Information Acquisition in Global Games of Regime Change

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Abstract

We study costly information acquisition in global games, which are coordination games where payoffs are discontinuous in the unobserved state and in the average action of agents. We show that only symmetric equilibria exist and we provide sufficient conditions for uniqueness. We then characterize the value of information in this context and link it to the underlying parameters of the model. We explore the notion of equilibrium efficiency, complementarities in information choices, and the trade-off between public and private information. We show that the unique equilibrium of the game is inefficient and that strategic complementarities in actions do not always translate into strategic complementarities in information choices. Finally, we find that public and private information can be complements. These results contrast findings in beauty contest models, which are coordination games where payoffs depend continuously on the quadratic distance between individual actions and both the unobserved state and the average action of agents. We argue that these disparities are a result of differences in the value of additional information across these two classes of models. Therefore, our results emphasize the importance of the type of payoff structures (continuous versus discrete) in coordination games.

Key words: global games, information acquisition, coordination, value of information, public information.

1 Introduction

Global games have been extensively applied to model economic phenomena featuring coordination problems, such as currency crises (Morris and Shin, 1998), bank runs (Goldstein and Pauzner, 2005), FDI decisions (Dasgupta, 2007), and political revolts (Edmond, 2013). In a global game the payoffs of agents depend on both the state of the economy and on the actions of others. However, agents only observe noisy private and public signals about this state and, in order to choose an optimal action, they have to make inferences about its true value

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and about the beliefs that other agents hold. This perturbation of the information structure of the game gives rise to a very rich sequence of higher order beliefs, which leads agents to coordinate on a unique equilibrium. This prediction contrasts the complete information model, which features multiple equilibria. While the original models have been extended along many directions, the precision of private signals has typically been exogenously given and set to be identical across agents. In this paper we fill this gap by introducing costly information acquisition into the standard global games framework.

Endogenizing information in a global game is a relevant endeavor from an applied point of view as well. Following Dasgupta (2007), one can think of an emerging economy that wants to attract Foreign Direct Investment. In this model, potential investors have to decide whether to invest or not invest. For the profits to be positive there has to be enough investment so that the liberalization program succeeds (due to increasing returns to aggregate investment), so investors will want to coordinate on their decisions.[1] The returns of the project depend also on the state of the emerging economy, which can be uncertain at the time of the investment decision. In this context, potential investors can acquire more precise information about the state of the emerging economy by buying reports from that will assess the profitability of this investment.

Introducing costly information acquisition into a global game gives rise to a set of natural questions with non-trivial implications. We focus on the following questions: do investors acquire the socially efficient amount of private information (i.e. do they over-acquire or under-acquire information)? Are there strategic complementarities in information choices (i.e. do investors want to learn what others learn)? What is the trade-off between private and public information in this context? Does more precise public information always crowd out private information acquisition? Does it increase the probability of a successful investment? And finally, does it increase welfare?

In order to answer these questions we first characterize an equilibrium in our model. Our findings indicate that, under mild conditions on parameters, there exists a unique equilibrium in symmetric strategies. This supports the commonly made assumption of identical precision levels across agents in global games. We define the value of additional information in our setup and analyze how it is affected by prior beliefs, the behavior of other players, and the cost of investment. We find that the value of additional information is determined by the extent to which it helps the agent to avoid two types of mistakes in the coordination game: investing when investment is not profitable, and not investing when investment is profitable.

Using these insights we address each of the questions raised above. We find that the unique equilibrium of the game is generically inefficient and that, depending on the characteristics of the economy, investors either over-acquire or under-acquire information. In terms of strategic motives in information acquisition, we find conditions under which strategic complementarities in information acquisition arise and conditions where this is not the case, so that the optimal precision choice of an agent is a non-monotone function of the precision

[1] See Hall et al (1986), Hall (1987), and Caballero and Lyons (1992) for evidence of increasing returns to scale in investment. Cooper (1999) provides an excellent overview of the literature on complementarities in macroeconomics.

choices of others.

We then study the effects of an increase in the precision of public information on welfare. Our analysis provides a novel perspective on this issue by analyzing the trade-off between public and private information acquisition. In our model public information affects outcomes not only through agents' actions in the coordination game, but also by changing their incentives to acquire private information. We provide conditions under which more precise public information crowds out private information. Surprisingly, we find cases in which more precise public information leads investors to acquire more precise private information, i.e. where private and public information are complements. Finally, we show that the effect of more precise public information on the probability of successful investment and welfare depends on the characteristics of the economy.

Our analysis highlights the differences between global games and the closely related family of beauty contest models (in the spirit of Morris and Shin, 2002).[2] First, we find that whether an improvement in public information is welfare enhancing or not depends crucially on the ex-ante beliefs about the state of the economy, while in beauty contest models it depends on the relative informativeness of private and public information (Morris and Shin, 2002, Colombo et al., 2014). Second, in beauty contest models strategic complementarities in actions always lead to strategic complementarities in information acquisition (Hellwig and Veldkamp, 2009, Colombo et al., 2014, Myatt and Wallace, 2014). In the case of global games, we state conditions under which strategic complementarities in actions translate into strategic complementarities in information acquisition and show that if these conditions are violated then information choices are not strategic complements. Finally, in beauty contest models with private information acquisition an increase in the precision of public information always decreases the incentives to acquire more precise private information (Tong, 2007, Colombo et al., 2014), whereas in our model private and public information can be complements.

The paper is structured as follows. In section 2 we set up the model with costly information acquisition and explain the assumptions we make to solve it. In section 3 we solve the model and present results about the non-existence of asymmetric equilibria, existence of symmetric equilibria, and conditions ensuring uniqueness of the symmetric equilibrium. In section 4 we investigate notions of efficiency of the unique equilibrium. In section 5 we investigate if strategic complementarities in the coordination game translate into strategic complementarities in information choices. In section 6 we ask whether an increase in the precision of public information is welfare enhancing or not. Section 7 compares our results to previous results on information acquisition in beauty contest models. Section 8 summarizes the related literature and section 9 concludes. All the proofs are relegated to the appendix.

[2] Beauty contest models are also coordination games of incomplete information, but differ from global games in many respects. Choice sets are continuous in beauty contest models (as opposed to binary, as in global games) and agents have a quadratic utility function that depends on the distance between individual actions and both the average action of the other players and the underlying state of the economy.

2 The Model

We consider a two period model where investors have to decide first how much information to purchase and then, given this information, whether or not to invest in a risky project. The first period, where investors choose the precision of their private signals, constitutes the novel part of the model. The second period is similar to a standard global games model, with the exception that investors observe signals with different precisions.

There is a continuum of investors in the economy indexed by i, where $i \in [0,1]$. The economy is characterized by a parameter θ that measures the strength of its economic fundamentals and that is unobserved by investors. Each investor has to make two decisions. First, he has to decide how much information to acquire about θ. Then he has to decide whether to invest in a risky project (I) or not invest (NI). If an investor decides to invest he incurs cost $T \in (0,1)$. The benefit to investing is uncertain and depends on the state θ and on p, the proportion of investors that choose to invest. Investment is successful if $p > 1-\theta$, i.e. if the proportion of investors who invest is high enough with respect to the state. The return of a successful investment for each investor who invests is 1, in which case he will get the payoff $1-T$. Otherwise, if investment is unsuccessful, his payoff will be $-T$. The payoff to not investing is certain and normalized to 0. The payoffs are summarized below:[3]

$$u(I,p,\theta) = \begin{cases} 1-T \text{ if } p \geq 1-\theta \\ -T \text{ if } p < 1-\theta \end{cases} \tag{1a}$$

$$u(NI,p,\theta) = 0 \tag{1b}$$

Whether individual investment is successful or not depends on the state of the economy and on the number of individual investments. One can interpret this need for enough aggregate investment as resulting from increasing returns to scale in investment.[4]

Investors do not observe the state of the economy θ. Instead, they share a common prior belief that $\theta \sim N\left(\mu_\theta, \tau_\theta^{-1}\right)$. In addition, at the beginning of period 2, each investor i observes a noisy private signal about the realization of θ, given by x_i:

$$x_i = \theta + \tau_i^{-1/2}\varepsilon_i \ , \ \forall i \in [0,1] \, ,$$

where $\varepsilon_i \sim N(0,1)$ is an idiosyncratic noise, *i.i.d.* across investors, and independent of the realization of θ, and τ_i is the precision of investor i's signal.

In period 1, each investor decides how much information about θ to acquire by choosing the precision of his signal, $\tau_i \in [\underline{\tau}, \infty)$. If an investor chooses not to acquire information he will observe a signal with a default precision $\underline{\tau}$. The cost associated with choosing a precision τ_i is given by $C(\tau_i)$, i.e. investors face a trade-off between informativeness and cost of signals. After observing their respective signals, investors decide simultaneously whether to invest in the project or not. The payoffs from investment decisions, given by (1a) and (1b), are realized at the end period 2.

[3]This payoff structure is standard in the global games literature (see for example Corsetti et al., 2004, Morris and Shin, 2004, Hellwig et al. 2006, Dasgupta, 2007).

[4]The payoffs are chosen to make the game analytically tractable. All the qualitative results still hold if we allow the benefit from investing to be an explicit function of both the state θ and aggregate investment.

2.1 Assumptions

Before solving the model we make two sets of assumptions. The first one considers the underlying parameters of the game, while the second one pertains to the cost function.

Assumption 1 (Concavity) *We assume the following:*

- $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta], \quad 0 < \underline{\tau}_\theta < \overline{\tau}_\theta < \infty$
- $\mu_\theta \in [\underline{\mu}_\theta, \overline{\mu}_\theta], \quad -\infty < \underline{\mu}_\theta < \overline{\mu}_\theta < \infty$
- $T \in [\underline{T}, \overline{T}], \quad 0 < \underline{T} < \overline{T} < 1$
- $\underline{\tau} > \frac{1}{2\pi}\overline{\tau}_\theta^2$

The lower bound for precision choices $\underline{\tau}$ is chosen high enough to ensure not only that the coordination game has always a unique equilibrium, but also that the ex-ante utility function is concave in the individual precision choice τ_i.[5] The details of determining $\underline{\tau}$ can be found in the online appendix.

Assumption 2 (Cost function) *We assume that the cost function $C(\cdot)$ is:*

- *strictly increasing in τ_i, $C'(\cdot) > 0$*
- *strictly convex in τ_i, $C''(\cdot) > 0$*
- $C'(\underline{\tau}) = 0$
- $\lim_{\tau_i \to \infty} C'(\tau_i) = \infty$

These assumptions imply that the cost function is strictly convex, a common assumption in the literature on information acquisition. We further assume that an infinitesimal improvement in precision is costless to ensure that the problem is non-trivial and that investors always acquire information. The last assumption ensures that investors will never choose to acquire perfect information.

In the paper, we consider an additive Gaussian information structure and model information acquisition as a continuous precision choice. As pointed out by Yang (2014), this is not necessarily the information structure that investors would choose if they had the flexibility to design their own information structure. Yang (2014) shows that, in a similar setup, investors would typically prefer to observe binary signals, for a given θ. An advantage of Yang's approach is that investors can choose not only how much information to acquire, but also the type of signal they observe and its informativeness for any value of fundamentals. This allows investors to coordinate on their signal structures, and not only on their informativeness or precision, as is typically assumed in the literature and in our model.

[5] As pointed out by Radner and Stiglitz (1984) the marginal value of information can be increasing for low levels of informativeness. We choose $\underline{\tau}$ in order to ensure concavity of the ex-ante utility function in τ_i.

Despite this limitation, assuming an additive information structure has several advantages in the context of our model. First, allowing for flexible information acquisition as in Yang (2014) introduces multiplicity of equilibria into the model, which makes it difficult to establish comparative statics results. In contrast, by choosing an additive structure we can guarantee a unique equilibrium. Second, an additive Gaussian information structure is more tractable and allows us to analyze the resulting equilibrium in greater detail, which would not be possible under flexible information acquisition. Finally, using an additive information structure allows us to compare our results with the existing literature, both on global games with exogenous information structures and on information acquisition in beauty contest models.[6]

3 Solving the Model

We solve the model using backward induction. We start in period 2, taking as given the precision choices made by investors in period 1. Once we characterize the equilibrium outcome at $t = 2$ we move to the first stage of the game to determine optimal information choices. Note that the coordination game played by investors at $t = 2$, except for the heterogeneity in the quality of investors' signals, corresponds to a standard global game (see, for example, Morris and Shin, 2004).

3.1 Solving the Model: $t = 2$

Let Γ be a distribution of precision choices τ_i, that is, $\Gamma(\tau)$ is the proportion of investors who choose precision $\tau_i \leq \tau$ in the first period. To make his decision, investor i has to take into account the distribution of τ_i's in the economy (Γ), his own precision level (τ_i), his signal (x_i), and his prior belief about θ. Following the literature, we show that there exists a unique equilibrium in monotone strategies and that this is the only type of equilibrium in the coordination game.

Assume that all investors follow monotone strategies and let $a_i(x_i; \tau_i, \Gamma)$ be investor i's strategy.[7] Then, $a_i(\cdot)$ is monotone if there exists $x_i^*(\tau_i, \Gamma)$ such that

$$a_i(x_i; \tau_i, \Gamma) = \begin{cases} I \text{ if } x_i \geq x_i^*(\tau_i, \Gamma) \\ NI \text{ if } x_i < x_i^*(\tau_i, \Gamma) \end{cases}$$

Note that the thresholds can differ across investors with different precision levels and that they also depend on Γ. We assume that all investors with the same precision level, τ_i, have the same threshold $x_i^*(\tau_i, \Gamma)$. As in the standard global games models, the equilibrium in

[6] Note that an additive information structure is a common modelling device not only in the context of global games or beauty contest models, but also in the broad literature on costly information acquisition. See Veldkamp (2011) for examples in macroeconomics and finance, or Hwang (1993) and Hauk and Hurkens (2001) for examples in industrial organization.

