



## Signaling drive over the long term

Wei Li\*

University of British Columbia, Canada

Department of Economics, 1015 Buchanan Tower, 997-1873 Main Mall, Vancouver, BC Canada

### ARTICLE INFO

#### Article history:

Received 4 November 2008

Received in revised form 29 May 2010

Accepted 24 August 2010

Available online 21 October 2010

#### JEL classification:

C70

D82

D23

#### Keywords:

Career concerns

Hidden heterogeneity

Effort supply

### ABSTRACT

An agent's productivity depends on his responsiveness to existing incentives ("drive"). Over the long term, this heterogeneity in drive may create significant incentives for the agent to work hard even with vanishingly small amount of existing incentives, explicit or implicit.

© 2010 Elsevier B.V. All rights reserved.

### 1. Introduction

First observed by Fama (1980) and later shown by Holmström (1999) in a seminal paper, an economic agent's concern for the market's perception of his productivity can create incentives to work hard even without any explicit incentives. Since then, career concerns models often assume that agents differ in their inherent ability, which affects their productivity by a constant in each period of employment (Gibbons and Murphy, 1992; Dewatripont et al., 1999). Although heterogeneity in ability is very important, another crucial dimension of agents' productivity is how strongly they respond to existing incentives: an agent is more productive if he is more motivated than others by a given incentive system.

In a two-period model, Kőszegi and Li (2008) show that this heterogeneity in agents' responsiveness to incentives ("drive"), modeled as an agent's marginal utility of income, motivates them to work harder than that warranted by the existing incentives alone. Because more driven agents are expected to respond more strongly to existing incentives, a high output indicates high drive, which translates into expectations about harder work, and consequently higher future productivity and wages. These drive-signaling incentives induce the

agents to work harder in the first period to signal their high valuation of the second period. In their model, however, the magnitude of existing incentives is important: the agent's effort supply dwindles to zero if the existing incentives are negligible. The current model studies how heterogeneity in drive affects the agent's effort supply over the long term, for instance, through an agent's career.<sup>1</sup> In the career concerns tradition, this model assumes that there are some imperfect explicit incentives; and that agents are partly motivated by the implicit incentives provided by a competitive labor market. That is, the agents perceived to be more productive are paid more in each period. Although the main model focuses on drive-signaling incentives alone, the result extends easily into the case where the agent is motivated by the implicit incentives provided by the market to signal his talent as in Holmström (1999).

This paper finds that, in an infinite horizon model in which the agent's marginal utility of income evolves over time, drive-signaling incentives have a self-reinforcing feature, making it possible for them to build on negligible explicit incentives. Intuitively, agents with higher drive respond more strongly than others to any given explicit incentives. In addition to the explicit incentives, however, this now provides drive-signaling incentives as well, to which more driven

\* Department of Economics, 1027 Buchanan Tower, 997-1873 East Mall, Vancouver, BC V6T 1Z1, Canada. Tel.: +1 604 822 2839.

E-mail address: [weiliubc@interchange.ubc.ca](mailto:weiliubc@interchange.ubc.ca).

<sup>1</sup> As a type of hidden heterogeneity, drive is a stand-in for the broader issue of the private information about the agents' preferences such as disutility of effort, or difference in the length of career.

agents also respond more strongly. In this way, further drive-signaling incentives are created. By the same logic, the drive-signaling incentive bootstraps itself. If this effect is strong enough – for which a necessary and sufficient condition is provided – then an arbitrarily small amount of explicit incentives leads to non-trivial levels of effort in steady state. It is worth emphasizing that for some parameter values, the agent exerts significant effort in the unique steady-state equilibrium of the model despite vanishingly small amount of existing incentives. In essence, people work to signal that they are driven, and that matters almost solely because they will then want to signal it again.

Both explicit incentives and the implicit incentives to signal one's talent are very important determinants of the agents' effort supply. [Kőszegi and Li \(2008\)](#) and this model, however, take the first step in showing agents' incentives to signal their unobserved heterogeneity in drive may matter even when existing incentives are quite weak. This is consistent with [Kuhn and Lozano \(2008\)](#), who show that employees' work hours are driven in part by "signaling to the labor market that one is productive or ambitious and thus securing a better job in another firm." Also, [Stoddard and Kuhn \(2008\)](#) suggest that the increase in effort for teachers in public schools – who are salaried and face very weak performance-based pay – may be attributed to the market's reward for hidden productivity-related characteristics instead of changes in pay policy.

