Whither Hotelling: Tests of the Theory of Exhaustible Resources*

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Abstract We review the empirical literature that extends and tests the Hotelling (1931) model of the optimal depletion of an exhaustible resource. The theory is briefly described to set the stage for the review of empirical tests and applications. Those tests can be roughly divided into two broad categories—descriptive and structural—and we discuss the strengths and weaknesses of each before presenting the empirical studies of optimal extraction under conditions of exhaustibility. We also discuss some econometric pitfalls that applied researchers face when attempting to test the model.

CONTENTS

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1 Introduction

A survey of Hotelling's (1931) model of resource extraction and tests of that theory seems particularly appropriate, since Hotelling's model has dominated the economics of exhaustible resources for many decades. Not only was Hotelling the first to derive the implications of finite reserves for the evolution of prices and consumption under an optimal plan, but he also showed that competitive markets will achieve the planner's solution. This very rosy picture is, of course, a special case of the first theorem of welfare economics, which states that competitive markets are Pareto efficient.

One might therefore conclude that, since the market will solve the resource–extraction problem, we should forget about it. Unfortunately this is not the case. Indeed, many aspects of real–world markets, such as imperfect competition, non–neutral taxation, and the absence of property rights can lead to severe intertemporal distortions. Although most of those complications were not considered by Hotelling, his model can easily be altered to assess many interesting and realistic features of fuel and non-fuel mineral markets. Our survey discusses how this can be done.

Although we discuss the theory and derive simple models of optimal extraction, we emphasize empirical tests of that theory.[1] In particular, we look at studies that use data in an attempt to assess how well the Hotelling model predicts observed outcomes. Those studies range from simple descriptive exercises to more complex structural models of optimal extraction in a dynamic setting. We do not attempt to provide a complete survey of the literature on optimal extraction, especially the numerous theoretical models. Instead, we indicate some of the more influential

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[1] A recent review of the theory is provided by Gaudet (2007)

articles where it aids the exposition and apologize to any who feel that their papers have been neglected.

The organization of the paper is as follows. In the next section, we discuss some aspects of the history of the economics of exhaustible resources. We do this to explain why interest in the subject has waned in recent years and to convince the reader that Hotelling still has something to say to us. In section 3, we derive the Hotelling model and some of its more important variants. We do this because the empirical tests that we discuss are theory driven. In particular, even the simplest descriptive studies are attempts to assess the theoretical predictions. Section 4 discusses some of the econometric pitfalls that empiricists face when attempting to test the Hotelling model. These include problems that are associated with determining if market prices are stationary, dealing with endogeneity, and measuring shadow prices. Section 5, which is the heart of the paper, discusses empirical tests of the simple Hotelling model and some of its more tested variants. We say 'tested' rather than 'interesting' because we are limited in our coverage by the literature. In other words, some aspects of the Hotelling model have been tested more than others, which means that evidence supporting or rejecting some theories is scant. Furthermore, we limit attention to studies that deal explicitly with exhaustibility as opposed to extraction more generally. Finally, section 6 contains concluding remarks and suggestions for future research.

2 Background

Hotelling's classic article, which was published in 1931, inaugurated the theory of the optimal extraction of an exhaustible resource and exhaustible–resource economics more generally. Moreover, perhaps more than any other article, it has dominated a sub-discipline of economics. Nevertheless, it wasn't until the 1970s that theorists began to take serious note. At that time, many researchers who had previously shown little interest in the subject developed more sophisticated models that modified the basic Hotelling assumptions to include realistic features of the world.

Although Hotelling derived several variants of his model, in particular he solved the monopolist's extraction problem and he considered extraction costs that increase as the resource base is depleted, since that time other researchers have introduced additional complicating factors, of which we mention a few. General equilibrium effects have been included by embedding the Hotelling model in a model of aggregate growth (Stiglitz 1974; Solow 1976). Exploration has been modeled by allowing augmentation of the resource base through discoveries (Pindyck 1978). Uncertainty about the size of reserves (Gilbert 1979) or future demand and costs (Pindyck 1980) has been introduced. Durability effects have been included by allowing recycling and stockpiling (Levhari and Pindyck 1981). Imperfect competition among producers has been considered using a dominant firm or a cartel model (Gilbert 1978; Salant 1976). Taxation effects have been modeled by introducing distortions due to non–neutral tax policy (Sweeney 1977; Dasgupta, Heal and Stiglitz 1980). Finally, technical change has been examined by considering cost–lowering technological improvements (Slade 1982).

In spite of the fact that there was an explosion of interest in the theory of exhaustible resources, as is typical in economics, empirical tests of the theoretical models lagged behind. Nevertheless, in the 1980s a number of researchers published papers containing tests of both the simple model and some of its more realistic variants (e.g., Heal and Barrow 1980; Smith 1981; Slade 1982; Farrow

1985; Miller and Upton 1985a).

By the 1990s, however, interest in the subject began to wane, and by 2000 the flow of new theories and tests had been reduced to a trickle. Furthermore, although it was common in the 1980s for economics departments to offer courses in the economics of exhaustible resources, most such courses are now offered by resource, agricultural, or mineral economics departments, if at all. It is thus safe to say that mainstream economics has neglected Hotelling, at least his theory of extraction.[2] In this section, we discuss why interest blossomed in the 1970s and waned twenty years later.

A major event, or sequence of events, almost surely accounts for the outpouring of theoretical and empirical research on the optimal depletion of an exhaustible resource — the energy–price shocks that began in the early 1970s and culminated at the end of that decade. Prior to that time, the real price of crude oil had remained relatively constant for decades. However, between 1972 and 1981, the real price increased five fold, from just under 14 to 71 (2008) dollars per barrel. At that time, the industrialized world, which was totally unprepared for such an occurrence, began to think seriously about resource depletion and the limits to growth.

We are now in the throes of another set of oil–price shocks. In fact, the situation is even more dramatic this time. Indeed, the yearly average real price of crude oil rose from a low of $13 per barrel in 1998, lower than the price in 1946 or at any time since then, to an all–time high of $145 in July of 2008, an eleven–fold increase in just one decade. Will this trigger a renaissance in the theory of

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[2] Of course, Hotelling wrote other seminal papers in, e.g., Industrial Organization (Hotelling 1929) and Public Economics (Hotelling 1938).

exhaustible resources? We think not.