[7] In what follows we assume that each investor conditions his strategy on the distribution of precision choices Γ, rather than on each investor j's precision choice τ_j, $j \neq i$. This assumption is without loss of generality since investors do not care about the identity of a particular investor j who chooses precision τ_j, but rather about the proportion of investors that choose a given precision level.

monotone strategies is characterized by two equations: a Payoff Indifference (PI) condition and a Critical Mass (CM) condition. The difference with respect to the standard setup is that in our model each type τ_i has a different PI condition.[8]

Consider first the CM condition, which requires that at state θ^* the mass of investors that invest is equal to the mass of investors needed for investment to succeed:

$$\int \Pr\left(x_i \geq x_i^*\left(\tau_i; \Gamma\right) | \theta^*\right) d\Gamma(\tau_i) = 1 - \theta^*$$

Next, consider investor i, whose precision level is τ_i. The PI condition states that when observing signal $x_i^*\left(\tau_i, \Gamma\right)$, investor i is indifferent between investing and not investing, that is:

$$\Pr\left(\theta > \theta^*\left(\Gamma\right) | x_i^*\left(\tau_i; \Gamma\right)\right) - T = 0 \tag{2}$$

An equilibrium in monotone strategies is characterized by a set of signal thresholds $\{x_i^*\left(\tau_i, \Gamma\right)\}_{i \in [0,1]}$ and a threshold level for fundamentals, $\theta^*\left(\Gamma\right)$, that solve the PI and CM equations simultaneously. For the case of a normal distribution, this system of equations can be simplified to one equation in one unknown, $\theta^*\left(\Gamma\right)$:

$$\int \Phi\left(\frac{\tau_\theta}{\tau_i^{1/2}}\left(\theta^*\left(\Gamma\right) - \mu_\theta\right) + \frac{\left(\tau_i + \tau_\theta\right)^{1/2}}{\tau_i^{1/2}}\Phi^{-1}\left(T\right)\right) d\Gamma(\tau_i) = \theta^*\left(\Gamma\right) \tag{3}$$

Each $\theta^*\left(\Gamma\right)$ that solves Equation (3) is then associated with a different equilibrium. The next proposition specifies conditions for this equation to have a unique solution and for no other non-monotone equilibria to exist.

Proposition 1 *Under Assumption A1, for any Γ, the coordination game has a unique equilibrium in which all investors use threshold strategies $\{x_i^*\left(\tau_i, \Gamma\right), i \in [0,1]\}$ and investment is successful if and only if $\theta \geq \theta^*\left(\Gamma\right)$.*[9]

Note that the above proposition is a generalization of standard uniqueness result in global games as established by Hellwig (2002) and Morris and Shin (2004) to the setting where agents are heterogenous with respect to the precision of their information. Armed with this result we move on to the first period to analyze investors' optimal choices of precision.

3.2 Solving the Model: $t = 1$

We now consider the first stage of the game in which investors choose the precision of the signal they will observe at the beginning of the second stage. We assume that each investor will act optimally in the second period and that he believes that all other investors will act optimally as well.

[8] See Hellwig (2002) for a detailed derivation of PI and CM conditions in the model where investors share the same precision.

[9] Assumption $A1$ is stronger than necessary. Proposition 1 holds as long as $\inf\left(supp(\Gamma)\right) > \frac{1}{2\pi}\tau_\theta^2$.

3.2.1 Ex-ante Utility

Denote by $G_{\tau_\theta}(\theta)$ the prior belief of investors regarding θ, and by $F_{\tau_i}(x|\theta)$ the conditional distribution of x_i given θ and given that the signal x_i has precision τ_i. Recall that all investors are ex-ante identical, i.e. they have the same ex-ante utility.

The ex-ante utility of investor i who chooses precision τ_i and faces a distribution of precision choices Γ, for any $(\tau_\theta, \mu_\theta, T)$, can be written as:[10]

$$U^i(\tau_i;\Gamma) = -\int_{-\infty}^{\theta^*}\int_{x_i^*}^{\infty} T dF_{\tau_i}(x|\theta)\, dG_{\tau_\theta}(\theta) - \int_{\theta^*}^{\infty}\int_{-\infty}^{x_i^*}(1-T)\, dF_{\tau_i}(x|\theta)\, dG_{\tau_\theta}(\theta) + \int_{\theta^*}^{\infty}(1-T)\, dG_{\tau_\theta}(\theta) - C(\tau_i) \tag{4}$$

The above expression has an intuitive interpretation. The last term is the cost associated with the precision choice τ_i. Recall that investment is successful if and only if $\theta \geq \theta^*(\Gamma)$, in which case, if an investor invests his payoff is $1-T$. Hence, the third term of the above expression is the expected payoff at time $t=1$ for an investor who can perfectly observe θ in the second period. However, for any $\tau_i < \infty$ an investor's information at $t=2$ is noisy. This means that the investor will sometimes make mistakes, either investing when investment is unsuccessful (Type I mistake) or not investing when investment is successful (Type II mistake). The first two terms capture the expected cost of these two mistakes, respectively. We denote the cost of these mistakes for an investor with precision τ_i who faces a distribution of precision choices Γ by $M(\tau_i;\Gamma)$ where:

$$M(\tau_i;\Gamma) = \int_{-\infty}^{\theta^*}\int_{x_i^*}^{\infty} T dF_{\tau_i}(x|\theta)\, dG_{\tau_\theta}(\theta) + \int_{\theta^*}^{\infty}\int_{-\infty}^{x_i^*}(1-T)\, F_{\tau_i}(x|\theta)\, dG_{\tau_\theta}(\theta).$$

To better understand how a higher precision is beneficial to investors, we abstract from the cost of precision and focus on its benefit captured in the first three terms of equation (4). We define this benefit as $B^i(\tau_i;\Gamma)$:

$$B^i(\tau_i;\Gamma) \equiv -M(\tau_i;\Gamma) + \int_{\theta^*}^{\infty}(1-T)\, dG_{\tau_\theta}(\theta).$$

From the above equation we see that more precise private information is valued by an investor to the extent that it allows him to avoid committing costly mistakes. The specific mechanism is formalized in the following lemma.

Lemma 1 *The benefit of an increase in the precision of private signals is equal to the reduction of the expected cost of mistakes due to a change in the ex-ante joint distribution of*

[10] See Section $A.3$ of the Appendix for derivations.

(θ, x_i) *implied by this increase, and is given by:*[11,12]

$$\frac{\partial B^i(\tau_i;\Gamma)}{\partial \tau_i} = \frac{1}{2\tau_i}\frac{1}{\tau_i+\tau_\theta}\tau_i^{1/2}\phi\left(\frac{x_i^*-\theta^*}{\tau_i^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^*-\mu_\theta}{\tau_\theta^{-1/2}}\right). \tag{5}$$

Equation (5) shows that, for a Gaussian noise structure, the value of additional information depends on the distance between x_i^* and θ^* and on the distance between θ^* and μ_θ (with larger distances decreasing the value of additional information), but it does not depend on the relative cost of mistakes.[13]

To provide intuition for this result, we first focus on the distance between θ^* and μ_θ. Consider the case when difference between θ^* and μ_θ is large and positive (the case when the difference is negative is analogous). In this case, an investor assigns a low ex-ante probability to a successful investment, since prior beliefs indicate that θ is unlikely to take value greater than θ^*. As such he assigns low probability to committing Type II mistake. Thus, in equilibrium, he rarely chooses to invest (sets a high x_i^*) and expects this action to be correct most of the time. Thus, in this case the value of additional information is low. The opposite is true when θ^* and μ_θ lies close to each other. In this case, from investor's perspective, both investment outcomes are almost equally likely. Therefore, he assigns a relatively high probability to committing both mistakes and, thus, he attaches a high value to additional information.

To analyze how the value of additional information varies with the distance between x_i^* and θ^*, we need to understand why in equilibrium x_i^* might be far away from θ^*. Consider the case where x_i^* is higher than θ^*. This occurs in equilibrium when T is high and μ_θ is low. In this case, the investor is mostly concerned about making a Type I mistake, since he expects investment to be unsuccessful (low μ_θ) and investing is costly (high T). Therefore, in equilibrium he chooses a high x_i^* in order to minimize a Type I mistake. An increase in the precision of his private signal allows the investor to reduce the total expected cost of both mistakes. However, since he was already avoiding the mistake that he cares relatively more about, the reduction in the expected cost of mistakes that accompanies the increase in his precision is not very valuable. This is in contrast to the case when x_i^* is close to θ^* which happens only if the investor initially cares about avoiding both types of mistakes. As a result, an increase in precision allows him to reduce the probability of committing both

[11] A higher precision of private signals changes the expected cost of mistakes in two ways. First, a higher τ_i changes the ex-ante joint distribution of (θ, x_i) by better aligning the realization of the signal x_i to the state θ. Second, it affects the threshold x_i^*. A decrease in x_i^*, holding everything else constant, leads to a higher expected cost of a Type I mistake and a lower expected cost of a Type II mistake, since investors now invest more aggressively. However, since x_i^* is chosen to equalize the benefit from a successful investment to the potential cost of an unsuccessful investment, the marginal change in x_i^* due to a change in τ_i has no effect on the expected utility. Therefore, the marginal benefit of a higher precision comes from the change in the ex-ante joint distribution of (θ, x_i) that better aligns signals x_i with fundamentals θ.

[12] In the proof of Lemma 1 (Section A.3 of the Appendix) we provide an expression for the reduction in the expected cost of each mistake. Equation (5) is obtained by adding those two expressions.

[13] This surprising result is a consequence of the equilibrium condition $T\Pr(\theta < \theta^*|x^*) = (1-T)\Pr(\theta > \theta^*|x^*)$ and the properties of the normal distribution.

mistakes at a similar pace. It follows that in this case the value of additional information is higher than in the case where x_i^* is far away from θ^*.

3.3 Equilibrium at $t = 1$

In period 1 investors choose the precision of their signals. The expected payoff to investor i from choosing precision τ_i when he faces a distribution of precision choices Γ and believes that all investors behave optimally at $t = 2$ is given by:

$$\begin{aligned}
U^i(\tau_i;\Gamma) &= B^i(\tau_i;\Gamma) - C(\tau_i) \\
\text{where } \theta^*(\Gamma) \text{ solves } &\int \Pr\left(x_i \geq x^*(\tau_i;\Gamma)\,|\theta^*(\Gamma)\right) d\Gamma = 1 - \theta^*(\Gamma) \\
\text{and } x^*(\tau_i;\Gamma) &= \frac{\tau_i + \tau_\theta}{\tau_i}\theta^*(\Gamma) - \frac{\tau_\theta}{\tau_i}\mu_\theta + \frac{(\tau_i + \tau_\theta)^{1/2}}{\tau_i}\Phi^{-1}(T)
\end{aligned}$$

With the above description of the investor's problem at time $t = 1$, we can now define a Perfect Bayesian Nash Equilibrium of the two-stage game:

Definition 1 *A pure strategy Perfect Bayesian Nash Equilibrium is a set of precision choices* $\{\tau_i^*, i \in [0,1]\}$*, decision rules for the second period* $\{a_i^*(x_i;\tau_i,\Gamma), i \in [0,1]\}$*, and a distribution of precision choices* Γ^* *such that:*

1. *Each investor's choice of precision* τ_i^* *is optimal, given* Γ^**:*

$$B^i(\tau_i^*;\Gamma^*) - C(\tau_i^*) \geq B^i(\widehat{\tau}_i;\Gamma^*) - C(\widehat{\tau}_i) \quad \forall \widehat{\tau}_i \in [\underline{\tau}, \infty);$$

2. *The distribution implied by the investors' choices is almost surely equal to the distribution* Γ^**;*

3. *All investors behave optimally in the second stage:*

$$a_i^*(x_i;\tau_i,\Gamma^*) = \begin{cases} I \;\; if \;\; x_i \geq x_i^*(\tau_i,\Gamma^*) \\ NI \;\; if \;\; x_i < x_i^*(\tau_i,\Gamma^*) \end{cases}$$

where

$$x_i^*(\tau_i,\Gamma^*) = \frac{\tau_i + \tau_\theta}{\tau_i}\theta^*(\Gamma^*) - \frac{\tau_\theta}{\tau_i}\mu_\theta + \frac{(\tau_i + \tau_\theta)^{1/2}}{\tau_i}\Phi^{-1}(T)$$

and $\theta^*(\Gamma^*)$ *solves:*

$$\int \Phi\left(\frac{\tau_\theta}{\tau_i^{1/2}}(\theta^*(\Gamma^*) - \mu_\theta) + \frac{(\tau_i + \tau_\theta)^{1/2}}{\tau_i^{1/2}}\Phi^{-1}(T)\right) d\Gamma^*(\tau_i) = \theta^*(\Gamma^*).$$

The first condition requires that investors choose the precision of their private signals optimally. The second condition is a standard consistency requirement. Finally, the third condition requires investors to follow equilibrium strategies, given their choice of precision τ_i and their beliefs about the equilibrium precision choices of others, Γ^*. In particular, this condition requires that an investor behaves optimally in the second period even in the case of an individual deviation in precision choices.

With the above definition we can now state our main existence result.[14]

Theorem 1 *Suppose that Assumptions* $(A1)$ *and* $(A2)$ *hold. Then:*

1. *There are no asymmetric equilibria in which investors choose different precision levels in the first stage;*

2. *There exists a symmetric equilibrium of the information acquisition game where all investors choose in period* 1 *the same precision* τ^* *and equilibrium in period* 2 *is characterized by a pair of thresholds* $\{\theta^*(\tau^*), x^*(\tau^*)\}$*;*[15]

3. *There exists* $\underline{\underline{\tau}} < \infty$ *such that if* $\underline{\tau} > \underline{\underline{\tau}}$*, then there is a unique equilibrium in the information acquisition game.*

The above theorem establishes the existence of symmetric equilibria and rules out the existence of asymmetric equilibria. Moreover, if the default precision level is high enough, there is a unique symmetric equilibrium. Notice that the condition we impose on $\underline{\tau}$, i.e. that the default precision of signals is high enough, is in the same spirit as the standard condition to ensure uniqueness of equilibrium in global games. Finally, note that Theorem 1 supports the commonly made assumption of identical precision levels across agents in the context of global games.

In what follows, we assume that the above condition for uniqueness of the two-stage game is satisfied and denote the unique equilibrium precision choice as τ^*.[16] Since in equilibrium all agents choose the same precision level, with a slight abuse notation, below we express the benefit function as $B(\tau_i;\tau)$ and the ex-ante utility function as $U(\tau_i;\tau)$ rather than $B(\tau_i;\Gamma)$ and $U(\tau_i;\Gamma)$, respectively. In the remaining part of the paper we investigate the properties of the unique equilibrium.

[14] For this result to be true we need quasiconcavity of the ex-ante utility function, net of the precision cost, and a unique equilibrium in the second stage. The assumptions made in Section 2 ensure that these conditions are met (see Lemma 2 in the Online Appendix).

[15] Since in a symmetric equilibrium all investors choose the same precision, we abuse notation slightly and write $x^*(\tau^*)$ and $\theta^*(\tau^*)$ instead of $x^*(\tau^*;\Gamma^*)$ and $\theta^*(\Gamma^*)$, where $\Gamma^* = 1_{\tau \geq \tau^*}$.