**2. An infinite horizon model**

A risk neutral principal employs a risk neutral agent to work for an infinite number of periods  $t = 1, 2, \dots, \infty$ . There is a constant discount factor  $\delta$ . In each period  $t$ , the agent produces output  $q_t = e_t + \epsilon_t$ , where  $e_t$  is his effort level, and  $\epsilon_t$  is an independently distributed noise term with mean zero and variance  $\sigma_\epsilon^2$ . Throughout this paper, each unit of output is valued at one.

Agents are assumed to differ in their drive, which is modeled here as their marginal utility of income  $m_t$ . Alternatively,  $m_t$  can be thought of as how much an agent values his career at time  $t$ . The higher is  $m_t$ , the more responsive is an agent to any given incentives in period  $t$ . Only the agent knows his marginal utility, the principal does not. Before period 1 production takes place, the agent knows his drive  $m_0$ , but the principal only knows that  $m_0 \sim N(\mu_m, \sigma_m^2)$ . Moreover, in a long horizon, it seems realistic to assume that the agent's marginal utility of income changes due to changes in circumstances, and that it is correlated across time. To model this perpetual uncertainty in drive,  $m_t$  is assumed to evolve according to

$$m_{t+1} = m_t + v_t, v_t \sim N(0, \sigma_v^2).$$

All the error terms  $\epsilon_t$  and  $v_t$  are assumed to be independently distributed. The agent's disutility of effort  $e$  is  $c(e) = 1/2ke^2$ , which is additively separable from his utility from consumption. Thus the agent chooses effort  $e_t$  in each period to maximize  $\sum_{t=1}^{\infty} \delta^{t-1} (m_t w_t - c(e_t))$ .

Slightly departing from the standard career concerns model, it is assumed that some existing incentives form part of the agent's compensation. Specifically, the agent is paid a wage  $w_t = b_t + \beta q_t$  in period  $t$ . The term  $b_t$  can be interpreted as the agent's "base salary," while  $\beta q_t$  is his performance-contingent "bonus", which represents the incentives already in place. The labor market is assumed to be perfectly competitive, and thus the principal makes zero expected profit in each period by offering the agent a base salary  $b_t$  equal to his expected output conditional on his past performances, and net of his expected bonus. The other wage-setting assumptions are in the career concerns tradition: contracting is period-by-period; and the principal is either prevented from using a fully explicit incentive contract, or does not wish to do so. [Section 4](#) shows that a similar result holds in a

variation of this model, where no explicit incentives are present but the agents differ in both ability and drive.

This paper looks for the (pure-strategy) linear rational expectations equilibria. In equilibrium, each type of agent chooses his effort level optimally given the principal's anticipated inferences, and the principal updates her beliefs about the agent's type in a Bayesian way, given her expectations about his behavior. Linear equilibria are natural candidates to consider because payoffs increase linearly with  $m_t$  and the cost function is quadratic. Because the agent's marginal benefit from exerting effort is proportional to  $m_t$ , his effort level is an affine function of  $m_t$  in each period:  $e_t = \alpha_t m_t$ . But since this problem is very difficult in general, the analysis focuses on the steady-state levels of  $\alpha_t$ , where  $\alpha_t$  is constant over time.

**3. Effort supply in steady state**

At the end of period  $t$ , the principal forms a more precise estimate about the agent's drive given his output  $q_t$ ; but the change in  $m_{t+1}$  introduces new uncertainty in period  $t + 1$ . Let  $h^t = (q_1, \dots, q_t)$  be the history of outputs up to time  $t$ , then the following result greatly simplifies the analysis:

**Lemma 1.**  $\alpha_t$  is a constant  $\alpha$  if and only if  $\text{Var}[\alpha m_t | h^{t-1}]$  is constant.

Thus the steady state consists of a pair of parameters: a steady-state responsiveness to incentive  $\alpha$  and a steady-state conditional variance of  $\alpha m_t$ , which is denoted as  $\sigma_{\alpha m}^2$ .

