit during period t. Thus this net output measure is consistent with Pigou's (1941; 273-274) preference for an output measure that is consistent with maintaining the physical capital stock. However, the problem with the net output measures of output and income, NDO^t and NDI^t, is the fact that the income measure does not accurately measure the nominal income generated by the production unit over the period; NDI^t omits the capital gains (or losses) that accrue to the initial capital stock held by the production unit. Adding these capital gains to NDI^t leads to period t *Comprehensive Net Domestic Income* generated by the producer over period t, $CNDI^t$, defined as follows:

$$\begin{aligned} CNDI^t &\equiv P_L^t Q_L^t + r^t P_K^{t-1} Q_K^{t-1} + \Pi^t \\ &= NDI^t + i^t P_K^{t-1} Q_K^{t-1} \ \text{ using the second line in (12).} \end{aligned} \tag{14}$$

The first line in (14) tells us comprehensive net income is equal to payments to labour $P_L^t Q_L^t$ plus interest and dividend payments to the owners of the production unit for tying up their capital for the period, $r^t P_K^{t-1} Q_K^{t-1}$, plus any pure profits Π^t that might have occurred.[16] The second line in (14) tells us that $CNDI^t$ is equal to NDI^t plus capital gains on the production unit's initial capital stock.

In order to determine the net output measure that matches up with the comprehensive measure of income defined by the first line in (14), we use the right hand side of (5) to

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[16] Rymes (1968) (1983) defined $r^t P_K^{t-1} Q_K^{t-1}$ as *waiting services* and advocated replacing the user cost of capital by waiting services.

eliminate Π^t from the right hand side of (14). We obtain the following expression for period t *Comprehensive Net Domestic Output*, $CNDO^t$ for the production unit:

$$\begin{aligned} CNDO^t &\equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t + \; P_K^t[Q_I^t - \delta^t Q_K^{t-1}] + \; i^t P_K^{t-1} Q_K^{t-1} && (15) \\ &= CVA^t + P_K^t[Q_I^t - \delta^t Q_K^{t-1}] + \; i^t P_K^{t-1} Q_K^{t-1} && \text{using (10)} \\ &= CVA^t + P_K^t[Q_K^t - Q_K^{t-1}] + i^t P_K^{t-1} Q_K^{t-1} && \text{using (3)} \\ &= CVA^t + P_K^t Q_K^t - (1 + i^t) P_K^{t-1} Q_K^{t-1} + i^t P_K^{t-1} Q_K^{t-1} && \text{using (4)} \\ &= CVA^t + P_K^t Q_K^t - P_K^{t-1} Q_K^{t-1} \\ &= NDO^t + i^t P_K^{t-1} Q_K^{t-1} \end{aligned}$$

The second last line in (15) tells us that our comprehensive measure of net domestic product for the production unit $CNDO^t$ is equal to comprehensive value added, CVA^t, plus the value of the end of period capital stock, $P_K^t Q_K^t$, less the value of the beginning of the period capital stock, $P_K^{t-1} Q_K^{t-1}$. This is a very straightforward definition of net (nominal) output. On the other hand, the net domestic measure of output, NDO^t, is equal to CVA^t plus the net change in the capital stock evaluated at end of period prices, $P_K^t[Q_K^t - Q_K^{t-1}]$. The last line in (15) shows that $CNDO^t$ is equal to NDO^t plus asset appreciation $i^t P_K^{t-1} Q_K^{t-1}$ if the asset inflation rate i^t is positive. If i^t is negative due to obsolescence or other reasons, then Comprehensive Net Domestic Output will be less than Net Domestic Output. Thus the comprehensive net income measure is a maintenance of financial capital approach to the measurement of income whereas the net income measure is a maintenance of real physical capital approach.