[16] To be more precise, we assume that $\underline{\tau}$ is not only high enough to ensure uniqueness of equilibrium, but also that it is high enough to imply that the slope of the best-response function is lower than $\frac{5}{6}$ (the uniqueness argument requires this slope to be less than 1). Since the slope of the best response function converges to zero for all $\tau > \underline{\tau}$ as $\underline{\tau} \to \infty$, such a lower bound exists. We need this additional requirement to prove Proposition 6.

4 Spillover Effects and the Inefficiency of Equilibrium

The information acquisition game exhibits spillover effects, since investors do not take into account the impact of their precision choices on the equilibrium investment outcome. In particular, an increase in the precision of all investors affects their utility through its impact on θ^*. However, since all investors take θ^* as given, they ignore this effect when choosing their individual level of precision. This, as we show below, leads to the unique equilibrium of the game being inefficient.

We define an efficient symmetric precision choice as the one that maximizes the ex-ante expected utility taking into account the spillover effects.

Definition 2 *We say that a precision choice τ^{**} is efficient if*

$$\tau^{**} \in max_{[\underline{\tau},\infty)} B^i(\tau,\tau) - C(\tau)$$

By the above definition, the precision choice τ^{**} is efficient if it allows investors to achieve the highest ex-ante utility when they coordinate their precision choices. Let $\boldsymbol{\tau}_i^*(\cdot)$ be investor i's best response function. The difference between the equilibrium precision τ^* and the efficient precision τ^{**} is that the former is chosen in a non-cooperative fashion, that is $\tau^* = \boldsymbol{\tau}_i^*(\tau^*)$, while the latter is chosen in a cooperative fashion. Hence, τ^{**} is not necessarily a best-response to all other investors choosing τ^{**}. Indeed, we show that generically $\tau^{**} \neq \boldsymbol{\tau}_i^*(\tau^{**})$.[17]

The solution to the problem in Definition 2 is either a corner solution, $\tau^{**} = \underline{\tau}$, or it satisfies the following necessary first order condition:

$$B_1^i(\tau^{**},\tau^{**}) + B_2^i(\tau^{**},\tau^{**}) - C'(\tau^{**}) = 0$$

This condition is necessary, but not sufficient, for the equilibrium to be efficient since in some cases $B^i(\tau,\tau) - C(\tau)$ is not a quasi-concave function of τ. We discuss this issue in more detail below.[18]

We first show that the unique equilibrium is typically inefficient. To state our result we define $\mu_\theta^E(T)$ as the unique solution to:

$$\mu_\theta = \Phi\left(\sqrt{\frac{\tau^*(\mu_\theta)}{\tau^*(\mu_\theta)+\tau_\theta}}\Phi^{-1}(T)\right) + \frac{1}{\sqrt{\tau^*(\mu_\theta)+\tau_\theta}}\Phi^{-1}(T)$$

where $\tau^*(\mu_\theta)$ is the equilibrium choice of precision, given that the mean of the prior is μ_θ. We show in the appendix (proof of Proposition 2) that in equilibrium $B_2^i(\tau^*,\tau^*) = 0$ if and only if $\mu_\theta = \mu_\theta^E$. Using this observation, we arrive at the following result:

Proposition 2 *Consider the equilibrium precision choice τ^*. For any $T \in (0,1)$, if $\mu_\theta \neq \mu_\theta^E(T)$ then the equilibrium precision choice is inefficient.*

[17] We show in the appendix that the set of arguments that maximizes the above expression is non-empty and that $\tau^{**} < \infty$.

[18] See also Section 3.2 in the Online Appendix.

Above we explained why the game features spillover effects, which lead investors to acquire an inefficient level of information in equilibrium. We investigate now whether investors over-acquire or under-acquire information. We say that investors globally over-acquire information if $\tau^* (\mu_\theta) > \tau^{**} (\mu_\theta)$. On the other hand, investors locally over-acquire information if a small decrease in precision from its equilibrium level would lead to an increase in welfare. The definitions are analogous for under-acquisition of information.

The following proposition fully characterizes the conditions under which investors locally under or over acquire information in equilibrium.

Proposition 3 *Consider the investors' equilibrium precision choices.*

1. *If $\mu_\theta > \mu_\theta^E (T)$ then investors locally over-acquire information.*
2. *If $\mu_\theta = \mu_\theta^E (T)$ (and $T \geq \frac{1}{2}$) then investors choose the locally efficient level of information.*
3. *If $\mu_\theta < \mu_\theta^E (T)$ then investors locally under-acquire information.*

To understand the intuition behind this proposition, recall that, when choosing their precision, investors take into consideration only their private benefit and cost. In particular, they choose a precision taking as given the equilibrium threshold θ^*, ignoring the effect their collective decisions have on the equilibrium probability of a successful investment. Thus, the social benefit of additional information tends to differ from the private benefit of a higher precision since the former also takes into account the effect of precision choices on θ^*.

When the investment threshold θ^* is decreasing in the precision of all investors in the neighborhood of the equilibrium precision choice τ^*, which happens when $\mu_\theta < \mu_\theta^E (T)$, then the marginal private benefit of extra information is lower than the marginal social benefit. This is because the marginal social benefit takes into account the positive effect of a higher private precision on investment. Since at the equilibrium precision the marginal private benefit is equal to the marginal cost of extra information, it follows that the social benefit of more precise information is strictly higher than its marginal cost. Thus, in this case it would be welfare improving if all investors acquired more information, i.e. investors are locally under-acquiring information in equilibrium. The opposite it true if the investment threshold θ^* is increasing in investors' precision choices in the neighborhood of the equilibrium precision choice τ^*, which happens when $\mu_\theta > \mu_\theta^E (T)$. In this case, the marginal private benefit is higher than the marginal social benefit and investors locally over-acquire information.

Finally, as is shown in the proof of Proposition 2, when $\mu_\theta = \mu_\theta^E (T)$ then the private and social marginal benefits of additional information are equal at the equilibrium precision level τ^*, and hence τ^* is an extremum point of the welfare function. However, this is not enough to conclude that agents acquire the locally efficient amount of information. In particular, it can be shown that when $T < \frac{1}{2}$, then τ^* corresponds to a local minimizer of the welfare function, while if $T \geq \frac{1}{2}$, then τ^* corresponds to a local maximizer of the welfare function.

The above intuition can also be used to understand when investors globally over-acquire or under-acquire information. In particular, if the investment threshold θ^* is monotone in τ, then the local results translate directly into global results. In this case, the marginal private benefit of additional information is always either lower (when θ^* is a decreasing function of τ), or always higher (when θ^* is an increasing function of τ) than the social marginal benefit of information. The difficulty of fully characterizing global results is due to the fact that θ^* can be a non-monotone function of private precision choices.[19]

In the online appendix (Proposition 9) we show that the local results translate into global results except for the case when $T < \frac{1}{2}$ and $\mu_\theta \in \left(\widehat{\mu}_\theta\left(T, \underline{\tau}, \tau_\theta\right), T\right)$, where

$$\widehat{\mu}_\theta\left(T, \underline{\tau}, \tau_\theta\right) = \Phi\left(\sqrt{\frac{\underline{\tau}}{\underline{\tau} + \tau_\theta}}\Phi^{-1}\left(T\right)\right) + \frac{1}{\sqrt{\underline{\tau} + \tau_\theta}}\Phi^{-1}\left(T\right).$$

In this case, it is possible for investors to locally under-acquire, but globally over-acquire information. This is because, for these parameters, θ^* is first increasing and then decreasing in the investors' precision choices. Thus, if the equilibrium precision is high, a small increase in investors' precision choices from the equilibrium level is welfare improving, since it leads to a lower investment threshold. At the same time, it is possible that from the planner's perspective it is optimal to acquire no information, since it is both costly and leads to a higher θ^*.Verifying this analytically, however, is difficult because the welfare function may not be quasiconcave.[20] Section 3 of the Online Appendix explores these issues in more detail.

5 Strategic Complementarities in Information Acquisition

We now investigate whether strategic complementarities in the coordination game translate into strategic complementarities in information acquisition. In the context of beauty contest games, Hellwig and Veldkamp (2009) have shown that this is indeed the case. In our model this is not always true.

Definition 3 *Let τ_i be investor i's precision choice, while τ is the precision choice of all the other investors. We say that information choices are strategic complements if for all $\tau_i \geq \underline{\tau}$*

[19] See Szkup (2014) for a complete characterization of conditions under which θ^* is monotone or non-monotone function of model parameters in global games.

[20] To understand why the welfare function may not be quasi-concave note that a higher precision has three separate effects on the welfare function. First, a higher precision allows investors to avoid costly mistakes. Second, a higher precision, through its effect on investment choices, affects the threshold θ^*. Finally, a higher τ is associated with a higher cost. If $T < 1/2$ and $\mu_\theta \in \left(\widehat{\mu}_\theta\left(T, \underline{\tau}, \tau_\theta\right), T\right)$, then θ^* is initially increasing and then decreasing in τ. Thus, a small increase in τ is not only costly, but it also lowers the probability of a successful investment. These two negative effects tend to reduce welfare. However, as τ keeps on increasing, the negative effect of a higher τ on investment decreases sharply. For intermediate values of τ (precision choices near the point where θ^* achieves the global maximum), the negative effect of a higher τ on investment becomes negligible. At this point, it is possible that the reduction in the expected cost of mistakes becomes the dominant effect and, as a result, the welfare function becomes increasing in τ. However, as τ increases further, the reduction in the expected cost of mistakes becomes smaller and smaller. Intuitively, if investors already have precise information then they are able to avoid committing mistakes to a large extent, and there is little value to additional information. As a result, the welfare function becomes again decreasing in τ, driven by the increasing cost of a higher precision. For more details, see Section 3 in the Online Appendix where we investigate numerically the non-quasiconcavity of the welfare function.

and for all $\tau \geq \underline{\tau}$ we have:

$$\frac{\partial^2 B^i(\tau_i, \tau)}{\partial \tau_i \partial \tau} > 0.$$

The above definition states that information choices are strategic complements if the value of additional information to investor i is increasing in the precision choices of the other investors for all pairs of $\{\tau_i, \tau\}$. Recall from Section 3.2 that the value of additional information to investor i is determined by the distance between x_i^* and θ^*, as well as the distance between θ^* and μ_θ. A change in the precision choice of the other investors, τ, affects investor i's incentives to acquire information by affecting these distances, and hence the value of additional information to investor i. As shown in the next proposition, there is no guarantee that strategic complementarities in information choices arise in our model.

Proposition 4 *Define*

$$\overline{\mu}^{SC}(\underline{\tau}, \tau_\theta, T) \equiv T + \frac{1}{\sqrt{\underline{\tau} + \tau_\theta}} \Phi^{-1}(T)$$

1. *Suppose that $T > \frac{1}{2}$.*
   - (a) *If $\mu_\theta \notin \left(T, \overline{\mu}^{SC}(\underline{\tau}, \tau_\theta, T)\right)$, then information choices are strategic complements.*
   - (b) *If $\mu_\theta \in \left(T, \overline{\mu}^{SC}(\underline{\tau}, \tau_\theta, T)\right)$, then there is a lack of strategic complementarities.*
2. *Suppose that $T = \frac{1}{2}$. Then information choices are always strategic complements.*
3. *Suppose that $T < \frac{1}{2}$.*
   - (a) *If $\mu_\theta \notin \left(\overline{\mu}^{SC}(\underline{\tau}, \tau_\theta, T), T\right)$, then information choices are strategic complements.*
   - (b) *If $\mu_\theta \in \left(\overline{\mu}^{SC}(\underline{\tau}, \tau_\theta, T), T\right)$, then there is a lack of strategic complementarities.*

The above proposition indicates that when $T \neq \frac{1}{2}$, for extreme values of the prior mean, information choices are strategic complements, while for intermediate values they are not. To see this, fix T and consider the case when μ_θ is low, so that θ^* is high and the distance between the two is large (the case for high μ_θ is analogous). In this case, an investor cares mainly about Type I mistakes, since he assigns a low ex-ante probability to a successful investment, so he attaches relatively low value to additional information. An increase in τ, the precision choice of other investors, leads to a decrease in θ^*. This is because when μ_θ is low, an increase in τ implies that investors assign a lower weight to the unfavorable information, represented by low μ_θ, and thus invest more often. However, a decrease in θ^* increases the expected probability of a successful investment. As a result, investor i shifts his concern from avoiding mainly a Type I mistake to avoiding both types of mistakes more evenly. This increases his demand for information.

To see why information choices might not be strategic complements consider the case when $T > \frac{1}{2}$ and $\mu_\theta \in \left(T, \overline{\mu}^{SC}(\underline{\tau}, \tau_\theta, T)\right)$ and assume that investor i has a low precision, τ_i, and that the precision of the rest of the investors, τ, is high.[21] When τ_i is low, investor i will care slightly more about a Type I mistake than the rest of the investors, since a high T implies that this mistake is relatively more costly, and his information is not as precise as that of the rest of the investors. When both τ and T are high, an additional increase in τ will increase θ^* (see the proof of Proposition 4), thus decreasing the probability of a successful investment. This, in turn, will make investor i shift his concern even further towards avoiding a Type I mistake, thus becoming less concerned about a Type II mistake. Since the value of additional information is higher when an investor cares about both types of mistakes, this adjustment in investor i's behavior makes him value additional information even less, which decreases his incentives to acquire information. An analogous argument holds when $T < \frac{1}{2}$ and μ_θ takes a value in $\left(\overline{\mu}^{SC}\left(\underline{\tau}, \tau_\theta, T\right), T\right)$.

6 Public Information and Welfare

In recent years, the effect of public information on welfare has attracted a lot of attention (see Morris and Shin, 2002, and the literature that followed). This motivates us to study, in the context of our model, the effects of the precision of public information on welfare. Going back to our example in the introduction, consider a government that, in order to encourage foreign direct investment, decides to provide investors with detailed information regarding the current state of the economy. This initial report provided by the government shapes the investors' prior beliefs about the state of the economy. In addition to this information, investors have the possibility to gather more information privately. It is of interest to understand what is the effect of the public information initially released by the government on investors' incentives to acquire private information, on the probability of successful investment, and on the ex-ante social welfare.

We interpret prior beliefs as public information and study how changes in the precision of this type of public information affect equilibrium strategies and outcomes.[22] Given our interpretation, we first investigate how an increase in the precision of public information effects investors' incentives to acquire private information. We then turn our attention to the effects on coordination among investors, and finally on the welfare implications of changes in the informativeness of the prior.