First, consider the evolution of the principal's beliefs of the agent's drive. At the beginning of period  $t$ , the conditional variance of  $\alpha m_t$  is  $\sigma_{\alpha m}^2$ . Then, the principal observes a noisy signal  $q_t = \alpha m_t + \epsilon_t$  of  $\alpha m_t$ , decreasing the conditional variance to  $\frac{\sigma_\epsilon^2 \sigma_{\alpha m}^2}{\sigma_\epsilon^2 + \sigma_{\alpha m}^2}$ . But then the marginal utility of income changes, increasing the conditional variance of  $\alpha m_{t+1}$  by  $\alpha^2 \sigma_v^2$ . For the principal's belief to have conditional variance  $\sigma_{\alpha m}^2$  at the beginning of period  $t + 1$ , they must exactly offset each other in steady state. Formally,

$$\frac{\sigma_\epsilon^2 \sigma_{\alpha m}^2}{\sigma_\epsilon^2 + \sigma_{\alpha m}^2} + \alpha^2 \sigma_v^2 = \sigma_{\alpha m}^2. \tag{1}$$

Second, consider the agent's incentive to work hard to increase output in period  $t$ . To begin with, it affects his wage  $w_{t+1} = E[q_{t+1} | h^t]$ , which consists of two parts: his expected period  $t + 1$  performance-contingent bonus  $\beta E[q_{t+1} | h^t]$ ; and his base salary, which is his expected wage net of expected bonus payments, given past outputs. Formally:

$$b_{t+1} = (1-\beta) \left[ \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_{\alpha m}^2} E[\alpha m_t | h^{t-1}] + \frac{\sigma_{\alpha m}^2}{\sigma_\epsilon^2 + \sigma_{\alpha m}^2} q_t \right].$$

Clearly, the agent's base salary  $b_{t+1}$  increases in his output  $q_t$ , because the principal thinks that the agent is more driven, and thus more productive in period  $t + 1$  and should be paid more. Therefore, if  $\alpha > 0$ , by increasing effort, the agent can increase the principal's belief that he is more driven. The incentive to manipulate the principal's beliefs about his drive is referred to as the agent's drive-signaling incentive.

In the long term, the agent's incentive to work hard in period  $t$  influences all his future expected wages. Thus his effort cost must be equal to the returns in terms of the increase in all his future wages. For any  $t' > t$ , in steady state, it can be shown that:

$$\frac{\partial b_{t'}}{\partial q_t} = (1-\beta) \frac{\sigma_{\alpha m}^2}{\sigma_{\alpha m}^2 + \sigma_\epsilon^2} \left[ \frac{\sigma_\epsilon^2}{\sigma_{\alpha m}^2 + \sigma_\epsilon^2} \right]^{t'-t-1}$$

Since the agent does not expect his marginal utility to change on average, he uses  $m_t$  in evaluating the future return to his effort in period  $t$ . Thus, discounting and summing  $\frac{\partial b_t}{\partial q_t}$  over  $t'$  and using  $e_t = \alpha m_t$ , the agent's effort supply is determined by the following equation:

$$k\alpha = \beta + (1-\beta) \frac{\delta \sigma_{am}^2}{(1-\delta)\sigma_e^2 + \sigma_{am}^2}. \quad (2)$$

Eqs. (1) and (2) are necessary and sufficient for the pair  $(\alpha, \sigma_{am}^2)$  to constitute a steady state. The following result focuses on the agent's effort supply with little explicit incentives.

**Theorem 1.** For  $\beta > 0$ , there exists a unique steady state which has positive average effort ( $\alpha > 0$ ). Furthermore,

1. Suppose  $(1-\delta)^2 \sigma_e^2 < \delta^2 \sigma_v^2$ . For  $\beta = 0$ , there are two steady states, one with zero effort ( $\alpha = 0$ ) and one with positive average effort ( $\alpha > 0$ ). As  $\beta \rightarrow 0$ , the steady state approaches the positive steady state corresponding to  $\beta = 0$ .
2. Suppose  $(1-\delta)^2 \sigma_e^2 \geq \delta^2 \sigma_v^2$ . For  $\beta = 0$ , the unique steady state has zero effort ( $\alpha = 0$ ). As  $\beta \rightarrow 0$ , the corresponding  $\alpha$  approaches zero.