Having estimates of the nominal income generated by a production unit is not the end of the story. In order to evaluate the contributions of a production sector to the creation of income, it is necessary to convert the nominal income measure into a real income measure. That is, the nominal measure of income should be divided by a consumer price index to convert nominal income flows into real income flows. We note that our suggested comprehensive measure of real income generated by a production unit (which is $CNDI^t$ deflated by a consumer price index) is exactly the income concept recommended by the accountant Sterling:

"It follows that the appropriate procedure is to (1) adjust the present statement to current values and (2) adjust the previous statement by a price index. It is important to recognize that *both* adjustments are necessary and that neither is a substitute for the other. Confusion on this point is widespread." Robert R. Sterling (1975; 51).

Sterling (1975; 50) termed his income concept *Price Level Adjusted Current Value Income*. Unfortunately, Sterling's income concept has not been widely endorsed in accounting circles due to difficulties in implementing it in an unambiguous manner. But conceptually, Sterling's income concept is consistent with our Comprehensive Net Domestic Product income concept that is deflated by a consumer price index.

Which income concept is "best"? The gross income concept clearly overstates sustainable consumption and so this concept can be dismissed. However, choosing between the physical and real financial maintenance perspectives is more problematical: reasonable economists could differ on this choice. The merits of the two perspectives were debated by Pigou and Hayek over 80 years ago. Pigou (1941; 273-274) favoured the maintenance of physical capital approach while Hayek (1941; 276-277) favoured the maintenance of real financial capital approach (the approach of Sterling). Hayek noted that obsolescence of a capital good[17] leads to a loss of income which is not captured in the maintenance of physical capital approach to income measurement but it is captured in the maintenance of financial capital approach. Moreover, the approach of Pigou does not capture the gains in income that are generated by increasing land prices. The amount of land could remain constant but increases in the price of business land that are greater than the change in the consumer price index should lead to an increase in the real income generated by a production unit but the physical approach neglects these real income gains. If a price increase in an asset is foreseen, then the revaluation term can be regarded as a positive contribution to the net revenues produced by the production unit under consideration; i.e., the unit "transports" the asset from a time when it is less valued to a time when it is more highly valued.

As Hicks (1946; 184) said in his income chapter: "What a tricky business this all is!"

[17] This is the case where i^t is negative.

Table 1 shows the relationship of the three alternative definitions of output and relationship of the three matching definitions of income or primary input, where $CVA^t \equiv P_Y^t Q_Y^t - P_Z^t Q_Z^t - P_{IP}^t Q_{IP}^t = VA^t - P_{IP}^t Q_{IP}^t$ is Comprehensive Value Added from (10), and $[r^t - i^t + (1+i^t)\delta^t]P_K^{t-1} = U^t$is the user cost of capital from (6):

Table 1: Alternative Output and Corresponding Income Concepts

Output Concepts	Income Concepts
$GDO^t = CVA^t + P_K^t Q_I^t$	$GDI^t = P_L^t Q_L^t + [r^t - i^t + (1+i^t)\delta^t]P_K^{t-1} Q_K^{t-1} + \Pi^t$
$NDO^t = GDO^t - (1+i^t)\delta^t P_K^{t-1} Q_K^{t-1}$	$NDI^t = P_L^t Q_L^t + [r^t - i^t]P_K^{t-1} Q_K^{t-1} + \Pi^t$
$CNDO^t = NDO^t + i^t P_K^{t-1} Q_K^{t-1}$	$CNDI^t = P_L^t Q_L^t + r^t P_K^{t-1} Q_K^{t-1} + \Pi^t$

4. Economy Wide Measures of Output, Input and Income

In this section, we extend the analysis to many types of capital and we also aggregate over production units. Suppose there are F production units in the economy, N types of capital, J classes of outputs (including outputs of capital goods) and M classes of intermediate inputs (excluding capital good purchases).[18] The counterparts to definitions (1)-(6) will be explained below.