One reason why interest in the Hotelling model has waned is that it is a very long–run model and attention has shifted to the here and now. Commodity, and in particular oil, markets have been so volatile in recent years that it is difficult to focus on long–run trends. To illustrate we present as an example some historical statistics for U.S. crude oil prices[3] in Figures 1 and 2. In Figure 1 we plot the log of annual average nominal and real (2008 dollar) price for the period 1949–2008. Even though the averaging process removes much variation in the data, it is clear that the price behaves very differently in the periods before and after the early 1970s. To examine price volatility in more detail, in Figure 2 we plot the annual coefficient of variation of the monthly average crude oil price for the period January, 1974 to August, 2008.[4] Although there are episodes of relatively high volatility throughout the period, the trend in volatility is clearly upward. Furthermore, it is notable that prices rose to a peak of $145 in mid July, 2008 but then fell back to about $55 four months later. Under such circumstances it is difficult to plan or to make sensible investment decisions. It is therefore no wonder that it is unfashionable to think about the very long run.

Although the most publicized, crude–oil markets are not alone in their volatility. For example, copper prices rose from under $1 a pound in 2003 to over $4 five years later. Furthermore, like oil, the rise was not steady but was charac-

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[3] The annual price is the U.S. Energy Information Association's "Crude Oil Domestic First Purchase Price" from the data files supporting the *EIA Annual Energy Review 2007*. We add the difference between Imported and Domestic Refinery Acquisition Prices to the First Purchase Price for the 1974–82 period to allow for the effects of price controls implemented in the United States at that time.

[4] To our knowledge, monthly data does not exist prior to this period.

terized by peaks and troughs. Attention has therefore shifted to assessing the consequences of high prices and predicting the occurrence of high–price periods. In particular, the links between oil prices and the macro economy have received much attention (e.g., Hamilton 1983) due to the presumption that high prices are linked to recessions and are thus 'bad.'

Are we running out of cheap sources of energy and non-fuel minerals? On the one hand, it is clear that many of the reserves that were easiest and cheapest to extract have been depleted. On the other hand, technological improvements have meant that extraction of previously uneconomic resources is now possible. Nevertheless, it seems imperative to have a long–run plan that encompasses diminished supplies of fuel and non-fuel minerals and to seek to find reasonable substitutes for those commodities. Unfortunately, excessive volatility makes sensible planning difficult.

To illustrate, consider consumer reaction to the oil–price shocks that occurred in the 1970s. At first, consumers were interested in buying smaller more fuel efficient cars. However, as the price fell back to 'normal' levels, interest in efficiency dwindled and, in spite of the fact that the average new car today has a much better mileage rating than it did several decades ago, one nevertheless observes an inordinate number of sports utility vehicles that are driven almost exclusively on city streets. Had high prices been sustained, through taxation or other means, it is likely that the stock of cars in North America would more closely resemble the stock in Europe. Furthermore, the revenue from taxation could have been invested in developing new technologies and substitute materials.

These are just ideas to keep in mind when reading our survey. In particular, we should not let extreme price volatility trick us into taking a short–run view.

It seems inevitable that relative prices of exhaustible resources will rise at some future time, not just in the short run but on a permanent basis. Furthermore, as Hotelling demonstrated, high prices need not be 'bad' but instead can result from an optimal plan. With this in mind, we turn to the theory of long–run price movements and tests of that theory.

3 The Basic Hotelling Model and Some Extensions

3.1 The Basic Model

The simple Hotelling model can be derived with the help of optimal control theory. Consider a mine owner who extracts an exhaustible resource that is sold in a competitive market. Let the market price, quantity extracted, and reserves remaining in time t, be $p(t)$, $q(t)$, and $R(t)$, and the constant discount rate be r. The extraction–cost function is assumed to depend on the rate of extraction, with $C(0) = 0$, $C'(q(t)) \geq 0$, and $C''(q(t)) > 0$. In other words, extraction costs are convex.

The producer's objective function, J, is his discounted profit stream,

$$J = \int_0^\infty e^{-rt}\pi(q(t))\,dt = \int_0^\infty e^{-rt}[p(t)(q(t) - C(q(t))]\,dt, \tag{1}$$

and he chooses a time path for extraction to maximize J, subject to the constraints $\dot{R(t)} = -q(t)$, $q(t) \geq 0$, $R(t) \geq 0$, and $R(0) = R_0$, where a dot over a variable denotes a time derivative. In other words, extraction depletes reserves, both must be non-negative, and initial reserves are R_0. The current–value Hamiltonian for this problem is $H = pq - C(q) + \lambda\dot{R} = pq - C(q) - \lambda q$, where λ is the shadow price on the resource constraint and the time argument has been suppressed. Among the necessary conditions for the solution to this dynamic optimization problem are the following three first–order conditions:

$$H_q = p - C_q - \lambda = 0 \quad \textit{or} \quad \lambda = p - C_q, \tag{2}$$

$$\dot{\lambda} = r\lambda \quad \textit{or} \quad \frac{\dot{p} - \dot{C}_q}{p - C_q} = r, \tag{3}$$

and

$$\dot{R} = -q, \tag{4}$$

where a subscript on a function denotes a partial derivative.

The first first–order condition says that the shadow price on the resource constraint is the profit on the marginal unit. In other words, an extra unit of the resource would yield a marginal profit equal to the market price net of marginal extraction cost. The second is the famous r–percent rule, which states that the shadow price must rise at the rate of interest, r. Since the producer discounts the future at the rate r, the shadow price is constant in present–value terms, which ensures that, at the margin, the producer is indifferent between extracting one unit today or at some time in the future. Finally, the third says that the constraint on the rate of depletion is satisfied.

Let us consider the second first–order condition further. First, Hotelling derived his r–percent rule under the assumption of zero extraction cost (i.e., $C(q) = 0$). Under that assumption, the shadow price equals the market price and both rise at the rate of interest. The producer, however, is a price taker with constant (zero) marginal costs. This means that the producer cannot choose q so as to equate his marginal profit with the shadow price. Instead, the industry price must evolve so as to make (2) true. In other words, aggregate consumption or industry demand in period t, $D(p(t))$, must equal aggregate production in that period, $Q(p(t))$. The production of individual firms, however, is not well defined.[5]

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[5] This is true of any competitive industry with constant marginal cost.