The first part of **Theorem 1** demonstrates just how powerful drive-signaling incentives can be. Even with very weak explicit incentives ( $\beta$  arbitrarily close to 0), agents exert a significant amount of effort with an infinite horizon. More importantly, **Theorem 1** says that high effort is not merely a possibility, but the *unique* steady-state equilibrium for some parameters. The intuition is that the need to prove one's drive feeds on itself. When  $\beta$  is positive, the steady-state  $\alpha$  is at least a small positive number, because more driven agents care more about the performance-contingent bonus and thus work at least slightly harder. But once agents with different levels of drive behave differently, they will be willing to work not only for the original bonus, but also to signal that they are driven. Moreover, more driven people respond more strongly to these drive-signaling incentives, further increasing the gap between the more and the less driven and strengthening their incentive to work, and so on.

The bootstrapping result implies that even though drive-signaling incentives rely on some existing incentives to build on, they can build on *very little* such incentives. In this model, as soon as explicit incentives create a small difference between agents of different drive, the above intuition kicks in, and drive-signaling incentives get a life of their own. This indicates that, at least when drive changes stochastically, the agent's incentive to work hard may not decline much even when existing incentives are very weak.<sup>2</sup>

The key condition for bootstrapping to occur is  $(1-\delta)^2 \sigma_e^2 < \delta^2 \sigma_v^2$ . Several factors contribute to the force of the self-reinforcing mechanism described above. First, if drive changes more from period to period ( $\sigma_v^2$  is large), or the agent is more patient ( $\delta$  close to 1) – both of which imply that there are greater incentives to signal one's drive – bootstrapping is more likely to occur. For instance, this implies that after periods of major changes in life, a more driven agent has a stronger incentive to signal that his new drive is high by working hard. This prediction is consistent with evidence on the motherhood wage penalty: [Waldfogel \(1997\)](#) and [Anderson et al. \(2003\)](#), among others, show that the penalty is due to hidden heterogeneity such as the perceived reduced drive, even after discounting the effect on tenure and experience. Second, if agents are more responsive to incentives ( $k$  is small), drive-signaling incentive both builds more quickly on itself and more quickly increases the difference between different types of agents (on which drive-signaling incentives

depend). In addition, if output is accurately observed ( $\sigma_e^2$  is small), it is easier to signal one's drive, making bootstrapping more likely to occur.

#### 4. Extension and concluding remarks

Many existing career concerns models focus on agents with heterogeneity in ability. Since most circumstances of interest feature heterogeneity along both dimensions, the main model is extended to show that significant effort can also build on vanishingly small amount of implicit incentives, such as the incentive to show one has high ability.

Agents now differ also in their ability: a high ability agent is more productive even without any incentives in place. In each period, an agent produces output  $q_t = a_t + e_t + \epsilon_t$ , where  $a_t$  is the agent's ability in period  $t$ . Before period 1 production, the principal and the agent have the same prior belief about his ability:  $a_0 \sim N(0, \sigma_a^2)$ . Over time, the agent's ability evolves according to the following process:

$$a_{t+1} = a_t + \eta_t, \eta_t \sim N(0, \sigma_\eta^2).$$

To focus on the interaction of drive-signaling incentives and career concerns incentives, it is assumed that no explicit incentives are in place. Thus the principal is restricted to fixed-wage contracts  $w_t = E[q_t | h^{t-1}]$  in each period. All other assumptions remain. The following result described the agent's steady-state effort supply when the heterogeneity in ability is very small.