In what follows we assume that $T = \frac{1}{2}$. This assumption implies that Type I and Type II mistakes are equally costly, and that investors care equally about coordinating with other investors on investing and on not investing. While not without loss of generality, this assumption simplifies the analysis substantially, allowing us to completely characterize the impact of an increase in the precision of public information on private information acquisition and on the probability of successful investment. The case when $T \neq \frac{1}{2}$ is discussed in detail

[21] By "lack of strategic complementarities" we refer to the situation where there exist pairs $\{\tau_i, \tau\}$ such that $\frac{\partial^2}{\partial \tau_i \partial \tau} B\left(\tau_i, \tau\right) < 0$, i.e., where an increase in the other investors' precision choices leads to lower incentives for investor i to further increase his own precision. This is different from strategic substitutabilities, which would correspond to the situation where for all τ_i and all τ we have $\frac{\partial^2}{\partial \tau_i \partial \tau} B\left(\tau_i, \tau\right) < 0$. It can be verified that in our model information choices cannot be strategic substitutes (see the proof of Theorem 1).

[22] This interpretation of public information is similar to Metz (2002) and Morris and Shin (2004).

in the Online Appendix.

One should note that our modelling of public information is different from the typical approach in the literature. Public information is commonly modelled as a separate public signal that is observed simultaneously with the private signal, and an increase in public information is modelled as an increase in the precision of this signal (see Morris and Shin, 2002, Colombo et al., 2014, among others). In those setups, agents choose the amount of private information before the public signal is realized. In our approach, investors condition their private information choices on the realization of the public signal, captured by μ_θ.[23] In order to facilitate the comparison of our model with the existing literature on beauty contest models, in section 7 we compare our results to a version of the beauty contest model with a proper prior, but without an explicit public signal.

6.1 Trade-off between Public and Private Information

To analyze the trade-off between public and private information, notice that more precise public information affects the value of acquiring private information through three different channels. First, more precise public information changes the joint density of $\{\theta, x_i\}$. Since this effect is independent of investors' behavior, we call this the passive information effect. Second, a change in τ_θ, by changing the informativeness of the prior, affects an individual investor's investment strategy, for any given precision choice. Since this effect involves a change in the investor's behavior we call it the active information effect. Finally, a change in τ_θ affects the equilibrium threshold θ^* through a change in the other investors' investment strategies. We call this the coordination effect.

In comparison, more precise private signals affect the value of acquiring more private information only through the passive and the active information effects. Not only is the coordination effect not present, but the passive information effect is also different. In particular, more precise private information better aligns the signals with the realization of the fundamental. In contrast, more precise public information increases the likelihood of fundamentals taking values closer to their mean. This subtle difference in the passive information effect can lead to complementarities between public and private information.

Proposition 5 *Let $T = \frac{1}{2}$. There exist cutoffs $\widehat{\mu}^-$ and $\widehat{\mu}^+$ such that $\widehat{\mu}^- < \frac{1}{2} < \widehat{\mu}^+$, and*

1. *if $\mu_\theta \notin \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$, then private and public information are substitutes,*

2. *if $\mu_\theta \in \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$, then private and public information are complements.*

To understand the intuition behind this proposition, we consider first the passive information effect (i.e., we keep θ^* and x^* constant). An increase in τ_θ increases the likelihood of the fundamental taking a value near μ_θ. If θ^* lies near μ_θ, this leads to a higher probability that the realization of θ will be close to the critical threshold θ^*. For a given precision of private

[23] This has the disadvantage of introducing sensitivity to the prior mean when studying the effects of changes in the precision of the prior. Unfortunately, introducing a separate public signal makes the analysis intractable.

information, such a change in the distribution of θ increases the ex-ante probability that an investor's signal will lie on the "wrong" side of θ^*, leading him to take the incorrect action.[24] In this case, an increase in the precision of public information increases the expected cost of mistakes, which increases the value of additional information. Therefore, when θ^* lies near μ_θ the passive information effect encourages investors to acquire more private information. Note that this effect is strongest when $\theta^* = \mu_\theta$, which happens exactly when $\mu_\theta = \frac{1}{2}$. The opposite is true when θ^* is far from μ_θ, which happens when μ_θ is far from $\frac{1}{2}$. In this case, an increase in τ_θ shifts the probability mass away from the values of θ at which investors are particularly susceptible to taking the incorrect action. In this case, the passive information effect discourages private information acquisition. This effect is particularly strong when θ^* is far from μ_θ, which happens when μ_θ is far from $\frac{1}{2}$.

The above argument explains why the passive information effect encourages information acquisition when μ_θ is close to $\frac{1}{2}$ and discourages it otherwise. What about the other effects? When $T = \frac{1}{2}$ the coordination effect will always discourage private information acquisition by increasing the gap between μ_θ and θ^*. Intuitively, when μ_θ is high (so that $\mu_\theta > \theta^*$), an increase in τ_θ reassures investors that the fundamentals are strong, which encourages investment and leads to a decrease in θ^*. Since now θ^* lies further away from μ_θ, the probability that the actual realization of the fundamental will be close to the critical threshold θ^* is lower. For a given precision of private information, such a change in θ^* decreases the ex-ante probability that an investor will take the incorrect action, reducing his incentives to acquire more private information. An analogous intuition applies to the case when μ_θ is low. By a similar logic, the active information effect pushes the investors' threshold, x^*, away from μ_θ (and away from θ^*), thus decreasing the probability that an investor will take the incorrect action that he wants to avoid the most.[25]

Note that the active information and the coordination effects become stronger as μ_θ moves away from $\frac{1}{2}$. In particular, the active information effect is strong when a small change in τ_θ leads to a large change in x^*. In turn, the change in x^* is large when x^* lies far from μ_θ since a small change in the precision of the prior has a large effect on investors' posterior beliefs, evaluated at the critical signal. Since investors choose the threshold signal to be close to μ_θ when μ_θ is close to $\frac{1}{2}$, it follows that the active information effect is strong when μ_θ is far from $\frac{1}{2}$ and weak when μ_θ is close to $\frac{1}{2}$. Finally, since the change in θ^* is driven by a change in x^*, the same intuition applies to the coordination effect.

6.1.1 The case $T \neq \frac{1}{2}$

One may wonder whether the above intuition extends to the case when $T \neq \frac{1}{2}$. In particular, are there values of μ_θ such that private and public information are complements when $T \neq \frac{1}{2}$? In the Online Appendix (Section 4) we show that there exist $\underline{T}$ and $\overline{T}$, $0 < \underline{T} < 1/2 < \overline{T} < 1$,

[24] The probability of taking the incorrect action is highest when θ lies close to θ^*, since an investor faces the highest likelihood of receiving a signal $x_i < \theta^*$, while in reality $\theta > \theta^*$, and vice versa.

[25] When $T = \frac{1}{2}$, the value of μ_θ determines which mistake investors care more about. When $\mu_\theta > \frac{1}{2}$ investors want to make sure that they invest when investment is successful and they choose $x^* < \theta^* < \mu_\theta$. The opposite is true when $\mu_\theta < \frac{1}{2}$, in which case investors prefer to coordinate on not investing and set $x^* < \theta^* < \mu_\theta$.

such that, for all $T \in \left(\underline{T}, \overline{T}\right)$, there are values of μ_θ for which private and public information are complements. Moreover, numerical simulations suggest that this result extends to all $T \in (0,1)$ (Figure 1).[26]

Understanding which effects drive the results when $T \neq \frac{1}{2}$ is more difficult, since it requires comparing the absolute magnitude of each of the three effects. However, our analytical results reported in the Online Appendix, suggest that, unless T takes extreme values, the passive information effect still plays an important role in driving the complementarity between public and private information. This is because the effect of the passive information effect on the incentives to acquire private information is the same regardless of the value of $T \neq \frac{1}{2}$. In particular, it is still true that whenever θ^* is close to μ_θ the passive information effect encourages information acquisition, while the opposite is true when θ^* lies far from μ_θ.

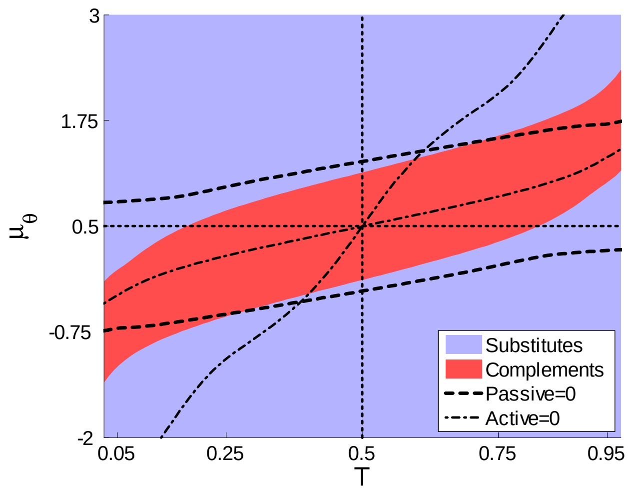

Figure 1: Relation between private and public information

Figure 1 further supports this claim. In Figure 1, the area between the two dashed lines corresponds to the region where the passive information effect is positive, and the area between the two dash-dotted lines corresponds to the area where the active information effect is positive.[27] We can see that, unless T takes extreme values, the region where private and public information are complements (the dark shaded region in Figure 1) lies in the interior of the region where the passive information effect is positive. This suggests that, unless T is very small or very large, the passive information effect is the key force driving the complementarity between public and private information.

When T takes on extreme values, the complementarity between private and public information can be driven by the active information effect. To understand why this is the case,

[26] See Section 4.3 in the Online Appendix for numerical robustness checks.

[27] The two dash-dotted lines intersect at $T = \frac{1}{2}$ since, as explained above, in this case the active information effect is always non-positive (and strictly negative when $\mu_\theta \neq \frac{1}{2}$).

recall that investors care both about the cost of mistakes (captured by T) and about the probability with which they commit these mistakes (captured by μ_θ). When T is very high, investors are mainly worried about committing a Type I mistake, even if committing such a mistake is not very likely from an ex-ante perspective (i.e., for high values of μ_θ). However, when μ_θ is high, an increase in τ_θ assures investors that investment will be successful, since it increases the probability that the realization of θ will be high. Hence, from the investors' ex-ante perspective, an increase in τ_θ increases the likelihood that investors commit a Type II mistake. This, in turn, makes investors shift their concern from avoiding mainly a Type I mistake to avoiding both types of mistakes. Since investors' incentives to acquire information are high when investors care about both types of mistakes, this increases their demand for information.

6.2 Effects of Increasing Public Information on Coordination

In the previous subsection we analyzed the relationship between an increase in τ_θ and investors' precision choices. In this subsection we study the effect of an increase in the precision of public information on the probability of a successful investment. To provide a complete analytical characterization of this result, we continue to assume that $T = \frac{1}{2}$.[28]

Proposition 6 *Let $T = \frac{1}{2}$ and suppose that the precision of public information increases.*

1. *If $\mu_\theta < \frac{1}{2}$, then the ex-ante probability of a successful investment decreases.*
2. *If $\mu_\theta = \frac{1}{2}$, then the ex-ante probability of a successful investment is unchanged.*
3. *If $\mu_\theta > \frac{1}{2}$, then the ex-ante probability of a successful investment increases.*

An increase in τ_θ affects the probability of a successful investment through three channels. First, it affects the ex-ante distribution of θ. Second, it affects directly the value of the threshold θ^* through an adjustment in investors' investment strategies (holding their precision choices constant). Third, it leads indirectly to a change in θ^* by affecting investors' precision choices. We find that the second effect is the dominant force that determines whether the probability of a successful investment increases or decreases. Therefore, to understand the intuition behind this result it is enough to understand the direction of a change in θ^* due to a change in τ_θ, holding precision choices constant.

To understand how a change in τ_θ affects θ^* recall that when making their decision, investors care about the expected value of θ, given by $\frac{\tau_\theta \mu_\theta + \tau x_i}{\tau_\theta + \tau}$. An increase in the precision of the prior leads an investor to assign a higher weight to his prior belief about θ and a lower weight to his private signal x_i. When μ_θ is high (i.e. $\mu_\theta > \frac{1}{2}$), investors expect that investment is likely to be successful and hence they set $x^* < \mu_\theta$. It follows that an increase in τ_θ would increase posterior expectations of investors who received signals around the threshold signal. As a consequence, these investors would now choose to invest, thus increasing the aggregate investment and lowering θ^*. The opposite is true for a low μ_θ ($\mu_\theta < \frac{1}{2}$). Finally, for $\mu_\theta = \frac{1}{2}$

[28] We explore the case when $T \neq \frac{1}{2}$ in Section 5.1 of the Online Appendix.

we have $x^* = \mu_\theta$, hence a change in τ_θ has no effect on θ^* and the direct effect is equal to zero.

6.3 Welfare Consequences of a Higher τ_θ

In this section, we turn our attention to welfare considerations of more precise public information. Since all investors are ex-ante identical and play a symmetric equilibrium, it is enough to analyze the ex-ante utility of a single investor in order to determine welfare consequences of an increase in transparency of public information. Recall that the ex-ante utility of an investor who plays a symmetric equilibrium with precision choice τ^* is given by:

$$U^i(\tau^*;\tau^*) = -\int_{-\infty}^{\theta^*(\tau^*)}\int_{x^*(\tau^*)}^{\infty} T dF_{\tau^*}(x|\theta)\, dG_{\tau_\theta}(\theta) - \int_{\theta^*(\tau^*)}^{\infty}\int_{-\infty}^{x^*(\tau^*)} (1-T)\, dF_{\tau^*}(x|\theta)\, dG_{\tau_\theta}(\theta) + \int_{\theta^*(\tau^*)}^{\infty} (1-T)\, dG_{\tau_\theta}(\theta) - C(\tau^*)$$

The total impact of a change in the precision of public information, after simplification, can be expressed as:

$$\begin{aligned}\frac{dU^i(\tau^*;\tau^*)}{d\tau_\theta} = &-\frac{d\theta^*}{d\tau_\theta}\left(1-F(x^*|\theta^*)\right) g_{\tau_\theta}(\theta^*) \\ &+\int_{\theta^*(\tau^*)}^{+\infty} \frac{\partial}{\partial\tau_\theta}(1-T)\left[1-F_{\tau^*}(x^*|\theta)\right] g_{\tau_\theta}(\theta)\, dx d\theta \\ &-\int_{-\infty}^{\theta^*(\tau^*)} \frac{\partial}{\partial\tau_\theta} T\left[1-F_{\tau^*}(x^*|\theta)\right] g_{\tau_\theta}(\theta)\, dx d\theta \end{aligned} \tag{6}$$

The above equation states that an increase in the precision of the prior affects welfare through two channels. First, it changes the threshold for fundamentals that determines if investment is successful by affecting the equilibrium strategies of investors in both stages of the game (x^* and τ^*). This is captured by the first line in equation (6). Second, a change in τ_θ affects welfare by changing the probability with which investors make correct decisions. This is captured by the last two terms of equation (6).