Define the *period t pure profits of production unit* f, Π_f^t, as follows, for $f = 1, \ldots, F$:

[18] To keep our notation as simple as possible, we have only one type of labour in the economy. The algebra can readily be generalized to many types of labour.

$$\Pi_f^t \equiv \sum_{j=1}^{J} P_{Yfj}^t Q_{Yfj}^t - \sum_{m=1}^{M} P_{Zfm}^t Q_{Zfm}^t - \sum_{n=1}^{N} P_{IPfn}^t Q_{IPfn}^t \quad (16)$$
$$- P_{Lf}^t\, Q_{Lf}^t + \sum_{n=1}^{N} P_{Kfn}^t Q_{Kfn}^t - (1+r^t) \sum_{n=1}^{N} P_{Kfn}^{t-1} Q_{Kfn}^{t-1}$$
$$= CVA_f^t - P_{Lf}^t\, Q_{Lf}^t + \sum_{n=1}^{N} P_{Kfn}^t Q_{Kfn}^t - (1+r^t) \sum_{n=1}^{N} P_{Kfn}^{t-1} Q_{Kfn}^{t-1}$$

The *Comprehensive Value Added* for production unit f, CVA_f^t, is defined as follows, for $f = 1, \ldots, F$:

$$CVA_f^t \equiv \sum_{j=1}^{J} P_{Yfj}^t Q_{Yfj}^t - \sum_{m=1}^{M} P_{Zfm}^t Q_{Zfm}^t - \sum_{n=1}^{N} P_{IPfn}^t Q_{IPfn}^t \quad (17)$$

The various price and quantity variables appearing on the right hand side of definitions (16) and (17) are defined as follows:

P_{Yfj}^t ≡ (unit value) price of output j sold by production unit f during period t;

Q_{Yfj}^t ≡ total quantity of output j produced by unit f during period t;

P_{Zfm}^t ≡ (unit value) price of intermediate input m purchased by unit f during period t;

Q_{Zfm}^t ≡ total quantity purchased of intermediate input m purchased by unit f during period t;

P_{IPfn}^t ≡ (unit value) price of one unit of investment good n *purchased* by unit f during period t;

Q_{IPfn}^t ≡ total number of units of the investment good n *purchased* by unit f during period t;

P_{Lf}^t ≡ wage rate for one hour of labour used by unit f during period t;

Q_{Lf}^t ≡ total hours worked for unit f in period t;

P_{Kfn}^t ≡ price of a unit of capital stock n held by unit f at the end of period t;

Q_{Kfn}^{t} $\equiv$ quantity of capital stock n held by unit f at the end of period t;

P_{Kfn}^{t-1} $\equiv$ price of a unit of the capital stock n held by unit f at the beginning of period t;

Q_{Kfn}^{t-1} $\equiv$ quantity of capital stock n held by unit f at the beginning of period t;

r^{t} $\equiv$ period t cost of capital for all production units.

The assumption that the cost of capital r^t is constant across all production units is only a very rough approximation to reality. We make this assumption because at a later stage of our analysis, we adapt our algebra to the problem of determining an economy wide *ex post* rate return on capital.

We have defined P_{IPfn}^{t} and Q_{IPfn}^{t} as the period t price and quantity of purchases of investment good n by production unit f. However, each production unit may also produce units of the investment good internally for its own use. Thus define $Q_{IIfn}^{t} \geq 0$ as the amount of internally produced investment good n by unit f and P_{IIfn}^{t} as the corresponding imputed price for a unit of this internally produced investment. Define period t *total investment* in the n^{th} capital stock by production unit f as the sum of purchased investment, Q_{IPfn}^{t}, plus internally produced investment, Q_{IIfn}^{t}:

$$Q_{Ifn}^{t} \equiv Q_{IPfn}^{t} + Q_{IIfn}^{t}; \quad f = 1, \ldots, F; n = 1, \ldots, N. \qquad (18)$$

As in the previous section, we assume that *geometric depreciation* applies to each capital stock. Thus we assume that the following relationships between beginning and end of period capital stocks and total investment hold:

$$Q_{Kfn}^{t} = (1 - \delta_n^{t}) Q_{Kfn}^{t-1} + Q_{Ifn}^{t}; \quad f = 1, \ldots, F; n = 1, \ldots, N. \qquad (19)$$

Note that the period t geometric depreciation rate for the n^{th} type of capital, $\delta_n^{t\,t}$, depends on t and n but not on f.