**Theorem 2.** A steady state always exists. Moreover, (1) suppose  $k^2(1-\delta)^2 \sigma_e^2 < \delta^2 \sigma_v^2$ . For  $\sigma_\eta^2 = 0$ , there are two steady states, one with  $\alpha = 0$  and one with  $\alpha > 0$ . For any  $\sigma_\eta^2 > 0$ , there is a unique steady state. As  $\sigma_\eta^2 \rightarrow 0$ , the steady state approaches the positive steady state corresponding to  $\sigma_\eta^2 = 0$ . (2) Suppose  $k^2(1-\delta)^2 \sigma_e^2 \geq \delta^2 \sigma_v^2$ . For  $\sigma_\eta^2 = 0$ , the unique steady state has  $\alpha = 0$ . For a sufficiently small  $\sigma_\eta^2$ , the steady state is unique. As  $\sigma_\eta^2 \rightarrow 0$ , the corresponding  $\alpha$  approaches 0.

Even when  $\sigma_\eta^2$  approaches zero, with an infinite horizon the agent exerts a significant amount of effort. The reason is that when  $\sigma_\eta^2 > 0$ , the steady-state  $\alpha$  is at least a small positive number, because more driven agents care more about convincing the principal of their higher ability, which increases their future wages, and thus work slightly harder. This gives rise to drive-signaling incentive which builds on itself. This result contrasts with [Holmström \(1999\)](#), in which the agent only has an incentive to work hard if the heterogeneity in ability is large. Because even if ability is unknown to begin with, once it is almost known – which will occur in an infinite horizon model when  $\sigma_\eta^2$  is sufficiently close to zero – new outputs do not influence the principal's belief about the agent's ability any more, and he ceases providing effort.

Together, **Theorems 1 and 2** show that an agent's incentive to work hard to signal he is driven may not disappear when only very weak explicit or implicit incentives are present. This suggests that economic models of the firm should consider heterogeneity in various dimensions of productivity, not only differences in ability. Moreover, agents in this model essentially try to show that they consider their careers very important, which is potentially important in any career concerns applications, including those studying effort supply and those studying reputational cheap talk ([Prendergast and Stole, 1996](#); [Ottaviani and Sorensen, 2006](#); [Li, 2007](#)).

#### Acknowledgments

I thank Botond Kőszegi and Edward Schlee for advice and encouragement. I am also grateful for comments from George Akerlof, Mathias Dewatripont, Glenn Ellison, Bengt Holmström, Michael Jansson,

<sup>2</sup> Note that for  $(1-\delta)^2 \sigma_e^2 < \delta^2 \sigma_v^2$ , bootstrapping creates a discontinuity. If  $\beta = 0$ , zero effort ( $\alpha = 0$ ) is a steady state, and possibly even the more reasonable one because it is the unique equilibrium for any finite-horizon approximation of the current model. But for  $\beta > 0$ , the unique steady state effort is not close to zero.

Jon Levin, Markus Möbius, Antonio Rangel, Emmanuel Saez, Chris Shannon, and Jean Tirole. This paper has also benefited from the suggestions given by seminar participants at UC Berkeley, Chicago GSB, MIT, UC Riverside, USC, Stockholm School of Economics, Universitat Pompeu Fabra, and the 9th World Congress of the Econometric Society.

**Appendix**

**Proof of Lemma 1.** Recall that the conditional variance of  $\alpha m_t$  is  $\sigma_{\alpha m_t}^2$ . From simple updating of normals, for any  $\tilde{t} > t + 1$ , we have

$$\frac{\partial b_{\tilde{t}}}{\partial q_t} = (1-\beta) \frac{\partial}{\partial q_t} E[\alpha m_{\tilde{t}} | h^{\tilde{t}-1}] = (1-\beta) \cdot \frac{\sigma_{\alpha m_t}^2}{\sigma_{\alpha m_t}^2 + \sigma_\epsilon^2} \cdot \prod_{s=t+1}^{\tilde{t}-1} \frac{\sigma_\epsilon^2}{\sigma_{\alpha m_s}^2 + \sigma_\epsilon^2}. \tag{3}$$

Thus, the total return to increasing effort in period  $t$  is:

$$kx = \beta + (1-\beta) \frac{\sigma_{\alpha m_t}^2}{\sigma_{\alpha m_t}^2 + \sigma_\epsilon^2} \left( \delta + \sum_{t+2}^{\infty} \delta^{\tilde{t}-t} \prod_{s=t+1}^{\tilde{t}-1} \frac{\sigma_\epsilon^2}{\sigma_{\alpha m_s}^2 + \sigma_\epsilon^2} \right). \tag{4}$$