Despite its simplicity, it is difficult to determine the sign of equation (6). One would expect, however, that the effect of a change in the probability of investment is dominant since, in coordination games, changes in public information have a disproportional effect on equilibrium play (see e.g. Morris and Shin, 2002, 2003). According to Proposition 6, the probability of a successful investment is increasing in the precision of public information when μ_θ is large, and decreasing when μ_θ is low. Thus, we should expect welfare to be increasing when μ_θ is high and decreasing when μ_θ is low.[29] In Section 5.2 in the Online Appendix we provide results of numerical simulations that support this intuition.

[29] Proposition 6 states also that around $\mu_\theta = \frac{1}{2}$ the effect of an increase in τ_θ on investment is close to zero, hence in that region welfare is determined mainly by the change on the probability of mistakes.

7 Discussion and Comparison to Beauty Contest Models

7.1 Discussion of results

We have explored the consequences of private information acquisition in global games and the properties of the unique equilibrium in our game. We found that the parameters T and μ_θ are key in determining the results. While the exact mechanism through which T and μ_θ affect our conclusions depends on the specific question under study, the main reason why these two parameters affect our results is the same. Intuitively, T and μ_θ determine whether investors worry more about committing a Type I or a Type II mistake. For example, when T is high, the cost of committing a Type I mistake (investing when investment is unsuccessful) is higher than the cost of committing a Type II mistake (not investing when investment is successful). On the other hand, a low μ_θ indicates that investment is unlikely to be successful, so investors are less likely to commit a Type II than Type I mistake. Therefore, a high T and a low μ_θ imply that x^* and θ^* are high and, in particular, higher than μ_θ. The opposite is true when T is low and μ_θ is high. Thus, the values of μ_θ and T determine the relative positions of x^*, θ^* and μ_θ.

The relative positions of x^*, θ^* and μ_θ are key for our conclusions since they determine the sign of a change in θ^* and x^* with respect to changes in private and public precision, and whether the distances between x^* and θ^* and θ^* and μ_θ increase or decrease in response to changes in τ or τ_θ. For example, whether investors over-acquire or under-acquire information depends on the effect that an increase in the precision of private information has on the investment threshold θ^*. On the other hand, whether an increase in the precision of public information crowds out private information acquisition depends on the effect that an increase in τ_θ has on the marginal value of private information, which is determined by the distance between x^* and θ^* and θ^* and μ_θ. Each individual result of the paper and the associated conditions on T and μ_θ can be understood in this way.

Finally, the point $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ in the $\{T, \mu_\theta\}$-space plays a special role. When $T = \frac{1}{2}$ investors care as much about a Type I mistake as they care about a Type II mistake, while when $\mu_\theta = \frac{1}{2}$ they assign an equal probability to the investment being both successful and unsuccessful. Thus, investors choose a threshold for their signal such that they expect to invest and not invest with equal probability, i.e., $x^* = \mu_\theta$, which in turn implies $\theta^* = \mu_\theta$. As a consequence, a marginal change in τ or τ_θ has no effect on x^* or θ^*.

7.2 Comparison to Beauty Contest Models

Our model is related to the literature on the role of information in beauty contest models. In this type of games, investors' payoffs depend on how closely their action is to the average action taken by others and to the unknown state. In the context of incomplete information games with private and public signals, these models were first analyzed by Morris and Shin (2002). Angeletos and Pavan (2007) provide a careful and thorough analysis of this framework with an exogenous information structure. More recently, Hellwig and Veldkamp (2009), Myatt and Wallace (2012), and Colombo et al. (2014) analyze the effects of adding costly information acquisition into this framework.

Although global games and beauty contest models share a lot of common features, our findings suggest that there are important differences between these two setups when introducing endogenous information. First, we find that whether an improvement in public information is welfare enhancing or not depends crucially on the ex ante beliefs about the state, while in beauty contest models it depends on the relative informativeness of private and public information (Morris and Shin, 2002, Colombo et al., 2014). Second, as shown by Tong (2007) and Colombo et al. (2014), in beauty contest models, an increase in the precision of public information always decreases investors' incentives to acquire private information and leads to a lower precision of private information in equilibrium. In contrast, in our model public and private information can be complements (see section 6.1). Finally, Hellwig and Veldkamp (2009) and Colombo et al. (2014) show that in a beauty contest model complementarities in actions always translate into complementarities in information acquisition. While this is true for a wide range of parameters in our model, we show that there are cases in which this result does not hold for global games (see section 5).[30]

The difference between our findings in the context of global games and the findings in beauty contest models is due to the different role that information plays in these two classes of models. In beauty contest models that feature strategic complementarities in actions, an individual values information because it allows him to better align his action both with the underlying fundamentals and with the actions of the other investors.[31] In global games, information is valuable to an investor because it allows him to avoid costly mistakes. In that sense, the investor does not care per se about how closely his action covaries with fundamentals and with the actions of others, but rather about whether or not he takes the correct action. That is, he cares about whether he observes a signal x_i greater than his threshold x_i^* when $\theta > \theta^*$, and whether he observes a signal x_i smaller than his threshold x_i^* when $\theta < \theta^*$. Thus, he cares about the tail probabilities of the conditional distribution of $x_i|\theta$, since these tail probabilities determine the investor's expected cost of mistakes.

To see why this difference between the two models leads to very different conclusions, consider an increase in the precision of the signal to all but one investor. In a beauty contest model, when other investors choose to acquire more precise private information their private signals become more anchored around the fundamentals. This increases the value of additional information to investor i, since now this extra information allows him to better

[30]Note that the way in which we introduce public information is slightly different from the way in which public information is modeled in the beauty contest models of Hellwig and Veldkamp (2009) and Colombo et al. (2014). In these models, public information is composed of a common prior and an additional public signal that is drawn once the state has been realized. In our case, public information is composed only of the common prior. In Colombo et al. (2014) changes in the precision of public information are modeled as changes in the precision of the aggregate public signal, which is a sum of the precision of the prior and the precision of the public signal. However, the qualitative results of Colombo et al. (2014) would be unchanged if public information was modeled only through the prior, so the comparisons between our model and their paper hold.

[31]Since our model features only strategic complementarities, we restrict our comparison to beauty contest models with strategic complementarities. Typically, beauty contest models can feature either strategic complementarities or substitutabilities in actions, depending on parameters.

align his action with both the fundamentals and the actions of others. In contrast, in a global game an investor cares about the change in the precision of the others only to the extent that this change affects the threshold for fundamentals θ^* (if a change in the precision of other investors had no effect on θ^* then his behavior would be unchanged!). In particular, what matters is how the adjustment in θ^*, implied by a change in the precision of others, increases or decreases the relevant tail probabilities, and hence the expected cost of mistakes. It turns out that the direction of this adjustment is governed by two parameters: the mean of the prior belief, μ_θ, and the cost of investment, T. Depending on these two objects, the change in θ^* implied by a change in the precision of other investors' signals can lead to an increase in the relevant tail probabilities, increasing the expected cost of mistakes. Hence, in global games, for some parameters strategic complementarities in actions fail to translate into strategic complementarities in information acquisition.

To summarize, we can conclude that the differences between our findings in the context of global games and the existing results for beauty contest models are due to the fact that the value of additional information is very different across these two models. In global games it is determined by the tail probabilities of the conditional joint distribution of $\{\theta, x_i\}$, while in beauty contest models it is determined by the covariances between investors' signals and the fundamentals.

8 Related Literature

Our work is related to three strands of literature: global games, information acquisition, and transparency. Global games were introduced by Carlsson and van Damme (1993) in their seminal work as an equilibrium refinement concept and further extended by Frankel et al. (2003). This technique was first applied by Morris and Shin (1998) to the context of currency crises and since then it has been extensively used to model economic phenomena featuring coordination problems (see Dasgupta, 2007, Edmond, 2013, Goldstein and Pauzner, 2005, or Morris and Shin, 2004, among others).

While the original global games models were static,[32] several authors extended these models to multi-stage games (Angeletos et al., 2007, and Dasgupta, 2007, among others). We contribute to this literature by considering a model in which investors have the choice to acquire more precise information before playing the standard one-shot global game. Unlike these papers, in our model investors make choices in the first period that influence the structure of the game they play in the second period, whereas in the above papers investors repeatedly play a static global game. In this respect our work is most closely related to Angeletos and Werning (2005) and Chassang (2008). However, none of these studies considers costly information acquisition and its impact on the coordination game.

Costly information acquisition has been analyzed by Nikitin and Smith (2008) and Zwart (2008) in the context of the Diamond and Dybvig (1983) model of bank runs. However, in these two studies information acquisition is modeled as a binary decision to acquire a private signal with a given precision, or not to acquire a signal at all, which is in contrast to our

[32] That is, they featured only one-shot coordination games.

setup where all investors observe private signals and have to choose their individual precision. Moreover, these papers do not analyze resulting inefficiencies in information choices, characterize strategic complementarities or discuss welfare implications of more precise public information. Yang (2014) studies flexible information acquisition in coordination games where agents can choose how much and what kind of information to acquire. This flexibility leads to rational inattentive choices and encourages efficient coordination, but it also restores multiplicity of equilibria. This is contrary to our findings where agents choose how much information of a given type to acquire, which gives rise to a unique inefficient equilibrium. Our analysis utilizes results established by Szkup (2014) who characterized comparative statics results with respect to public and private precision in global games models. Finally, Szkup and Trevino (2014) consider a discrete version of our model and test its predictions experimentally.

The notion of transparency in the context of speculative attack models has been addressed in the global games literature by Heinemann and Illing (2002) and Bannier and Heinemann (2005). There are several key differences between these studies and our work. First of all, their notion of transparency is different from ours. They interpret an increase in transparency as an increase in the precision of private signals. In contrast, we follow closely the literature on transparency in beauty contest models and interpret higher transparency as a higher precision of the public signal. Secondly, in the above papers players only make decisions in the one-stage coordination game.[33] In our model investors have the option to choose the precision of the private signal they receive before playing the coordination game.

9 Conclusions

In this paper we analyze the role of endogenous information in a global games model. We show that in these games investors are prone to making two types of mistakes: investing when investment is not profitable, or not investing when investment is profitable. We study the effect that precision choices have on the incidence of these two types of mistakes in the coordination game and analyze how the value of more precise information is affected by prior beliefs, the behavior of other players, and the cost of investment.

We characterize conditions under which our game has a unique equilibrium and analyze several aspects of it. First, we show that in general the choice of precision made by investors in the unique equilibrium is inefficient. Depending on the parameters of the model, investors acquire too much or too little information. We also show that even though there are strategic complementarities in actions in the second stage, contrary to the findings of Hellwig and Veldkamp (2009) for beauty contest models, the strategic complementarities in actions do not always translate into strategic complementarities in information acquisition.

We also consider the effects of an increase in the precision of the prior on the incentives to acquire private information, on the probability of a successful investment, and on welfare. We characterize the cases where more precise public information crowds out the acquisition

[33] Bannier and Heinemann (2005) present a two-stage model in which a governmental agency chooses first the precision of private signals, and then agents play a global game in the second stage with an exogenously given precision.

of private information and the cases where private and public information might be complements. We find that an increase in transparency might increase or decrease the probability of a successful investment and welfare, depending on the initial conditions in the economy.

Our model abstracts from considerations of a strategic government who can choose the precision of public information based on its own signal. A strategic government in a global games setup has been analyzed in a standard speculative attack game by Angeletos et al. (2006), Angeletos and Pavan (2011), and Goldstein et al. (2011). Exploring the issue of strategic release of information in the model with endogenous information acquisition is an important direction for further research. A shortcoming of the global games results is that they restrict the precision of public information, relative to private information, to ensure uniqueness of equilibria. It would be interesting to investigate whether endogenous information acquisition can mitigate this critique. This issue is left for future research.

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A Appendix

In this appendix we provide all the proofs and derivations that have been omitted in the main body of the paper. The appendix is divided into five sections. In Section $A.1$ we state without proofs preliminary results that are used to establish results reported in the paper and in the proofs of the following subsections. The proofs of these results can be found in the online appendix. In Section $A.2$ we provide proofs of results stated in Section 3.1 of the paper (proof of Proposition 1). In Section $A.3$ we provide results stated in Section 3.2 of the paper (derivations of equation (4) and proofs of Lemma 1 and Theorem 1). In Sections $A.4$ and $A.5$ we provide the proofs of results reported in Sections 4 and 5. Finally, Section $A.6$ contains the proofs of results stated in Section 6.

A.1 Preliminary Results

A.1.1 Results used in Section 3

Lemma A.1 *Consider the benefit function, $B^i(\tau_i;\Gamma)$.*

1. *$B^i(\tau_i;\Gamma)$ is strictly increasing in τ_i;*
2. *$\frac{\partial B^i}{\partial \tau_i}$ is bounded from above;*
3. *$\lim_{\tau_i\to\infty} \frac{\partial B^i}{\partial \tau_i} = 0$;*
4. *For $\tau_i > \underline{\tau}$, $\frac{\partial^2 B^i}{\partial \tau_i^2} < 0$.*

Proof. See Lemma 2 in the Online Appendix. ■

Lemma $A.1$ establishes that the benefit function for investor i is increasing in his precision choice, bounded, and concave. We will use this result in the proof of Theorem 1.

A.1.2 Results used in Section 4

Lemma A.2 *Denote by $\tau^*(\mu_\theta)$ the equilibrium precision choice as a function of μ_θ. Then for each T, there exists a unique μ_θ, call it $\mu_\theta^E(T)$, that solves*

$$\mu_\theta = \widehat{\mu}^\tau(T, \tau^*(\mu_\theta), \tau_\theta)$$

where

$$\widehat{\mu}^\tau(T, \tau^*(\mu_\theta), \tau_\theta) = \Phi\left(\sqrt{\frac{\tau^*(\mu_\theta)}{\tau^*(\mu_\theta)+\tau_\theta}}\Phi^{-1}(T)\right) + \frac{1}{\sqrt{\tau^*(\mu_\theta)+\tau_\theta}}\Phi^{-1}(T)$$

Moreover, for each $\mu_\theta > \mu_\theta^E(T)$ we have $\mu_\theta > \widehat{\mu}^\tau(T, \tau^*(\mu_\theta), \tau_\theta)$ and for all $\mu_\theta < \mu_\theta^E(T)$ we have $\mu_\theta < \widehat{\mu}^\tau(T, \tau^*(\mu_\theta), \tau_\theta)$.

Proof. See Lemma 4 in the Online Appendix. ■

We use Lemma $A.2$ to show that the information choice in the symmetric equilibrium is generically inefficient.