Second, (3) determines the rate of change of price, not its level. In particular, (2), (3) and (4) define a pair of differential equations, for which two boundary conditions are required to determine a particular solution. The initial stocks define one boundary condition ($R(0) = R_0$). Under complete exhaustion of the resource, the other boundary condition is that all stocks are extracted. The level of price is then determined by the equality of cumulative consumption and cumulative production over the lifetime of the industry, a relationship that can be expressed as

$$\int_0^\infty D(p(t))\,dt = \int_0^\infty Q(p(t))\,dt = \mathbf{R_0}, \tag{5}$$

where $\mathbf{R_0}$ denotes aggregate industry reserves in period 0.

Finally, Hotelling showed that the monopolist's problem is similar. One merely substitutes marginal revenue for price in the first–order conditions. To illustrate, the shadow price λ, or marginal value, becomes marginal revenue net of marginal cost, which increases at the rate of interest.

3.2 Depletion

In order to understand how the simple model can be modified to include realistic features of resource markets, consider first the possibility that extraction costs depend, not only on current extraction, but also on remaining reserves. The new cost function, which is $C(q, R)$ with $C_R < 0$, captures the notion that the best or cheapest ores will be extracted first. Depletion thus involves moving to successively higher–cost ores. Although least–cost–first is an assumption here, it is an optimal plan in many models (e.g., Solow and Wan 1976).

When depletion is introduced, first–order conditions (2) and (4) are unchanged.

Equation (3), however, becomes

$$\dot{\lambda} = r\lambda + C_R \quad or \quad \frac{\dot{p} - \dot{C}_q}{p - C_q} = r + \frac{C_R}{\lambda}. \tag{6}$$

Since $C_R < 0$, the shadow price increases at a slower rate in (6) than in (3). This is true because extraction today leads to higher costs tomorrow, and the owner internalizes this externality.

We now have two predictions that can be taken to data — shadow prices should increase, either at the rate of interest or at a slower rate. However, casual inspection of price data reveals that, for many commodities, prices have fallen over long periods, and the models that we have derived thus far cannot explain falling prices. There are, however, simple and realistic assumptions under which market prices can fall. We discuss three of these: models with exploration, technical change, and recycling.

3.3 Exploration

We have thus far assumed that reserves are known in period 0 and that they cannot be augmented. However, oil and mining companies spend vast amounts on exploration in an attempt to find new deposits. Furthermore, if the extraction cost function is of the form $C(q, R)$, with $C_R < 0$, new discoveries, by augmenting the reserve base, lower costs. We will examine this model formally, paying particular attention to the effect of exploration on market prices.

Suppose that we amend the previous model by adding a stock of cumulative discoveries, D, with rate of change $\dot{D}$. The firm can exert an exploratory effort e at cost $C^2(e)$, which is assumed to be convex. New additions, or equivalently the rate of change of discoveries, evolve according to the rule $\dot{D} = f(e, D)$, with $f_e > 0$ and $f_D < 0$. In other words, exploratory effort leads to additional discoveries but the rate at which deposits are found falls as cumulative discoveries increase. This is true because exploration is sampling without replacement — once a deposit has been found, it cannot be found again. Finally, the equation for the evolution of reserves, which must be amended to include new discoveries, becomes $\dot{R} = f(e, D) - q$.

Although not necessary, it is simpler to work with an extraction cost function of the form $C(q, R) = C^1(R)q$. With this cost function, marginal cost is constant within a period but changes as reserves are discovered and/or depleted. Under these assumptions, one can show that (see, e.g., Pindyck 1978)

$$\dot{p} = r[p - C^1(R)] + C^{1'}(R)f(e, D) = r(p - C_q) + C_{qR}\dot{D}. \tag{7}$$

To understand the implications of exploration for prices, one must compare equation (7) to the constant marginal cost, no–exploration case. One can show that the comparable equation for that case is

$$\dot{p} = r(p - C_q). \tag{8}$$

With equation (8), market prices increase over time. Equation (7), however, contains a second term, $C_{qR}\dot{D}$. New discoveries, $\dot{D}$, cannot be negative. However, when marginal extraction costs rise as reserves are depleted (i.e., $C_{qR} < 0$) as assumed, the second term is negative.

Pindyck 1978) argues that early on costs fall rapidly, since cumulative discoveries are small and exploratory effort is very productive. Later, however, when most deposits have been found, costs fall slowly, if at all. This can give rise to U–shaped price paths that fall initially but rise eventually, which is a third testable hypothesis. We will see that there are other theoretical models that give rise to U–shaped price paths and numerous tests of that hypothesis.

3.4 Technical Change

The evolution of an industry or an economy is characterized by two important factors — growth and technical change — and, in our view, the latter is more important than mere growth in size. Indeed, new techniques and processes have revolutionized our world, and the fuel and non-fuel mineral industries are no exceptions. We will therefore examine how changes in technology affect the evolution of an exhaustible–resource industry, paying particular attention to the effect on market prices.

We assume that technical change enters the extraction cost function, which becomes $C(q, R, t)$ with $C_t < 0$. In other words, costs fall over time as technology improves. Although not necessary, the analysis is facilitated by assuming that technical change is Hicks neutral, and that there are constant returns to scale in production. Under those assumptions, the cost function takes the form $C(q, R, t) = h(t)C^1(R)q$.

It can be shown (see, e.g., Slade 1982) that the new first–order conditions yield

$$\dot{p} = r(p - C_q) + C^1(R)h'(t) = r(p - C_q) + C_{qt}. \qquad (9)$$

Compared to (8), the equation for the rate of change of price in the absence of technical change, equation (9) contains an additional term that represents the rate at which marginal costs fall due to changes in technology. In particular, since this term is negative, prices can fall. Slade (1982) argues that such is apt to be the case early on when scarcity rents (λ) are small. However, as reserves are depleted, prices eventually rise, leading to U–shaped price paths.