Let  $y = \frac{kx-\beta}{1-\beta}$ . For  $\alpha_t$  to be constant, the total return to increasing effort in period  $t + 1$  has to be the same. This leads to the following recursion:

$$y = \frac{\sigma_{\alpha m_t}^2}{\sigma_{\alpha m_t}^2 + \sigma_\epsilon^2} \left[ \delta + \delta \frac{\sigma_\epsilon^2}{\sigma_{\alpha m_{t+1}}^2} y \right]. \tag{5}$$

After some manipulation, we can put the above in the following form:

$$\delta \left( \frac{1}{\sigma_{\alpha m_t}^2} - \frac{1}{\sigma_{\alpha m_{t+1}}^2} \right) = \text{constant} - \frac{1-\delta}{\sigma_{\alpha m_t}^2} \tag{6}$$

Now we prove by contradiction. Suppose the left hand side (LHS) of Eq. (6) is not zero. Suppose first that it is positive, then  $\sigma_{\alpha m_{t+1}}^2 > \sigma_{\alpha m_t}^2$ . Take the corresponding expression for  $t + 1$ :

$$\delta \left( \frac{1}{\sigma_{\alpha m_{t+1}}^2} - \frac{1}{\sigma_{\alpha m_{t+2}}^2} \right) = \text{constant} - \frac{1-\delta}{\sigma_{\alpha m_{t+1}}^2} \tag{7}$$

Since the right hand side (RHS) of Eq. (7) is greater than that of Eq. (6), we have  $\sigma_{\alpha m_{t+2}}^2 > \sigma_{\alpha m_{t+1}}^2$ . Furthermore, the reciprocal of the conditional variance is decreasing at an increasing rate. But that is impossible, since the reciprocal is bounded from below by zero.

Now suppose that the LHS of Eq. (6) is negative. Then,  $\sigma_{\alpha m_{t+1}}^2 < \sigma_{\alpha m_t}^2$ , and the RHS of Eq. (7) is smaller than that of Eq. (6). Therefore, the reciprocal of  $\sigma_{\alpha m_t}^2$  is increasing at an increasing rate. This implies that  $\sigma_{\alpha m_t}^2 \rightarrow 0$  as  $t \rightarrow \infty$ . But since  $m_t$  changes every period (by random variables of given variance), the principal's beliefs cannot become arbitrarily precise. This completes the proof.

**Proof of Theorem 1.** Eqs. (1) and (2) define two curves of  $\alpha$  as a function of  $\sigma_{\alpha m}^2$ . Call these curves  $f(\sigma_{\alpha m}^2)$  and  $g_\beta(\sigma_{\alpha m}^2)$ , where  $g_\beta(\sigma_{\alpha m}^2)$

denotes a family of functions indexed by  $\beta$ . Since  $\sigma_{\alpha m}^2 \geq 0$ , steady states are intersections of the two curves in the positive domain. Note that for any  $\beta > 0$ ,  $g_\beta(0) > f(0)$ . Also,  $g$  is bounded while  $f$  is not. Thus they intersect by continuity.

Next we prove uniqueness for  $\beta > 0$ . For  $\sigma_{\alpha m}^2 > 0$ , we can take the derivative of  $f$  and  $g$ , and put them in the following form:

$$f'(\sigma_{\alpha m}^2) = \frac{1}{\sigma_v} \cdot \frac{\sqrt{\sigma_\epsilon^2 + \sigma_{\alpha m}^2} - \frac{\sigma_{\alpha m}^2}{2\sqrt{\sigma_\epsilon^2 + \sigma_{\alpha m}^2}}}{\sigma_\epsilon^2 + \sigma_{\alpha m}^2} = \frac{f(\sigma_{\alpha m}^2)}{\sigma_{\alpha m}^2} - \frac{f(\sigma_{\alpha m}^2)}{2(\sigma_\epsilon^2 + \sigma_{\alpha m}^2)}$$

$$g'_\beta(\sigma_{\alpha m}^2) = \frac{g_\beta(\sigma_{\alpha m}^2) - \frac{\beta}{k}}{\sigma_{\alpha m}^2} - \frac{g_\beta(\sigma_{\alpha m}^2) - \frac{\beta}{k}}{(1-\delta)\sigma_\epsilon^2 + \sigma_{\alpha m}^2}.$$