Lemma A.3 *Consider $\frac{\partial\theta^*}{\partial\tau}$.*

1. *If $\mu_\theta < \widehat{\mu}^\tau(T, \tau, \tau_\theta)$ then $\frac{\partial\theta^*}{\partial\tau} < 0$*
2. *If $\mu_\theta = \widehat{\mu}^\tau(T, \tau, \tau_\theta)$ then $\frac{\partial\theta^*}{\partial\tau} = 0$*
3. *If $\mu_\theta > \widehat{\mu}^\tau(T, \tau, \tau_\theta)$ then $\frac{\partial\theta^*}{\partial\tau} > 0$*

where

$$\widehat{\mu}^\tau(T, \tau, \tau_\theta) = \Phi\left(\sqrt{\frac{\tau}{\tau+\tau_\theta}}\Phi^{-1}(T)\right) + \frac{1}{\sqrt{\tau+\tau_\theta}}\Phi^{-1}(T)$$

Proof. See Lemma 5 in the Online Appendix. ■

Lemma $A.3$ is used to determine whether investors over-acquire or under-acquire information. For details and intuition behind this result we refer an interested reader to Szkup (2014).

A.1.3 Results used in Section 5

Lemma $A.4$ establishes what happens to the equilibrium threshold θ^* as τ tends to infinity.

Lemma A.4 *As $\tau \to \infty$, the threshold $\theta^* \to T$.*

Proof. See Lemma 10 in the Online Appendix. ■

Lemma $A.4$ characterizes the "global" behavior of threshold θ^* as a function of τ. This result is key for our analysis and has been established by Szkup (2014).

Lemma A.5 *Let $\underline{\tau}$ be the precision of private information that investors are initially endowed with.*

1. *Suppose that $T > \frac{1}{2}$.*
   (a) *If $\mu_\theta \leq T$ then θ^* is decreasing, for all $\tau > \underline{\tau}$,*
   (b) *If $\mu_\theta \in \left(T, \widehat{\mu}^\tau \left(T, \underline{\tau}, \tau_\theta\right)\right)$ then θ^* is initially decreasing, and then increasing in τ,*
   (c) *If $\mu_\theta \geq \widehat{\mu}^\tau \left(T, \underline{\tau}, \tau_\theta\right)$ then θ^* is increasing, for all $\tau > \underline{\tau}$.*
2. *Suppose that $T = \frac{1}{2}$.*
   (a) *If $\mu_\theta < \frac{1}{2}$ then θ^* is decreasing, for all $\tau > \underline{\tau}$,*
   (b) *If $\mu_\theta = \frac{1}{2}$ then θ^* is constant in τ,*
   (c) *If $\mu_\theta > \frac{1}{2}$ then θ^* is increasing, for all $\tau > \underline{\tau}$.*
3. *Suppose that $T < \frac{1}{2}$.*
   (a) *If $\mu_\theta < \widehat{\mu}^\tau \left(T, \underline{\tau}, \tau_\theta\right)$ then θ^* is decreasing, for all $\tau > \underline{\tau}$,*
   (b) *If $\mu_\theta \in \left(\widehat{\mu}^\tau \left(T, \underline{\tau}, \tau_\theta\right), T\right)$ then θ^* is initially increasing and then decreasing in τ,*
   (c) *If $\mu_\theta \geq T$ then θ^* is increasing for all $\tau > \underline{\tau}$.*

   where
   $$\widehat{\mu}^\tau \left(T, \tau, \tau_\theta\right) = \Phi\left(\sqrt{\frac{\tau}{\tau + \tau_\theta}} \Phi^{-1}(T)\right) + \frac{1}{\sqrt{\tau + \tau_\theta}} \Phi^{-1}(T)$$
   and
   $$\begin{aligned} &\text{if } T > \frac{1}{2} \quad \text{then for all } \ \tau \geq \underline{\tau},\ \widehat{\mu}^\tau \left(T, \tau, \tau_\theta\right) > T \\ &\text{if } T < \frac{1}{2} \quad \text{then for all } \tau \geq \underline{\tau},\ \ \widehat{\mu}^\tau \left(T, \tau, \tau_\theta\right) < T \end{aligned}$$

Proof. See Lemma 6 in the Online Appendix. ■

A.1.4 Results used in Section 6

Lemma A.6 *Let $T = \frac{1}{2}$ and consider θ^*.*

1. *If $\mu_\theta < \frac{1}{2}$, then $\theta^* > \frac{1}{2}$, $\frac{\partial \theta^*}{\partial \tau_\theta} > 0$, and $\frac{\partial \theta^*}{\partial \tau} < 0$;*
2. *If $\mu_\theta = \frac{1}{2}$, then $\theta^* = \frac{1}{2}$, $\frac{\partial \theta^*}{\partial \tau_\theta} = 0$, and $\frac{\partial \theta^*}{\partial \tau} = 0$;*
3. *If $\mu_\theta > \frac{1}{2}$, then $\theta^* < \frac{1}{2}$, $\frac{\partial \theta^*}{\partial \tau_\theta} < 0$, and $\frac{\partial \theta^*}{\partial \tau} > 0$.*

Proof. See Lemma 12 in the Online Appendix. ■

Lemma A.7 *Let $T = \frac{1}{2}$ and consider $\frac{\partial \tau^*}{\partial \mu_\theta}$.*

1. *If $\mu_\theta < \frac{1}{2}$ then $\frac{\partial \tau^*}{\partial \mu_\theta} > 0$;*
2. *If $\mu_\theta = \frac{1}{2}$ then $\frac{\partial \tau^*}{\partial \mu_\theta} = 0$;*
3. *If $\mu_\theta > \frac{1}{2}$ then $\frac{\partial \tau^*}{\partial \mu_\theta} < 0$;*

Proof. See Lemma 13 in the Online Appendix. ■

A.2 Solving the model: $t = 2$

Proposition 1 *For any given Γ, suppose that* $\inf(supp(\Gamma)) > \frac{1}{2\pi}\tau_\theta^2$*. Then the coordination game has a unique equilibrium in which all investors use threshold strategies $x_i^*(\tau_i, \Gamma)$ and investment is successful if and only if $\theta \geq \theta^*$.*

Proof. We will show that for any Γ such that $\inf(supp(\Gamma)) > \frac{1}{2\pi}\tau_\theta^2$ there exists a unique equilibrium in monotone strategies. To show that there are no other type of equilibria one can use the procedure of iterative deletion of dominated strategies (see e.g. Morris and Shin, 2004). Since this step is standard in the literature, we do not repeat it here.

Suppose that in the second stage of the game the distribution of precision choices among investors is given by some distribution function $\Gamma(\tau)$ with bounded support.[34] Assume that all investors follow monotone strategies and that those investors who chose the same precision level τ set the same threshold $x^*(\tau)$, above which they invest. Moreover, let $\theta^*(\Gamma)$ be the threshold level for fundamentals, such that if $\theta > \theta^*(\Gamma)$ then investment is successful.

An equilibrium in monotone strategies has to satisfy the following Payoff Indifference condition

$$\Pr\left(\theta \geq \theta^*(\Gamma) \,|\, x^*(\tau_i)\right) = T \;, \forall \tau_i \in supp(\Gamma) \tag{7}$$

as well as the Critical Mass condition

$$\Pr\left(x_i \geq x^*(\tau_i) \,|\, \theta^*(\Gamma)\right) = 1 - \theta^*. \tag{8}$$

Equation (7) implies that in the case of a Normal distribution, for all $i \in [0,1]$ we have

$$x^*(\tau_i) = \frac{\tau_i + \tau_\theta}{\tau_i}\theta^* - \frac{\tau_\theta}{\tau_i}\mu_\theta + \frac{(\tau_i + \tau_\theta)^{1/2}}{\tau_i}\Phi^{-1}(T)$$

Substituting $x^*(\tau_i)$ into Equation (8) and re-arranging, we get

$$\int \Phi\left(\frac{\tau_\theta}{\tau_i^{1/2}}\theta^* - \frac{\tau_\theta}{\tau_i^{1/2}}\mu_\theta + \frac{(\tau_i + \tau_\theta)^{1/2}}{\tau_i^{1/2}}\Phi^{-1}(T)\right) d\Gamma = \theta^*$$

[34] The bounded support assumption follows from Assumptions $(A1)$, $(A2)$, and Lemma $A.1$.

We need to show that there exists a unique θ^* that solves the above equation. It is sufficient to show that the slope of the LHS is always strictly less than 1. Note that

$$\begin{aligned}
&\frac{\partial}{\partial\theta^*}\int\Phi\left(\frac{\tau_\theta}{\tau_i^{1/2}}\theta^*-\frac{\tau_\theta}{\tau_i^{1/2}}\mu_\theta+\frac{(\tau_i+\tau_\theta)^{1/2}}{\tau_i^{1/2}}\Phi^{-1}(T)\right)d\Gamma\\
&=\int\phi\left(\frac{\tau_\theta}{\tau_i^{1/2}}\theta^*-\frac{\tau_\theta}{\tau_i^{1/2}}\mu_\theta+\frac{(\tau_i+\tau_\theta)^{1/2}}{\tau_i^{1/2}}\Phi^{-1}(T)\right)\frac{\tau_\theta}{\tau_i^{1/2}}d\Gamma\\
&\leq\int\frac{1}{\sqrt{2\pi}}\frac{\tau_\theta}{\tau_i^{1/2}}d\Gamma\\
&\leq\frac{1}{\sqrt{2\pi}}\frac{\tau_\theta}{\underline{\tau}^{1/2}}\\
&<1
\end{aligned}$$

where the last inequality follows from our maintained assumption that $\frac{\tau_\theta}{\underline{\tau}^{1/2}}<\sqrt{2\pi}$, which guarantees a unique equilibrium in the second stage. A unique θ^* implies in turn a unique threshold $x^*(\tau_i)$. It follows that for an arbitrary distribution of precision choices, we have a unique equilibrium in monotone strategies for the second stage of the game. ■

A.3 Solving the model: $t=1$

Derivation of Equation (4) The ex-ante utility is given by

$$U^i(\tau_i;\Gamma,\tau_\theta,\mu_\theta,T)=\int_{\theta=-\infty}^{+\infty}\int_{x_i\geq x_i^*(\tau_i;\Gamma)}\left[1_{\{\theta\geq\theta^*(\Gamma)\}}-T\right]dF_{\tau_i}(x|\theta)\,dG_{\tau_\theta}(\theta)-C(\tau_i)$$

Notice that

$$\begin{aligned}
&\int_{\theta=-\infty}^{+\infty}\int_{x_i\geq x_i^*(\tau_i;\Gamma)}\left[1_{\{\theta\geq\theta^*(\Gamma)\}}-T\right]dF_{\tau_i}(x|\theta)\,dG_{\tau_\theta}(\theta)-C(\tau_i)\\
&=\int_{\theta^*}^{\infty}\int_{x^*}^{\infty}(1-T)\,dF(x|\theta)\,dG(\theta)-\int_{-\infty}^{\theta^*}\int_{x^*}^{\infty}T\,dF(x|\theta)\,dG(\theta)-C(\tau_i)\\
&=-\int_{-\infty}^{\theta^*}\int_{x^*}^{\infty}T\,dF(x|\theta)\,dG(\theta)-\int_{\theta^*}^{\infty}\int_{-\infty}^{x^*}(1-T)\,dF(x|\theta)\,dG(\theta)\\
&\quad+\int_{\theta^*}^{\infty}(1-T)\,dG(\theta)-C(\tau_i)
\end{aligned}$$

which is the expression reported in Equation (4).

Lemma 1 *The benefit, in terms of expected utility, of an increase in the precision of private signals is equal to the reduction of the expected cost of mistakes due to a change in the ex-ante joint distribution of (θ,x_i), implied by this increase, and is equal to:*

$$\frac{\partial B^i(\tau_i,\Gamma)}{\partial\tau_i}=\frac{1}{2\tau_i}\frac{1}{\tau_i+\tau_\theta}\tau_i^{1/2}\phi\left(\frac{x_i^*-\theta^*}{\tau_i^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^*-\mu_\theta}{\tau_\theta^{-1/2}}\right)$$

Proof. Differentiating $B^i(\tau_i, \Gamma)$ with respect to τ_i we get

$$-\underbrace{T\left(\int_{-\infty}^{\theta^*} \frac{\partial x_i^*}{\partial \tau_i} f_{\tau_i}(x_i^*|\theta)\, g_{\tau_\theta}(\theta)\, d\theta + \int_{-\infty}^{\theta^*} \frac{1}{2}\frac{x^* - \theta}{\tau_i} f_{\tau_i}(x_i^*|\theta)\, g_{\tau_\theta}(\theta)\, d\theta\right)}_{\text{Reduction in the expected cost of Type I mistake}}$$

$$-\underbrace{(1-T)\left(\int_{\theta^*}^{\infty} \frac{\partial x_i^*}{\partial \tau_i} f_{\tau_i}(x_i^*|\theta)\, g_{\tau_\theta}(\theta)\, d\theta + \int_{\theta^*}^{\infty} \frac{1}{2}\frac{x^* - \theta}{\tau_i} f_{\tau_i}(x_i^*|\theta)\, g_{\tau_\theta}(\theta)\, d\theta\right)}_{\text{Reduction in the expected cost of Type II mistake}}$$

Evaluating the above integrals we obtain the following expression for the expected reduction in the expected cost of Type I mistake:

$$\begin{aligned}&-T\frac{\partial x_i^*}{\partial \tau_i} f_{\tau_i,\tau_\theta}(x_i^*)\, G_{\tau_i,\tau_\theta}(\theta^*|x_i^*) - \frac{1}{2\tau_i} T f_{\tau_i,\tau_\theta}(x_i^*)\, G_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\, x^* \\ &-\frac{1}{2\tau_i} T E\left[\theta^*|x_i^*\right]_{\tau_i,\tau_\theta}(x_i^*)\, G_{\tau_i,\tau_\theta}(\theta^*|x_i^*) + \frac{1}{2\tau_i} T f_{\tau_i,\tau_\theta}(x_i^*)\, g_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\, (\tau_i + \tau_\theta)^{-1/2},\end{aligned}$$

where $f_{\tau_i,\tau_\theta}(x_i^*)$ is the unconditional distribution of x_i, $g_{\tau_i,\tau_\theta}(\theta^*|x_i^*)$ is the conditional distribution of θ given x_i and $(\tau_i + \tau_\theta)^{-1/2}$ is the standard deviation of the conditional distribution of θ given x_i.