3.5 Durability, Recycling, and Inventories

Unlike the mineral fuels, which once consumed cannot be reused, many non-fuel minerals can be recycled. Indeed, commodities like gold are rarely discarded. This implies that the stock of the commodity that is in circulation is important rather than the flow of current extraction. Furthermore, that stock depreciates only slowly.

One can model this situation formally by introducing a new state variable, S, the stock in circulation, with $\dot{S} = q - \delta S$ and $S(0) = 0$, where δ is the depreciation rate or rate at which the stock is lost. Following Levhari and Pindyck (1981), we also introduce an inverse–demand relationship, $f(S)$, the marginal value of the flow of services from holding one unit of the stock, with $f'(S) < 0$. In other words, the marginal value is greater when the stock is smaller.

In equilibrium, the marginal value of holding a unit should equal the marginal cost, which has three terms: the opportunity cost of the cash investment, rp, the monetary value of the depreciation, δp, and the capital gain, $\dot{p}$ (which is a negative cost). One can thus write $f(S) = p(r + \delta) - \dot{p}$, which can be rearranged to obtain

$$\dot{p} = -f(S) + p(r + \delta). \tag{10}$$

Levhari and Pindyck argue that, since S increases initially but falls eventually, the price path is U shaped. However, if the cost function is $C(q)$, the shadow price obeys the r–percent rule.

Finally, although many fuels cannot be recycled, both fuel and non-fuel minerals can be stored, and inventory holding is an important aspect of commodity markets. Bresnahan and Suslow (1985) show that, if storage costs are zero and inventories are positive, the market price will follow the r–percent rule, at least in the short run. Indeed, this is simply an equilibrium condition in the asset market.

In this section, we have emphasized the behavior of market and shadow prices. Of course, the pattern of extraction is interesting as well. In simple models, q falls monotonically to zero. In more complex models, however, the extraction profile can be non-monotonic. We have emphasized prices because price is the variable that has received closest scrutiny in the empirical literature.

4 Econometric Issues

There are many econometric pitfalls that the applied researcher must deal with in attempting to test the Hotelling model. Unfortunately, the treatment of those problems in the research that we discuss below is not always satisfactory. Rather than point to flaws in individual papers, however, we discuss three topics that pose problems in many applications: the assumption of stationarity (nonstationarity), the issue of endogeneity, and the measurement of shadow prices.

4.1 Nonstationarity

A time series, x, is said to be stationary if the dependence between x_t and x_{t-j} depends on the distance between observations, j, but not on location, t. This means, in particular, that the mean and variance do not change over time. Many time series have a single unit root, which means that the first difference, $x_t - x_{t-1}$, is stationary. Unfortunately, when one runs regressions that involve nonstationary variables and does not difference those variables, the results obtained, such as tests of significance, are incorrect, and spurious relationships can be found.

This problem is important when attempting to assess trends in time–series data such as prices. To illustrate, consider the time–series model with a linear trend

$$p_t = \alpha_0 + \alpha_1 t + z_t, \qquad z_t = \beta z_{t-1} + \epsilon_t. \tag{11}$$

When $\beta = 1$ ($\beta < 1$) p is nonstationary (stationary). Unfortunately, it is difficult to distinguish between these possibilities when β is close to one, as is often the case for commodity prices, and different researchers have taken different stances on this issue.

We believe that many commodity prices are stationary. Our belief is not based on tests for unit roots but rather on variance–ratio tests that reveal the extent to which price shocks are persistent or transitory (see Pindyck 1999 for coal, crude oil, and natural gas, and Slade 2001 for copper). Those tests show that many prices are mean reverting to a trend, but that the rate of mean reversion is very slow and the trend can shift over time. However, many researchers would disagree with us and claim that commodity prices have a unit root and must therefore be differenced.

There is also a theoretical basis for our objection to the unit-root approach to examining the time–series properties of commodity prices in the context of Hotelling's model. The predictions of Hotelling's model and most extensions point to non–stationary price behavior. The issue is easy enough to see from the simplest of Hotelling's specifications, in which $\dot{p}/p = r$. Converting this to a discrete–time time–series model we have

$$p_t = (1+r)p_{t-1} + z_t,$$

which is clearly nonstationary (as $(1+r) > 1$), but at the same time is not a unit–root process.[6] Standard unit root tests for whether $\beta < 1$ or $\beta = 1$ are

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[6] If shocks are multiplicative in this model, $ln(p(t))$ has a unit root. However, most researchers

not designed to handle the explosive AR case predicted by theory and clearly are not designed to distinguish a unit root in z_t from the nonstationarity caused by eventual exhaustion of the resource.

Finally, theory predicts that certain types of shocks, such as to the level of reserves, have permanent effects, as the entire price path is affected by such shocks. Other shocks, such as strikes or business cycle fluctuations, may be of a temporary nature but, to the extent to which they affect output and hence remaining reserves, can have long-term price effects. The relative importance of permanent and transitory shocks is then an empirical issue. However, we would not expect unit–root tests to shed much light on these issues.

4.2 Endogeneity

Endogeneity is a ubiquitous problem in applied work, and tests of the Hotelling model are no exception. In order to illustrate this problem, consider estimating an extraction cost function. Suppose that this function is $C(q, R, t, v)$, where q is output, R is remaining reserves, t is time, and v is a vector of factor prices. The endogeneity problem arises when trying to estimate the cost function to obtain estimates of marginal cost (C_q and C_R) required for testing the Hotelling rule, since q is generally endogenous.

The standard solution to the endogeneity problem is to find a set of variables (instruments) that are correlated with the endogenous right–hand–side variable but not with the error in the estimating equation. Often the instruments are lagged endogenous variables that are assumed to be predetermined. This solution fails when considering extraction cost functions since, with finite or increasing–

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work with price levels.

cost reserves, extraction today affects extraction in all future time periods, implying that lagged endogenous variables are not predetermined.

Another possible solution to the endogeneity problem is to look for contemporaneous variables that are good instruments. In particular, when estimating a cost function, it is customary to look for demand–side variables that shift q. To determine if this is a legitimate remedy here, one must consider what the error represents. Typically it contains unobserved or omitted factor prices. Unfortunately, like output prices, factor prices are apt to be correlated with demand–side variables such as industrial production, which means that demand shifters are often not valid instruments.