Define economy wide *aggregate investment* and *aggregate capital stocks* for each asset n, Q_{In}^{t}, Q_{Kn}^{t-1} and Q_{Kn}^{t} as follows:

$$Q_{In}^{t} \equiv \sum_{f=1}^{F} Q_{Ifn}^{t}\,; \quad Q_{Kn}^{t-1} \equiv \sum_{f=1}^{F} Q_{Kfn}^{t-1}\,; \quad Q_{Kn}^{t} \equiv \sum_{f=1}^{F} Q_{Kfn}^{t}. \tag{20}$$

Now sum equations (19) over production units f. Using definitions (20), we obtain the following relationship between aggregate capital stocks and aggregate total investment:

$$Q_{Kn}^{t} = (1-\delta_n^t)Q_{Kn}^{t-1} + Q_{In}^{t}; \quad n = 1, \ldots, N. \tag{21}$$

Define *aggregate labour input* Q_L^t by summing labour input over production units:

$$Q_L^t \equiv \textstyle\sum_{f=1}^{F} Q_{Lf}^{t}. \tag{22}$$

Definitions (23)-(25) define (unit value) *prices* for aggregate beginning and end of period capital stocks, Q_{Kn}^{t-1} and Q_{Kn}^{t} for each asset n, $n = 1, \ldots, N$, and for aggregate labour input Q_L^t:

$$P_{Kn}^{t-1} \equiv \sum_{f=1}^{F} P_{Kfn}^{t-1} Q_{Kfn}^{t-1} / Q_{Kn}^{t-1}\,; \tag{23}$$

$$P_{Kn}^{t} \equiv \sum_{f=1}^{F} P_{Kfn}^{t} Q_{Kfn}^{t} / Q_{Kn}^{t}\,; \tag{24}$$

$$P_L^t \equiv \sum_{f=1}^{F} P_{Lf}^t Q_{Lf}^t / Q_L^t . \qquad (25)$$

Once the aggregate beginning and end of period prices for each of the N capital stocks has been defined, we can define the N *aggregate asset inflation rates* i_n^t by using the following definitions:

$$1 + i_n^t \equiv P_{Kn}^t / P_{Kn}^{t-1} ; \quad n = 1, \ldots, N \qquad (26)$$

Recall that the period t pure profits of production unit f, Π_f^t, were defined by (16). Define *economy wide aggregate profits* for period t, Π^t, by summing profits over production units:

$$\begin{aligned}
\Pi^t &\equiv \sum_{f=1}^{F} \Pi_f^t && (27)\\
&= \sum_{f=1}^{F}\left[CVA_f^t - P_{Lf}^t Q_{Lf}^t + \sum_{n=1}^{N} P_{Kfn}^t Q_{Kfn}^t - (1+r^t)\sum_{n=1}^{N} P_{Kfn}^{t-1} Q_{Kfn}^{t-1}\right]\\
&\qquad\qquad\qquad\qquad \text{using (16)}\\
&= \sum_{f=1}^{F} CVA^t - \sum_{f=1}^{F} P_{Lf}^t Q_{Lf}^t + \sum_{n=1}^{N}\sum_{f=1}^{F} P_{Kfn}^t Q_{Kfn}^t - (1+r^t)\sum_{n=1}^{N}\sum_{f=1}^{F} P_{Kfn}^{t-1} Q_{Kfn}^{t-1}\\
&= CVA^t - P_L^t Q_L^t + \sum_{n=1}^{N} P_{Kn}^t Q_{Kn}^t - (1+r^t)\sum_{n=1}^{N} P_{Kn}^{t-1} Q_{Kn}^{t-1}\\
&\qquad\qquad\qquad\qquad \text{using (20) and (22)-(25)}\\
&= CVA^t - P_L^t Q_L^t + \sum_{n=1}^{N}(1+i_n^t) P_{Kn}^{t-1} Q_{Kn}^t - (1+r^t)\sum_{n=1}^{N} P_{Kn}^{t-1} Q_{Kn}^{t-1}\\
&\qquad\qquad\qquad\qquad \text{using (26)}\\
&= CVA^t - P_L^t Q_L^t + \sum_{n=1}^{N}(1+i_n^t) P_{kn}^{t-1}[(1-\delta_n^t) Q_{Kn}^{t-1} + Q_{In}^t]\\
&\qquad - (1+r^t)\sum_{n=1}^{N} P_{Kn}^{t-1} Q_{Kn}^{t-1} \qquad \text{using (21)}\\
&= CVA^t - P_L^t Q_L^t + \sum_{n=1}^{N}\left(1+i_n^t\right) P_{Kn}^t Q_{In}^t\\
&\qquad - \sum_{n=1}^{N}[(1+r^t) - (1+i_n^t)(1-\delta_n^t)] P_{Kn}^{t-1} Q_{Kn}^{t-1}\\
&= CVA^t - P_L^t Q_L^t + \sum_{n=1}^{N} P_{Kn}^t Q_{In}^t - \sum_{n=1}^{N} U_n^t Q_{Kn}^{t-1}
\end{aligned}$$