The reduction in the expected cost of Type II mistake is similarly given by

$$\begin{aligned}&-(1-T)\frac{\partial x_i^*}{\partial \tau_i} f_{\tau_i,\tau_\theta}(x_i^*)\left(1 - G_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\right) - \frac{1}{2\tau_i}(1-T) f_{\tau_i,\tau_\theta}(x_i^*)\left(1 - G_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\right) x^* \\ &-\frac{1}{2\tau_i}(1-T) E\left[\theta^*|x_i^*\right] f_{\tau_i,\tau_\theta}(x_i^*)\left(1 - G_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\right) + \frac{1}{2\tau_i}(1-T) f_{\tau_i,\tau_\theta}(x_i^*)\, g_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\, (\tau_i + \tau_\theta)^{-1/2}\end{aligned}$$

Using the PI condition and the fact that in equilibrium $\left(1 - G_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\right) = T$, the first three terms in the reduction of each mistake cancel out. Thus, the benefit, in terms of expected utility, of an increase in the precision of private signals is equal to

$$\frac{1}{2\tau_i} f_{\tau_i,\tau_\theta}(x_i^*)\, g_{\tau_i,\tau_\theta}(\theta^*|x_i^*)\, (\tau_i + \tau_\theta)^{-1/2},$$

or equivalently,

$$\frac{\partial B^i(\tau_i, \Gamma)}{\partial \tau_i} = \frac{1}{2\tau_i}\frac{1}{\tau_i + \tau_\theta}\tau_i^{1/2}\phi\left(\frac{x_i^* - \theta^*}{\tau_i^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right)$$

which is the expression in the text. ■

Theorem 1 *Suppose that Assumptions* (A1) *and* (A2) *hold. Then:*

1. *There are no asymmetric equilibria in which investors choose different precision levels in the first stage;*

2. *There exists a symmetric equilibrium of the information acquisition game where all investors choose in period* 1 *the same precision* τ^* *and equilibrium in period* 2 *is characterized by a pair of thresholds* $\{\theta^*(\tau), x^*(\tau)\}$

3. *There exists* $\underline{\underline{\tau}} < \infty$ *such that if* $\underline{\tau} > \underline{\underline{\tau}}$*, then there is a unique equilibrium in the information acquisition game.*

Proof. We first argue that there are no asymmetric equilibria. Suppose that Γ is non-degenerate. By Proposition 1 we know that for any Γ there exists a unique equilibrium in monotone strategies in the second stage of the game. Since all investors are infinitesimally small, it follows that no investor can influence the outcome of the second stage and hence all investors take the equilibrium outcome as given. Moreover, Lemma $A.1$, together with Assumption $(A2)$, implies that each investor's problem at $t = 1$ has a unique solution.[35] Since all investors are ex-ante identical, this implies that they face the same decision problem and that the optimal solution is the same for all investors. It follows that the distribution of investors' precision choices is degenerate.

Next, we show that there exist symmetric equilibria. Denote by τ the precision choice of all other investors and let $\tau_i^*(\tau)$ be the optimal precision choice of investor i, given that all other investors choose precision τ. By the Theorem of the Maximum it follows that $\tau_i^*(\tau)$ is a continuous function of τ. Since $C'(\underline{\tau}) = 0$ we know that $\tau_i^*(\underline{\tau}) > \underline{\tau}$. Assumption $(A2)$ implies that there exists $\overline{\tau} < \infty$ such that investors will never find it optimal to a choose precision level $\tau_i > \overline{\tau}$. Therefore, we conclude that $\tau_i^*(\tau)$ is a continuous function mapping $[\underline{\tau}, \overline{\tau}]$ into itself. By Brouwer's Fixed Point Theorem we know that $\tau_i^*(\tau)$ has a fixed point, which we call τ^*. This fixed point of $\tau_i^*(\tau)$ is a symmetric equilibrium since if an investor believes that all other investors choose τ^*, his best response is to choose τ^* himself.

Finally, we show that if the lowest possible precision choice, $\underline{\tau}$, is high enough, then the symmetric equilibrium is unique. To establish this result we show that the slope of the best response function at the symmetric precision choice τ is always positive and tends to zero as $\tau \to \infty$.

The derivative of investor i's best response function with respect to τ, the precision choice of all other investors, is given by:

$$\frac{\partial \tau_i^*(\tau)}{\partial \tau} = -\frac{\frac{\partial^2}{\partial \tau_i \partial \tau} B^i(\tau_i^*(\tau), \tau)}{\frac{\partial^2}{\partial (\tau_i)^2} B^i(\tau_i^*(\tau), \tau) - \frac{\partial^2}{\partial (\tau_i)^2} C(\tau_i^*(\tau))}$$

where

$$\frac{\partial^2 B^i(\tau_i^*(\tau), \tau)}{\partial \tau_i \partial \tau} = -\frac{1}{2}\frac{1}{\tau_i}\frac{1}{\tau_i + \tau_\theta}\tau_i^{1/2}\phi\left(\frac{x_i^* - \theta^*}{\tau_i^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right)\frac{1}{2}\frac{\tau_\theta^2}{\tau^{1/2}(\tau + \tau_\theta)}\frac{(x_i^* - \mu_\theta)(x^* - \mu_\theta)}{\left(\frac{\tau_\theta}{\tau^{1/2}} - \frac{1}{\phi(\Phi^{-1}(\theta^*))}\right)}$$

If $\tau_i = \tau$, then $x_i^* = x^*$ and the above expression is necessarily positive. Thus, at the symmetric equilibrium, the slope of the best response function is positive.

[35] See the Online Appendix.

Let $E = \{\tau | \tau = \tau_i^* (\tau)\}$ be the set of all symmetric equilibrium precision choices. Then the numerator of $\frac{\partial \tau_i^*(\tau)}{\partial \tau}$ is positive for all $\tau \in E$ since $\tau_i = \tau$. By Lemma $A.1$ and Assumption $A2$, we know that the denominator is negative and hence it follows that

$$\frac{\partial \tau_i^* (\tau)}{\partial \tau} > 0, \ \forall \tau \in E$$

Note that $\forall \tau \in E$, by the convexity of the cost function (Assumption ($A2$)), we have the following result:

$$\left. \frac{\partial \tau_i^* (\tau)}{\partial \tau} \right|_{\tau_i = \tau} \leq \frac{\frac{\partial^2}{\partial \tau_i \partial \tau} B^i (\tau_i, \tau) |_{\tau_i = \tau}}{-\frac{\partial^2}{\partial (\tau_i)^2} B^i (\tau_i^* (\tau), \tau) |_{\tau_i = \tau}}$$

After computing the relevant derivatives, the above inequality can be expressed as:

$$\frac{\partial \tau_i^* (\tau)}{\partial \tau} \leq \frac{\frac{1}{2} \frac{\tau^{1/2} \tau_\theta^2}{(\tau + \tau_\theta)} (x_i^* - \mu_\theta) (x^* - \mu_\theta)}{-\left[\frac{3\tau + \tau_\theta}{2(\tau + \tau_\theta)} - \frac{\tau_\theta}{2(\tau + \tau_\theta)} (x^* - \mu_\theta) (x^* - \theta^*)\right] \left(\frac{\tau_\theta}{\tau^{1/2}} - \frac{1}{\phi(\Phi^{-1}(\theta^*))}\right)}$$

Note that

$$\lim_{\tau \to \infty} \frac{1}{2} \frac{\tau^{1/2} \tau_\theta^2}{(\tau + \tau_\theta)} (x_i^* - \mu_\theta) (x^* - \mu_\theta) = 0$$

$$\lim_{\tau \to \infty} \frac{\tau_\theta}{\tau^{1/2}} - \frac{1}{\phi (\Phi^{-1} (\theta^*))} = -\frac{1}{\phi (\Phi^{-1} (T))} < 0$$

$$\lim_{\tau \to \infty} \left[\frac{3\tau + \tau_\theta}{2 (\tau + \tau_\theta)} - \frac{\tau \tau_\theta}{2 (\tau + \tau_\theta)} (x^* - \mu_\theta) (x^* - \theta^*)\right] = \frac{3}{2}$$

where the last expression follows from the fact that $\lim_{\tau \to \infty} (x^* - \theta^*) = 0$. Hence, we conclude that

$$\lim_{\tau \to \infty} \frac{\partial \tau_i^* (\tau)}{\partial \tau} |_{\tau_i = \tau} \leq 0$$

It follows that for a given set of parameters $\{T, \mu_\theta, \tau_\theta\}$ there exists $\underline{\tau} (T, \mu_\theta, \tau_\theta) < \infty$ such that $\forall \tau \geq \underline{\tau}$ we have $\frac{\partial \tau_i^*(\tau)}{\partial \tau} |_{\tau_i = \tau^*} < 1$. Since $\{T, \mu_\theta, \tau_\theta\} \in [\underline{T}, \overline{T}] \times [\underline{\mu}_\theta, \overline{\mu}_\theta] \times [\underline{\tau}_\theta, \overline{\tau}_\theta]$ is a compact subset of $\mathbb{R}^3$, and since $\frac{\partial \tau_i^*(\tau)}{\partial \tau} |_{\tau_i = \tau}$ is continuous in $\{\tau, T, \mu_\theta, \tau_\theta\}$, it follows that there exists a value of $\underline{\tau}$, independent of $\{T, \mu_\theta, \tau_\theta\}$, such that $\frac{\partial \tau_i^*(\tau)}{\partial \tau} |_{\tau_i = \tau^*} < 1, \forall \tau \geq \underline{\tau}$. ∎

A.4 Spillover Effects and the Inefficiency of Equilibrium

Lemma A.8 *There exists an efficient choice of precision, τ^{**}.*

Proof. The efficient choice of precision, if it exists, is a solution to the following problem:

$$\max_{[\underline{\tau}, \infty)} B^i (\tau, \tau) - C (\tau)$$

The first derivative of the above equation is given by

$$B_1^i(\tau,\tau) + B_2^i(\tau,\tau) - C'(\tau)$$

From Lemma $A.1$ we know that $\frac{\partial B^i}{\partial \tau_i}$ is bounded from above and that $\lim_{\tau\to\infty} B_2^i(\tau,\tau) = 0$. Finally, by Assumption $(A2)$ $\lim_{\tau\to\infty} C'(\tau) = \infty$. Hence, there exists $\overline{\tau}^E$ such that no $\tau > \overline{\tau}^E$ can be a solution to the above maximization problem. But this implies that we are looking for a maximum of a continuous function over a compact subset of $\mathbb{R}$, hence $B^i(\tau,\tau) - C(\tau)$ must attain a maximum in $\left[\underline{\tau}, \overline{\tau}^E\right]$. Since $B^i(\tau,\tau) - C(\tau)$ is differentiable, it has to be the case that either the efficient precision choice τ^{**} satisfies the first order condition or $\tau^{**} = \underline{\tau}$. ■

Proposition 2 *Consider the equilibrium precision choice τ^*. For any $T \in [0,1]$, if $\mu_\theta \neq \mu_\theta^E(T)$ then the equilibrium precision choice is inefficient.*

Proof. Recall that the equilibrium precision choice satisfies

$$\frac{\partial B^i(\tau^*,\tau^*)}{\partial \tau_i} - C'(\tau^*) = 0$$

while the efficient choice of precision τ^{**} is either equal to $\underline{\tau}$ or, if $\tau^{**} > \underline{\tau}$, it satisfies

$$\frac{\partial B^i(\tau^*,\tau^*)}{\partial \tau_i} - \frac{\partial B^i(\tau^*,\tau^*)}{\partial \tau} - C'(\tau) = 0.$$

Therefore, a necessary condition for the equilibrium choice to be efficient is that $\frac{\partial U(\tau^*,\tau^*)}{\partial \tau} = 0$. Note that

$$\frac{\partial B^i(\tau^*,\tau^*)}{\partial \tau} = -\frac{\partial \theta^*}{\partial \tau}\left[1 - \Phi\left(\frac{x_i^* - \theta^*}{\tau_i^{-1/2}}\right)\right]\phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right)$$

Hence, $\frac{\partial B^i(\tau^*,\tau^*)}{\partial \tau} = 0$ if and only if $\left.\frac{\partial \theta^*}{\partial \tau}\right|_{\tau=\tau^*} = 0$.

However,

$$\left.\frac{\partial \theta^*}{\partial \tau}\right|_{\tau=\tau^*} = 0 \iff \mu_\theta = \Phi\left(\sqrt{\frac{\tau^*(\mu_\theta)}{\tau^*(\mu_\theta) + \tau_\theta}}\Phi^{-1}(T)\right) + \frac{1}{\sqrt{\tau^*(\mu_\theta) + \tau_\theta}}\Phi^{-1}(T)$$

From Lemma $A.2$ we know that for each T there exists a unique μ_θ that satisfies the above equation, which implies that generically the equilibrium precision choice is inefficient. ■

Proposition 3 *Consider the investors' equilibrium precision choices.*

1. *If $\mu_\theta > \mu_\theta^E(T)$ then investors locally over-acquire information*

2. *If $\mu_\theta = \mu_\theta^E(T)$ (and $T \geq \frac{1}{2}$) then investors choose the locally efficient level of information*

3. *If $\mu_\theta < \mu_\theta^E(T)$ then investors locally under-acquire information*

Proof. Note first that the derivative of investor i's ex-ante utility function U^i with respect to the precision choice of other investors, τ, is given by

$$U_2^i(\tau_i;\tau) = B_2^i(\tau_i;\tau) = -\frac{\partial\theta^*}{\partial\tau}\left(1-\Phi\left(\frac{\theta^*-x_i^*}{\tau_i^{-1/2}}\right)\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^*-\mu_\theta}{\tau_\theta^{-1/2}}\right)$$

and, thus, its sign is determined by $\frac{\partial\theta^*}{\partial\tau}$. Next, recall that the equilibrium precision choice satisfies

$$B_1^i(\tau^*;\tau^*) - C'(\tau^*) = 0$$

On the other hand, the first derivative of the planner's objective function with respect to τ is given by

$$B_1^i(\tau;\tau) + B_2^i(\tau;\tau) - C'(\tau)$$

It follows that if $B_2^i(\tau^*;\tau^*) > 0$ then a small increase in the investors' precision choices τ would increase each investor's ex-ante utility, i.e. investors locally under-acquire information. Similarly, when $B_2^i(\tau^*;\tau^*) < 0$ then a small decrease in τ would lead to a higher welfare, i.e. investors locally over-acquire information.