Although the endogeneity problem is not insurmountable, it requires ingenuity in finding appropriate instruments.

4.3 Measuring Shadow Prices

If extractive firms purchased unextracted ore from its owner each period in a competitive environment, we would observe a competitive–market price for the in situ resource. Of course, this rarely happens; usually vertically–integrated mining firms own the rights to extract from a large deposit and do not regularly purchase the ore input. Consequently, an important variable for tests of the Hotelling model, λ, is rarely observed and must be inferred, usually by applying the definition obtained from a particular theoretical model.[7]

With most of the models discussed in the previous section, the shadow price, λ, is the market price, p, net of marginal cost, C_q. Market prices are observable and pose no measurement problem.[8] However it is difficult if not impossible to

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[7] This is analogous to the issue of determining a user cost of capital.

[8] This depends on how far down the supply chain they are observed. The more processed the

measure true marginal cost. In particular, it is not always obvious which inputs are variable and which are fixed. To illustrate, labor is usually considered to be variable. However, many workers in extractive industries are employed under contract rather than on a day–to–day basis, which implies that a substantial portion of the work force is quasi fixed and should be excluded from marginal–cost calculations.

This problem is not unique to the Hotelling model, and various solutions have been adopted in the literature. We discuss one solution and indicate why it is inappropriate here. Industrial Organization economists frequently must estimate marginal costs, and many have given up on direct measures. Instead, they retrieve marginal costs (either numbers or functions) from first–order conditions for equilibrium in the market. With structural tests of the Hotelling model, in contrast, the equilibrium condition is the object of the tests, and it is therefore inappropriate to assume that it holds.

5 Testing and Evaluating

5.1 Tests of the Hotelling Model

General Comments As discussed in section 3, the basic Hotelling model predicts that, in the absence of extraction cost, the market price p of an exhaustible-resource commodity will rise at the rate of interest r, $\dot{p}/p = r$. When marginal extraction cost is nonzero, the shadow price or marginal profit, $\lambda = p - C_q$, rises at the rate of interest, $\dot{\lambda}/\lambda = r$. Moreover, when extraction costs depend on the level of reserves remaining R, the shadow price rises at a rate that is less

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commodity on which we observe price, the more complicated the cost function is that must be determined in order to compute λ.

than the rate of interest. This lower rate of shadow-price appreciation, which is given by $\dot{\lambda}/\lambda = r + C_R/\lambda$, reflects the user cost associated with deterioration of the quality of ore mined in the future. Furthermore, several factors were shown to yield U–shaped market–price paths that decline initially but rise eventually. Finally, when markets are imperfectly competitive one must replace price in the above equations with marginal revenue. These are some of the predictions that have been taken to data.

The tests that have been performed have mainly been of two sorts: descriptive and structural. The first class assesses outcomes that are associated with the market equilibrium without having to specify the nature of that equilibrium. Its advantage is that there is no need to commit to a specific model. Instead, one can assess which models are consistent with the data and which are not. Its shortcoming is that one cannot perform formal tests. The second class tests a specific model by estimating structural equations. Its strength is that formal tests can be performed. Its shortcoming is that it imposes more structure (e.g., on the cost function and the nature of competition in the market), and that structure may be inappropriate. We report findings from both classes.

Descriptive Studies Most descriptive studies have examined the behavior of mineral commodity prices. Moreover, since the predictions of Hotelling's model are long–run, many tests make use of a century or more of data on the prices of different fuel and non-fuel minerals.

Barnett and Morse (1963) were perhaps the first to analyze mineral-commodity prices formally. They looked at relative price trends in an attempt to uncover evidence of natural–resource scarcity, and they concluded that, because real prices had fallen over time, scarcity was not a problem. Other researchers who have examined price trends, however, are not in complete agreement with Barnett and Morse. For example, Smith (1978) looked at the stability of the coefficients of estimated price–trend relationships and decided that the data are too volatile to support definitive conclusions.

Those studies set the stage for somewhat more formal descriptive assessments. For example, Heal and Barrow (1980) related metal price movements to interest rates and found that the results are not supportive of the Hotelling model. In particular they discovered that changes in interest rates, not interest–rate levels, predict prices.[9]

Many researchers have assessed the possibility that price paths might be U–shaped. Perhaps the first was Slade (1982), who based her descriptive tests on the idea that price declines might be due to technical change, as in equation (9). She found that, although fitted linear trends were negative for many mineral commodities, quadratic trends revealed evidence of upturns in the real prices of mineral commodities that began in the 1970's.

Subsequently, many other researchers examined the issue of quadratic trends and reached a variety of conclusions. The principal factor that differs across studies, which perhaps accounts for the different conclusions drawn, is the econometric technique used. For example, Moazzami and Anderson (1994), who estimated an error–correction model, found evidence of U–shaped price paths, whereas Berck and Roberts (1996), who estimated both difference and trend–stationary models, found evidence of U–shapes under the former but not the latter.[10] Finally,

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[9] For more on the relationship between interest rates and prices, see Smith (1981) and Agbeyegbe (1989).

[10] See also Ahrens and Sharma (1997), who found some trend and some difference stationary

Pindyck (1999), who estimated a model in which prices revert to a quadratic trend that shifts over time, found U shapes.[11] The evidence is thus mixed. Nevertheless, the idea that real prices have risen in recent years receives stronger support.

There are many possible further refinements that could be undertaken. For example, Figures 1 and 2 suggest that not only do trends shift over time but also variances are non-constant. Indeed, there are periods of both high and low volatility that suggest using an ARCH or GARCH model (see, e.g., Engle 1982; Bollerslev 1986).

Structural Models More formal tests of the Hotelling model rely on estimates of some combination of an industry–wide demand function, a production, profit, or cost function for the extractive firm or industry, and a first–order condition (e.g., an Euler equation) that is associated with dynamic-profit maximization. Examples include Stollery (1983), Farrow (1985), Halvorsen and Smith (1984, 1991), Young (1992), and Chermak and Patrick (2001).