It is easy to see that  $f'(\sigma_{\alpha m}^2) > g'_\beta(\sigma_{\alpha m}^2)$  whenever  $f(\sigma_{\alpha m}^2) \geq g_\beta(\sigma_{\alpha m}^2)$ . Since  $g$  is increasing in  $\beta$  and  $g'$  is decreasing in it, the same is true for any positive  $\beta$  as well. Together with the fact that  $f$  and  $g$  intersect, this implies that the intersection is unique. This steady state obviously has  $\alpha > 0$ .

For  $\beta = 0$ ,  $\alpha = \sigma_{\alpha m}^2 = 0$  is clearly a steady state. Since  $g_0(0) = f(0)$ , the argument above implies that a positive steady state exists if  $g_0(0) > f'(0)$ , and does not exist if  $g_0(0) < f'(0)$ . When  $g_0(0) = f'(0)$ , it is easy to prove that  $f'(\sigma_{\alpha m}^2) > g_0(\sigma_{\alpha m}^2)$  for any positive  $\sigma_{\alpha m}^2$ . Thus, a positive steady state exists if and only if  $g'_0(0) > f'(0)$ . Now, we have  $g'_0(0) = \frac{\delta}{k(1-\delta)\sigma_\epsilon^2}$  and  $f'(0) = \frac{1}{\sigma_v \sigma_\epsilon}$ . Thus,  $g'_0(0) > f'(0)$  if and only if  $k^2(1-\delta)^2\sigma_\epsilon^2 < \delta^2\sigma_v^2$ . Finally, the convergence results follow from the continuity of  $f$  and  $g_\beta$  (in both  $\sigma_{\alpha m}^2$  and  $\beta$ ).

**Proof of Theorem 2.** Similar to above and omitted. Available upon request.

**References**

Anderson, D.J., Binder, M., Krause, K., 2003. The motherhood wage penalty revisited: experience, heterogeneity, work effort, and work-schedule flexibility. *Industrial & Labor Relations Review* 56 (2), 273–294.

Dewatripont, M., Jewitt, I., Tirole, J., 1999. The economics of career concerns, part I: comparing information structures. *Review of Economic Studies* 66, 183–198.

Fama, E.F., 1980. Agency problems and the theory of the firm. *Journal of Political Economy* 88 (2), 288–307.

Gibbons, R., Murphy, K.J., 1992. Optimal incentive contracts in the presence of career concerns: theory and evidence. *Journal of Political Economy* 100 (3), 468–505.

Holmström, B., 1999. Managerial incentive problems: a dynamic perspective. *Review of Economic Studies* 66 (1), 169–182 first published in 1982.

Kőszegi, B., Li, W., 2008. Drive and talent. *Journal of the European Economic Association* 6 (1), 210–236.

Kuhn, P., Lozano, F., 2008. The expanding workweek? Understanding trends in long work hours among U.S. men, 1979–2006. *Journal of Labor Economics* 26 (2), 311–343.

Li, W., 2007. Changing one's mind when the facts change: incentives of experts and the design of reporting protocols. *Review of Economic Studies* 74 (4), 1175–1194.

Ottaviani, M., Sorensen, P., 2006. Reputational cheap talk. *The Rand Journal of Economics* 37 (1), 155–175.

Prendergast, C., Stole, L., 1996. Impetuous youngsters and jaded old-timers: acquiring a reputation for learning. *Journal of Political Economy* 104 (6), 1105–1134.

Stoddard, C., Kuhn, P., 2008. Incentives and effort in the public sector: have U.S. education reforms increased teachers' work hours. *Economics of Education Review* 27 (1), 1–13.

Waldfogel, J., 1997. The effect of children on women's wages. *American Sociological Review* 62 (2), 209–217.