The period t user cost for the nth type of capital, U_n^t, is defined by (28) and the period t aggregate *Comprehensive Value Added* for the economy, CVA^t is defined by (29):

$$U_n^t \equiv [r^t + i^t + \delta^t(1+i^t)] P_{kn}^t; \quad n = 1, \ldots, N; \qquad (28)$$

$$CVA^t \equiv \sum_{f=1}^{F} CVA_f^t = \sum_{f=1}^{F} \left\{ \sum_{j=1}^{J} P_{Yfj}^t Q_{Yfj}^t - \sum_{m=1}^{M} P_{Zfm}^t Q_{Zfm}^t - \sum_{n=1}^{N} P_{IPfn}^t Q_{IPfn}^t \right\} \quad (29)$$

It can be seen that the definition of aggregate pure profits given by (27) is the counterpart to (7), the definition of profits for a single production unit that was derived in the previous section. The decomposition (27) has the same structure as (7) except that now there are N assets instead of a single asset. It is straightforward to define aggregate gross domestic output, net domestic output and comprehensive net domestic output. Corresponding to the single production unit case summarized in Table 1, equations (30)-(32) show the relationship of the three alternative definitions of aggregate output and equations (33)-(35) show the relationship of the three matching definitions of income or primary input.

$$GDO^t = CVA^t + \sum_{n=1}^{N} P_{Kn}^t Q_{In}^t ; \quad (30)$$

$$NDO^t = CVA^t + \sum_{n=1}^{N} P_{Kn}^t [Q_{Kn}^t - Q_{Kn}^{t-1}] \quad (31)$$

$$= GDO^t - \sum_{n=1}^{N} (1 + i_n^t)\delta^t P_{Kn}^{t-1} Q_{Kn}^{t-1} ;$$

$$CNDO^t = CVA^t + \sum_{n=1}^{N} P_{Kn}^t Q_{Kn}^t - \sum_{n=1}^{N} P_{Kn}^{t-1} Q_{Kn}^{t-1} \quad (32)$$

$$= NDO^t + \sum_{n=1}^{N} i_n^t P_{Kn}^{t-1} Q_{Kn}^{t-1} ;$$

$$GDI^t = P_L^t Q_L^t + \sum_{n=1}^{N} [r^t - i_n^t + (1 + i_n^t)\delta^t] P_{Kn}^{t-1} Q_{Kn}^{t-1} + \sum_{f=1}^{F} \Pi_f^t ; \quad (33)$$

$$NDI^t = P_L^t Q_L^t + \sum_{n=1}^{N} [r^t - i_n^t] P_{Kn}^{t-1} Q_{Kn}^{t-1} + \sum_{f=1}^{F} \Pi_{\mathrm{f}}^t ; \quad (34)$$

$$CNDI^t = P_L^t Q_L^t + \sum_{n=1}^{N} r^t P_{Kn}^{t-1} Q_{Kn}^{t-1} + \sum_{f=1}^{F} \Pi_{\mathrm{f}}^t . \quad (35)$$

Note that in (30), gross investments Q_{In}^t are valued at the end of period prices for the corresponding capital stocks, P_{Kn}^t. Similarly, in (31), period t net investment in asset n, $Q_{Kn}^t - Q_{Kn}^{t-1}$, is valued at the end of period prices for the corresponding capital stocks, P_{Kn}^t.