There are many possible ways that the estimated structural equations can be used to test the Hotelling model. We list three here:

- One can estimate a cost function to obtain C_q and use it in conjunction with market prices (or marginal revenue obtained from an estimated demand function if imperfect competition is suspected) to calculate shadow prices, λ. It is then possible to test if those shadow prices increase at the rate of interest, or at a slower rate if the cost function depends on remaining reserves, or if they fall. A modified version of this approach is taken by

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series.

[11] See also Lee, List, and Strazicich (2006), who found structural breaks in deterministic trends.

Stollery (1983), who finds support for the Hotelling model with a discount rate of 15%.

- One can augment the first method to include a first–order condition such as (6). When this is done, the model can be tested by examining whether estimated parameters such as $\hat{r}$ make sense from an economic point of view as well as whether estimated shadow prices behave as predicted by the particular theory that is being tested. This approach is taken by Farrow (1985), Halvorsen and Smith (1984, 1991), Young (1992), and Chermak and Patrick (2001). The results of the these structural tests are quite mixed, with researchers finding falling shadow prices and/or negative interest rates. Most interpret these findings as unsupportive of the Hotelling model.
- One could estimate the building blocks, demand and cost, and use those equations to solve for the market equilibrium that is implied by dynamic profit maximization. It would then be possible to test if observed price and output paths lie within the confidence intervals that surround the paths predicted by the model. As far as we know, this has not be done.

A few comments are in order. First, most researchers who have estimated cost or profit functions for individual mines or mining industries assume that the technology of mining involves extracting unprocessed ore, n, which is combined with other inputs to produce metal, q. In other words, both mining and refining are modeled. n, which is transferred inside a vertically integrated firm, is treated as a quasi–fixed factor in the production of q. Once the firm's technology is known, shadow prices or rental rates, λ, can be approximated by one of two methods. They can be calculated as the difference between price and marginal cost, $p - C_q$, as in Stollery (1983), Farrow (1985), and Young (1992), or as the shadow price of the unpriced ore to the vertically integrated metal producer, $-C_n$, as in Halvorsen and Smith (1984, 1991) and Chermak and Patrick (2001). These two estimates of λ, however, do not measure the same thing. The first is the shadow price of one unit of contained metal in situ, whereas the second is the shadow price of one unit of ore of the current grade, also in situ.

Second, rejection of the Hotelling model is not an absolute rejection. Instead, it is a rejection of a particular variant. For example, falling shadow prices (which are equivalent to finding negative interest rates) are inconsistent with simple versions of Hotelling's model but not with other formulations.

Finally, the first–order conditions that we derived earlier are expressed in continuous time. For estimation purposes, however, one normally converts those equations into discrete–time analogs. When this is done, the discrete–time equations contain expected values of future realizations of variables. It is standard to assume that expectations are formed rationally (i.e., that decision makers use all currently available information in forming their forecasts). Estimation therefore often makes use of Generalized Method of Moments, which is an instrumental–variables technique. Unfortunately, the difficulties that are associated with finding appropriate instruments are at least as great here as those mentioned earlier.

Methods That Use Market Prices The studies described above are based on proxies for shadow prices that rely on econometric estimation. An alternative that is sometimes possible makes use of market proxies that rely on sales of undeveloped resources or assets of mining companies.

The model that was proposed and estimated by Miller and Upton (1985a) is perhaps the best known market–based application. They exploit a less widely known implication of Hotelling's analysis, which they call the Hotelling Valuation Principle (HVP). Specifically, they show that in a competitive market, the value of reserves in any currently operating, optimally managed mineral deposit should depend solely on the current spot price net of marginal-extraction cost, regardless of when the reserves will be extracted. They tested their model using stock–market valuations of the oil and gas reserves of a sample of US companies and found that the data are consistent with their Principle. Some subsequent tests, however, have found that the HVP over values mineral assets (see, e.g., Cairns and Davis 1998; Miller and Upton 1985b).

Not all market–determined shadow prices are based on stock–market valuations. Some resources are sold in the ground, and there is thus a market price for the unextracted resource. This is true, for example, of timber, which is sold unharvested. Livernois, Thille, and Zhang (2006) examine old–growth timber, which is nonrenewable, and use stumpage price bids in timber auctions as their measure of shadow prices.[12] Their structural tests are fairly supportive of Hotelling's model.

Transactions involving oil reserves were used by Adelman and Watkins (2005, 2008) to compute implied values for shadow prices. They argue that the shadow prices are lower than expected. There is not a discernible upward trend in their data, however the time series is not long, beginning only in 1982.

Based on the limited evidence to date, it thus appears that tests that use market–based proxies for shadow prices lead to conclusions that are more optimistic than do those that use econometrically estimated proxies.

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[12] The use of stumpage prices as proxies for shadow prices was suggested by Johnson and Libecap (1980).

5.2 Applications of the Hotelling Model

Many empirical researchers have examined the behavior of market and shadow prices in order to test the validity of the Hotelling model. There are other issues, however, that are also important but have received much less attention. Some of these are not tests but are instead applications. In other words, the Hotelling model is used as a tool in the evaluation of some other issue. In this subsection, we examine some of those applications, using one or two studies to illustrate possible approaches to each problem. As before, we limit attention to research that is framed in the context of exhaustibility. To illustrate, many applied economists have examined the effects of tax policy on resource extraction. Most of that work, however, is not set in the context of finite reserves and is therefore not discussed here.

Exploration Incorporating exploration into a Hotelling model yields further testable hypotheses. In particular, as suggested by Devarajan and Fisher (1982), one can manipulate the first–order conditions from a model that incorporates exploration to obtain an alternative measure of scarcity rent or the shadow price on the resource constraint. This proxy is full marginal discovery cost, which includes not only the direct marginal cost of discovering an extra unit of reserves but also the scarcity rent on exploration prospects.[13]

Devarajan and Fisher (1982) used data on pre–OPEC oil and gas discovery costs in the US to assess this relationship and found that direct discovery costs rose prior to the 1970s. They interpret the positive trend as a leading indicator

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[13] The model with exploration contains two shadow prices. Using the notation from section 3, λ_1, the multiplier on the constraint $\dot{R} = -q + \dot{D}$, is the scarcity rent on the resource in situ, whereas λ_2, the multiplier on the constraint $\dot{D} = f(w, D)$ is the rent on exploration prospects.

of impending scarcity. Lasserre (1985) performed a similar analysis with somewhat better data on discovery costs for oil in Alberta. In particular, he used information on bonuses as a proxy for the rent on exploration prospects — the second component of full marginal discovery cost. He concluded that not only had direct discovery cost been rising but also bonus money was a significant (approximately 20%) and rising proportion of full marginal cost, a further confirmation of impending scarcity.

Scarcity and Growth The link between resource scarcity and economic growth, in particular whether finite stocks of exhaustible resources will constrain growth, is a controversial question that has been debated by many, with Neoclassical economists taking more optimistic views that rely on substitution possibilities and technical change to relax the resource constraint (see, e.g., Stiglitz 1974) and Neomalthusians assuming more pessimistic positions, relying instead on the laws of thermodynamics to argue that sustained growth is neither possible nor desirable (see, e.g., Georgescu–Roegen 1971; Daly 1974). It is therefore not surprising that empiricists have also attempted to evaluate possible constraints on growth due to exhaustibility.

Although not made explicit in the studies, the early focus on demand and substitution between man–made capital (K) and exhaustible resources (R) (e.g., Berndt and Wood 1975) can be seen as an attempt to evaluate growth possibilities. Indeed, Stiglitz (1974) showed that the feasibility of sustainable growth depends crucially on the size of the elasticity of substitution between K and R. Unfortunately, the findings of the demand studies were not very optimistic, with substitution possibilities estimated to be very limited or nonexistent, which is not surprising for many raw material commodities.

More recently, empirical researchers have embedded partial–equilibrium Hotelling models in steady–state growth contexts to test if observed price patterns can be reproduced by estimated models. For example, Lin and Wagner (2007) derive conditions on the parameters of their model that imply that there will be no trend in resource commodity prices. Their model, which incorporates technical progress and depletion effects on the supply side, yields restrictions on the rate of technical change, the cost–increasing effect of depletion, and the rate of growth and price elasticity of demand that must be satisfied for prices to remain constant in real terms. They test those restrictions using data on 14 minerals and find that about half satisfy their condition.

Pricing Risk The tests that we have presented thus far are either embedded in a world of certainty or involve risk–neutral agents. It is standard, however, for risk–averse investors to tradeoff risk and return, and mining investment decisions are no exceptions. It is therefore desirable to have a model that prices risk as well as exhaustibility. Slade and Thille (1997) develop such a model and estimate it using data for a panel of Canadian copper mining firms. Specifically, they derive the rate of return that investors require to hold mining assets when the rate of technical change of the cost function is an exogenous risky process. Their model, which incorporates a capital–asset–pricing model (CAPM) into a Hotelling model of optimal extraction, yields a first–order condition for the expected rate of shadow–price appreciation of the form

$$\frac{\frac{E(d\lambda)}{dt}}{\lambda} = r + \frac{C_R}{\lambda} + \beta(r^m - r). \tag{12}$$

Comparing equations (6) and (12), we see that the latter contains an additional term, which is the risk premium from the CAPM.[14] Slade and Thille find that neither the coefficient restrictions that are implied by the Hotelling model nor those implied by the CAPM are rejected by the data. Moreover, their estimate of β is negative, which means that mining assets are good hedges against poor performance of financial assets. The degree of risk diversification that is implied by their estimate, however, seems too large to be the whole story.[15]

Strategic Behavior A priori it is not clear if imperfect competition distorts extraction profiles and, if it does, whether the gains from cartelization are large or small. For example, Stiglitz (1976) demonstrated that, with constant demand elasticity and zero extraction costs, monopoly and competitive price (and thus profit) paths are identical. Furthermore, if reserves are homogeneous and finite, monopoly and competitive price (and thus profit) paths must cross. Under such circumstances, there is little scope for monopoly profits. However, when depletion effects are introduced, total recovery can depend on market structure and monopoly profits can be everywhere higher. The relevant question is then 'how profitable is cartel formation for a given industry?'

This question is addressed by Pindyck (1977) using a model of a cartel with a competitive fringe. Pindyck calibrates his model for the oil industry (with cartel OPEC), the bauxite industry (with cartel IBA), and the copper industry (with cartel CIPEC). Comparing the cartel and competitive solutions for each industry, he concludes that the gains from cartelization are large for the first two but small for the third. These differences are accounted for by the market share of the cartel

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[14] For a discussion of the CAPM, see Brennan (1987).

[15] For more on combining Hotelling with CAPM, see Young and Ryan (1996).

in each industry as well as by the speeds of adjustment of consumer demand and fringe supply. Indeed, not only do the cartels in the first two industries account for larger fractions of their respective markets but also adjustments to changed conditions are slower in those markets, allowing for greater short–term gains.[16]

Ellis and Halvorsen (2002) look at a different issue — decomposing the gap between price and marginal cost into two components: the rent that is associated with exhaustibility and the rent that is associated with market power. They estimate their model for the largest firm in the nickel industry and find that monopoly power accounts for the lion's share of the gap.

It therefore seems that, at least in some industries, not only are substantial monopoly profits earned but also distortions relative to competitive trajectories can be large. However, the conditions that facilitate the successful exercise of market power vary by industry.

Resource Taxation Mining industries are subject to many forms of taxation and government regulation, including royalties, severance taxes, depletion allowances, and price controls, most of which are distortionary. Furthermore, those taxes can be levied at any stage of production (e.g., mining, refining, or fabrication). Finally, they can be economically large. For example, depletion allowances were set at one third of total revenues and price controls in the US kept domestic prices below one half of world prices.

As with monopoly power, if reserves are homogeneous and finite, taxes can only move extraction from one period to another — the extraction path tilts but the

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[16] Salant (1982) calibrates a model of the world oil industry that allows for several Cournot players that can be cartels (e.g., OPEC) or non–OPEC countries (e.g., Mexico) and a competitive fringe of small players.

area under that path remains constant. However, when costs rise with cumulative extraction, ultimate recovery can be distorted. Under those circumstances, it is especially important to evaluate the size of distortions for particular deposits and industries.