Equation (27) tells us that GDO^t is equal to GDI^t. Thus the right hand side of (30) is equal to the right hand side of (33). Set all of the firm pure profits equal to 0 and solve the equation that results from equating (30) to (33) for r^t. This is a linear equation which always has a solution. If we use ex post asset inflation rates in this equation, the r^t solution to this equation is the *ex post average rate of return* on all assets employed in the production sector. If we have smoothed the asset inflation rates, the resulting r^t can be regarded as an approximation to the *cost of capital* for the economy. Once the cost of capital is known, user costs and waiting services can be constructed and the various measures of output and input defined above can be constructed using available data.

It is not necessary to have industry or firm specific data on outputs and intermediate inputs. Each output produced by a domestic production unit is delivered to final domestic demand (i.e., to a household, government agency or exported) and each intermediate input used by a domestic producer (including purchases of investment goods from other domestic producers or from foreign suppliers) is supplied by a domestic producer or is imported. Thus it can be seen that aggregate comprehensive value added is equal to the value of household expenditures on consumer goods and services (valued at producer prices)[19] plus the value of government (net) purchases of goods and services from the private production sector plus the value of exports (before export taxes) less the value of imports (after import taxes). Suppose we have period t price and quantity indexes for these four components of final demand, say P_C^t, P_G^t, P_x^t and P_M^t for prices and Q_C^t, Q_G^t, Q_x^t and Q_M^t for quantities. Investment goods which are produced by a domestic producer and purchased by another domestic producer cancel out as we aggregate over production units. Produced investment goods which are not purchased by domestic producers end up in exports and purchased

[19] Jorgenson and Griliches (1973) noted the importance of using prices that producers face in productivity studies. If an output of a domestic producer is taxed, then the producer only gets the before tax price to add to revenue; if an imported good or service is taxed, then the producer faces the after tax price and the after tax value of the input should be added to producer cost.

investment goods which are imported end up in imports.[20] Then it can be seen that the following equality holds:

$$CVA^t = P_C^t Q_C^t + P_G^t Q_G^t + P_X^t Q_X^t - P_M^t Q_M^t. \tag{36}$$

Hence, our definition of aggregate comprehensive value added from the production perspective in (29) is consistent with the standard definition of value added from the expenditure perspective in (36), and can therefore be constructed using final demand prices and quantities. Thus the economy wide various output and input measures defined above can be computed using standard macroeconomic data for an economy.

5. Conclusion

We have systematically introduced alternative output, input and income concepts, for both individual production units (such as firms) and at aggregate levels. The differences in definitions have their roots in an Austrian model of production (Böhm-Bawerk 1891) and the debate between Pigou (1941) and Hayek (1941) on the maintenance of physical versus financial capital.

This paper contributes to the literature by making clear the definitions and their relationships, highlighting how each provides a different perspective. For example, each definition of output (at both individual production unit and aggregate levels) provides a different perspective of production. Use of price deflated versions of these output concepts in productivity studies will typically lead to different perspectives on productivity performance. Similarly for the primary income/income concepts.

[20] However, there is a problem with taxed intermediate inputs that are produced domestically and purchased by a domestic final demander. The tax revenue raised by this internal commodity tax does not cancel out as we aggregate over units. For more on the treatment of taxes in the production accounts, see Diewert (2006).

Researchers using firm level data can use the results on individual production units from section 3 to provide an enhanced view of sources of firm performance. More importantly for economic management, the aggregate measures presented in section 4 could be calculated by national statistical offices, providing macroeconomists and productivity researchers additional information that can be used to better inform policy.

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