Slade (1984) develops a model of an extractive firm in a competitive industry that incorporates various stages of production in the vertically integrated firm as well as varying grades of ore mined. The model is estimated for a US copper mining firm that owned only one mine. Company annual reports therefore provide time–series data for that mine. After estimation, the firm's optimal intertemporal behavior is determined under various assumptions about taxation and government controls. Comparisons of those solutions with the tax–free situation can then be used to evaluate the magnitude and time pattern of distortions. The effects that are uncovered include extraction paths that cross as well as changes in cumulative ore extraction and metal production. However, the latter two effects dominate. Moreover, tax policy can change ultimate ore extraction and metal processing intensity in opposite directions, and the directions of those changes depend on the stage of production at which the tax is levied. Finally, the size of distortions is estimated to be large. To illustrate, in the simulations a 10% royalty causes an 8% decline in cumulative metal production over the lifetime of the mine.

To summarize, in this subsection, we have discussed further tests and applications of the Hotelling model. Unfortunately, there are many interesting issues that we have not covered. Our neglect of those issues is not due to their lack of importance. It is due instead to lack of coverage in the empirical literature or to coverage that does not fit well with the goals of our survey.

6 Conclusions

We have attempted to survey the large empirical literature that tests and applies the Hotelling model, and it should be clear by now that there are many ways of doing this (i.e, many economic and econometric models), each with strengths and weaknesses. In concluding, we wish to emphasize two points that we feel transcend specific models and tests.

- Distortions relative to the planner's solution can be large. These can result from imperfect competition, distortionary taxation, risk aversion, and/or the inappropriate assignment of property rights, among other things. It is not clear, however, if those departures are sufficient to warrant government intervention, which is also distortionary. The answer to that question requires careful consideration of the circumstances in each market.
- The often cited fact that the Hotelling model is frequently rejected by the data (see, e.g., Krautkraemer 1998) must be interpreted with caution. Indeed, rejection usually means failure of a simple variant, and incorporating real–world detail can considerably improve performance. Furthermore, given substantial differences across markets and firms, a one–size–fits–all modeling approach and/or the use of very aggregate data are unlikely to be very illuminating.

Is empirical testing of the Hotelling model a dead issue? We think not. However, it is clearly imperative to distinguish between short–run volatility and long–run trends. In other words, we must be able to separate the signal from the noise. There are many reasons why mineral commodity prices are so volatile, including inelastic demand and supply at high prices as well as strong links with industrial production and the overall performance of the economy.

A further reason is related to the discrete and lumpy nature of many decisions. For example, the Hotelling model is based on the assumption that q is chosen continuously and costlessly. In reality, however, there are substantial costs associated with mine entry, exit, and temporary openings and closings, and those costs, combined with investment delays, introduce considerable inertia into production decisions. One possible approach to modeling the discrete and lumpy nature of extraction and the associated price and supply volatility would be to combine the theory of real options (see, e.g., Brennan and Schwartz 1985) with a Hotelling model of depletion.

Another important issue is the nature of technical change, which can also be discrete and can result, not only in substantial cost savings but can also change deposits from uneconomic resources to economic reserves. For example, fluid catalytic cracking drastically reduced petroleum refining costs and the advent of froth flotation transformed many uneconomic sulfide ores into valuable reserves. Nevertheless, in most extant studies, technical change is modeled as a smooth process or as a sequence of small events.

Finally, exploration and discovery of previously unknown deposits is clearly important for modeling depletion (or the lack thereof). However, there has been little empirical work that incorporates models of exploration and discovery into a Hotelling framework.

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Figure 1: Annual Crude Oil Prices (Log scale)

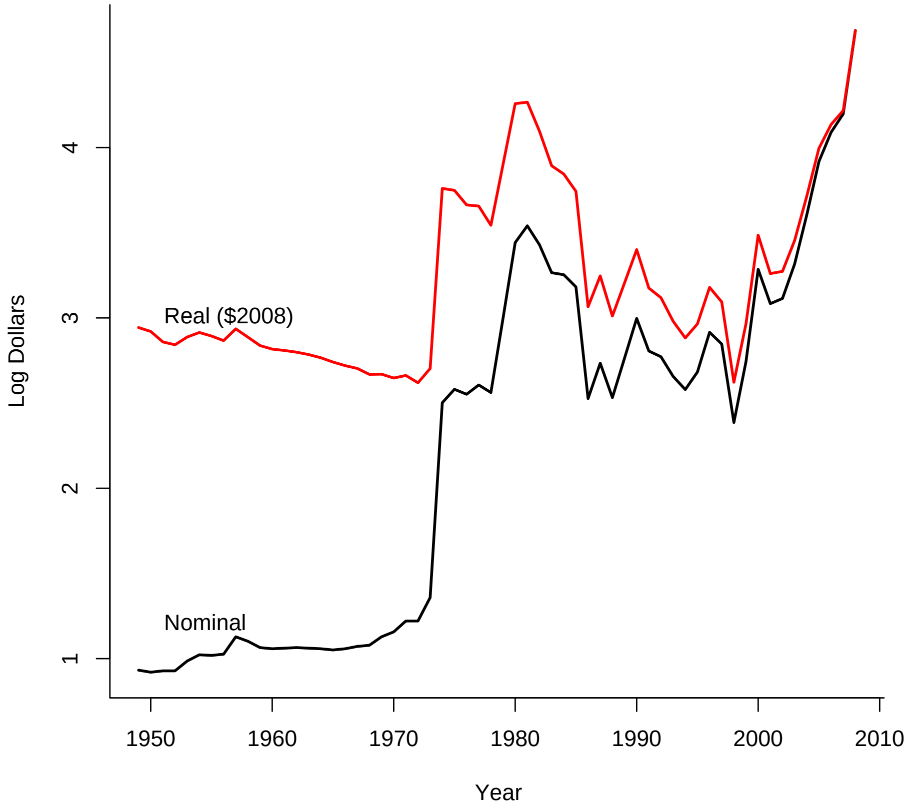

Figure 2: Annual Coefficient of Variation of Crude Oil Price