The above discussion implies that in order to establish whether investors locally over-acquire or under-acquire information we need to establish the sign of $\partial\theta^*/\partial\tau|_{\tau=\tau^*.}$ From Lemma $A.3$ we know that if $\mu_\theta > \widehat{\mu}^\tau(T,\tau,\tau_\theta)$ then $\partial\theta^*/\partial\tau > 0$, and if $\mu_\theta < \widehat{\mu}^\tau(T,\tau,\tau_\theta)$ then $\partial\theta^*/\partial\tau < 0$. From Lemma $A.4$ we know that if $\mu_\theta > \mu^E(T)$ then $\mu_\theta > \widehat{\mu}^\tau(T,\tau^*(\mu_\theta),\tau_\theta)$ and if $\mu_\theta < \mu^E(T)$ then $\mu_\theta < \widehat{\mu}^\tau(T,\tau^*(\mu_\theta),\tau_\theta)$. It follows that for all $\mu_\theta > \mu^E(T)$ we have $\partial\theta^*/\partial\tau|_{\tau=\tau^*.} > 0$ and for all $\mu_\theta < \mu^E(T)$ we have $\partial\theta^*/\partial\tau|_{\tau=\tau^*.} < 0$. Thus, if $\mu_\theta > \mu_\theta^E(T)$ then $B_2^i(\tau^*;\tau^*) < 0$, implying that a small decrease in precision from its equilibrium level would actually increase investors' welfare. On the other hand, if $\mu_\theta < \mu_\theta^E(T)$ then $B_2^i(\tau^*;\tau^*) > 0$ implying that a small decrease in precision from its equilibrium level would actually increase investors' welfare.

Finally, consider the case where $\mu_\theta = \mu^E(T)$. In that case $\partial\theta^*/\partial\tau|_{\tau=\tau^*.} = 0$. If $T \geq \frac{1}{2}$ then Lemma $A.5$ implies that an increase or a decrease in τ will lead to an increase in θ^* and hence it will have a negative impact on investor i's utility. Therefore, it follows that investors acquire the locally efficient level of information. On the other hand, when $T < \frac{1}{2}$ then Lemma $A.5$ implies that both an increase and a decrease in τ will lead to a decrease in θ^*, hence it will have a positive impact on investor i's utility. It follows that investors acquire an inefficient level of information and either a decrease or an increase in their precision choices would lead to an increase in welfare. ■

A.5 Strategic Complementarities in Information Acquisition

Proposition 4 *Define*

$$\overline{\mu}^{SC}\left(\underline{\tau}, \tau_\theta, T\right) \equiv T + \frac{1}{\sqrt{\underline{\tau} + \tau_\theta}} \Phi^{-1}\left(T\right)$$

1. *Suppose that* $T > \frac{1}{2}$.
   (a) *If* $\mu_\theta \notin \left(T, \overline{\mu}^{SC}\left(\underline{\tau}, \tau_\theta, T\right)\right)$ *then information choices are strategic complements.*
   (b) *If* $\mu_\theta \in \left(T, \overline{\mu}^{SC}\left(\underline{\tau}, \tau_\theta, T\right)\right)$ *then there is a lack of strategic complementarities.*
2. *Suppose that* $T = \frac{1}{2}$. *Then information choices are always strategic complements.*
3. *Suppose that* $T < \frac{1}{2}$.
   (a) *If* $\mu_\theta \notin \left(\overline{\mu}^{SC}\left(\underline{\tau}, \tau_\theta, T\right), T\right)$ *then information choices are strategic complements.*
   (b) *If* $\mu_\theta \in \left(\overline{\mu}^{SC}\left(\underline{\tau}, \tau_\theta, T\right), T\right)$ *then there is a lack of strategic complementarities.*

Proof. Consider investor i and let τ_i be his precision choice and let τ be the precision choice of all other investors. Recall that the cross-partial derivative of the ex-ante utility function, with respect to τ_i and τ, is given by:

$$\frac{\partial^2 U}{\partial \tau_i \partial \tau} = -\frac{1}{\tau_i} \frac{1}{\tau_i + \tau_\theta} \tau_i^{1/2} \phi\left(\frac{x_i^* - \theta^*}{\tau_i^{-1/2}}\right) \tau_\theta^{1/2} \phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right) \left[x_i^* - \mu_\theta\right] \frac{\partial \theta^*}{\partial \tau}$$

From the above expression we see that a higher precision chosen by other investors increases investor i's incentives to acquire information if and only if $\frac{\partial \theta^*}{\partial \tau}$ and $x_i^* - \mu_\theta$ are of opposite signs. We investigate the conditions when this is the case.

We consider first the special case of $T = \frac{1}{2}$. In that case, since

$$x_i^* - \mu_\theta = \frac{\tau_i + \tau_\theta}{\tau_i}\left(\theta^* - \mu_\theta\right) \text{ and } \frac{\partial \theta^*}{\partial \tau} \propto -\frac{1}{\tau}\left(\theta^* - \mu_\theta\right)$$

it follows immediately that $\frac{\partial \theta^*}{\partial \tau}$ and $x_i^* - \mu_\theta$ are of opposite signs and, thus, $\frac{\partial^2 U}{\partial \tau_i \partial \tau} > 0$. Since the slope of the best response function evaluated at the symmetric precision choice is positive (see the proof of Theorem 1), it follows that an increase in the precision choices by others encourages investor i to acquire more information.

Next, consider the case when $T > \frac{1}{2}$. In this case

$$x_i^* - \mu_\theta = \frac{\tau_i + \tau_\theta}{\tau_i}\left(\theta^* - \mu_\theta\right) + \frac{\sqrt{\tau_i + \tau_\theta}}{\tau_i} \Phi^{-1}\left(T\right)$$

and

$$\frac{\partial \theta^*}{\partial \tau} \propto -\frac{1}{\tau}\left(\theta^* - \mu_\theta\right) - \frac{1}{\tau} \frac{1}{\sqrt{\tau + \tau_\theta}} \Phi^{-1}\left(T\right)$$

Suppose first that $\mu_\theta \leq T$. From Lemma $A.5$ we know that $\frac{\partial\theta^*}{\partial\tau} < 0$, for all τ. Moreover, by lemma $A.4$ we know that $\theta^* \to T$. Thus, θ^* must converge to T from above implying that

$$\begin{aligned} x_i^* - \mu_\theta &= \frac{\tau_i + \tau_\theta}{\tau_i}\left(\theta^* - \mu_\theta\right) + \frac{\sqrt{\tau_i + \tau_\theta}}{\tau_i}\Phi^{-1}\left(T\right) \\ &\geq \frac{\tau_i + \tau_\theta}{\tau_i}\left(T - \mu_\theta\right) \\ &\geq 0 \end{aligned}$$

Therefore, when $T > \frac{1}{2}$ and $\mu_\theta \leq T$ then $\frac{\partial\theta^*}{\partial\tau} < 0$, while $x_i^* - \mu_\theta > 0$. Thus, in this case precision choices are strategic complements.

Now, assume that $\mu \geq \overline{\mu}^{SC}$, where

$$\overline{\mu}^{SC} = T + \frac{1}{\sqrt{\underline{\tau} + \tau_\theta}}\Phi^{-1}\left(T\right)$$

Note that, when $T > \frac{1}{2}$ then

$$\overline{\mu}^{SC} > \widehat{\mu}\left(T, \underline{\tau}, \tau_\theta\right).$$

Therefore, by Lemma $A.5$, we know that if $\mu \geq \overline{\mu}^{SC}$, then for all $\tau \in [\underline{\tau}, \infty)$, $\frac{\partial\theta^*}{\partial\tau} > 0$. Since $\lim_{\tau\to\infty}\theta^*\left(\tau\right) = T$, it follows that θ^* converges to T from below, i.e., for all $\tau \in [\underline{\tau}, \infty)$ we have $\theta^*\left(\tau\right) < T$. This implies that

$$x_i^* - \mu_\theta \leq \frac{\tau_i + \tau_\theta}{\tau_i}\left(T - \overline{\mu}^{SC}\right) + \frac{\sqrt{\tau_i + \tau_\theta}}{\tau_i}\Phi^{-1}\left(T\right) \leq 0$$

where the last inequality is strict for all $\tau_i > \underline{\tau}$. Thus, when $T > \frac{1}{2}$ and $\mu_\theta > \overline{\mu}^{SC}$ we have $x_i^* - \mu_\theta \leq 0$ and $\frac{\partial\theta^*}{\partial\tau} > 0$, hence information choices are strategic complements.

Next, we show that if $\mu_\theta \in \left(T, \overline{\mu}^{SC}\right)$, then information choices are not strategic complements. Fix $\mu_\theta \in \left(T, \overline{\mu}^{SC}\right)$ and note that, since $\mu_\theta > T$, by Lemma $A.5$ we know that $\frac{\partial\theta^*}{\partial\tau} > 0$ for large enough τ. Define

$$\varepsilon = \overline{\mu}^{SC} - \mu_\theta$$

Since $\lim_{\tau\to\infty}\theta^*\left(\tau\right) = T$, it follows that for large enough τ we have

$$\theta^* > T - \frac{\varepsilon}{2}$$

Then

$$\begin{aligned} \theta^* - \mu_\theta &> T - \frac{\varepsilon}{2} - \left(\overline{\mu}^{SC} - \varepsilon\right) \\ &= T - \frac{\varepsilon}{2} - \left(T + \frac{1}{\sqrt{\underline{\tau} + \tau_\theta}}\Phi^{-1}\left(T\right) - \varepsilon\right) \\ &= \frac{\varepsilon}{2} - \frac{1}{\sqrt{\underline{\tau} + \tau_\theta}}\Phi^{-1}\left(T\right) \end{aligned}$$

thus,

$$x_i^* - \mu_\theta > \frac{\tau_i + \tau_\theta}{\tau_i}\left[\frac{\varepsilon}{2} - \frac{1}{\sqrt{\underline{\tau} + \tau_\theta}}\Phi^{-1}(T)\right] + \frac{\sqrt{\tau_i + \tau_\theta}}{\tau_i}\Phi^{-1}(T)$$
$$= \frac{\tau_i + \tau_\theta}{\tau_i}\frac{\varepsilon}{2} + \frac{\sqrt{\tau_i + \tau_\theta}}{\tau_i}\left[-\frac{\sqrt{\tau_i + \tau_\theta}}{\sqrt{\underline{\tau} + \tau_\theta}} + 1\right]\Phi^{-1}(T)$$

It follows that for all τ_i close to $\underline{\tau}$, we have

$$x_i^* - \mu_\theta > 0$$

Hence, for τ_i close to $\underline{\tau}$ and τ large enough we have $x_i^* - \mu_\theta > 0$ and $\frac{\partial \theta^*}{\partial \tau} > 0$. This in turn implies that there are pairs $\{\tau_i, \tau\}$ such that a marginal increase in τ decreases investor i's incentives to acquire information, hence for $\mu_\theta \in \left(T, \overline{\mu}^{SC}\right)$ information choices are not strategic complements.

An analogous argument can be used to prove the result when $T < \frac{1}{2}$. ■

A.6 Transparency and Welfare

A.6.1 Trade-off between Public and Private Information

Proposition 5 *Let $T = \frac{1}{2}$. There exist cutoffs $\widehat{\mu}^-$ and $\widehat{\mu}^+$, $\widehat{\mu}^- < \frac{1}{2} < \widehat{\mu}^+$ such that:*

1. *if $\mu_\theta \notin \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$ then private and public information are substitutes.*
2. *if $\mu_\theta \in \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$ then private and public information are complements*

Proof. We are interested in the sign of the effect of an increase in the precision of public information on private information acquisition, i.e. we want to determine under which conditions

$$\frac{d\tau_i^*}{d\tau_\theta} = \frac{1}{1 - \left.\frac{\partial \tau_i^*}{\partial \tau}\right|_{\tau = \tau^*}} \left.\frac{\partial \tau_i^*}{\partial \tau_\theta}\right|_{\tau_i = \tau^*}$$

is positive or negative.

First, note that, as shown in the proof of Theorem 1,

$$\frac{1}{1 - \left.\frac{\partial \tau_i^*}{\partial \tau}\right|_{\tau = \tau^*}} > 0$$

Thus, it is enough to determine the sign of $\left.\frac{\partial \tau_i^*}{\partial \tau_\theta}\right|_{\tau_i = \tau^*}$, where

$$\left.\frac{\partial \tau_i^*}{\partial \tau_\theta}\right|_{\tau_i = \tau^*} = -\frac{\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i = \tau^*}}{\left.\frac{\partial U^2}{\partial \tau_i^2}\right|_{\tau_i = \tau^*}}$$

By Lemma $A.1$, $\frac{\partial^2 U}{\partial^2 \tau_i}$ is negative and, therefore, the sign of $\frac{\partial \tau_i^*}{\partial \tau_\theta}$ is determined by the sign of $\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i = \tau^*}$

$$
\begin{aligned}
\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i=\tau^*} = & -\frac{1}{2}\frac{1}{\tau^*}\frac{1}{\tau^*+\tau_\theta}{\tau^*}^{1/2}\phi\left(\frac{x^*-\theta^*}{{\tau^*}^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^*-\mu_\theta}{\tau_\theta^{-1/2}}\right)\left[\frac{1}{\tau^*+\tau_\theta}-\frac{1}{2\tau_\theta}+\frac{1}{2}\left(\theta^*-\mu_\theta\right)^2\right] \\
& -\frac{1}{2}\frac{1}{\tau^*}\frac{1}{\tau^*+\tau_\theta}{\tau^*}^{1/2}\phi\left(\frac{x^*-\theta^*}{{\tau^*}^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^*-\mu_\theta}{\tau_\theta^{-1/2}}\right)\left[\tau^*\left(x^*-\theta^*\right)\left.\frac{\partial x_i^*}{\partial \tau_\theta}\right|_{\tau_i=\tau^*}\right] \\
& -\frac{1}{2}\frac{1}{\tau^*}\frac{1}{\tau^*+\tau_\theta}{\tau^*}^{1/2}\phi\left(\frac{x^*-\theta^*}{{\tau^*}^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^*-\mu_\theta}{\tau_\theta^{-1/2}}\right)\left[\tau_\theta\left(x^*-\mu_\theta\right)\left.\frac{\partial \theta^*}{\partial \tau_\theta}\right|_{\tau_i=\tau^*}\right]
\end{aligned}
$$

The above expression implies that a change in τ_θ affects the investors' incentives to acquire information through three channels: (i) by changing the joint density of $\{\theta, x_i\}$ (passive information effect, captured by the term in the first line), (ii) by changing the informativeness of the prior, it affects the investor's investment strategy (active information effect, captured by the second line), (iii) it affects the change in the equilibrium threshold θ^*, since an increase in the precision of the prior affects also the other investors' investment strategies (coordination effect, captured by the third line of the above expression). We investigate under which conditions each of these effects encourages an individual investor to acquire more or less information.

The above observations are independent of the value of T. However, from now on we assume that $T = \frac{1}{2}$ to simplify substantially the analysis, while not affecting the underlying logic of our arguments. From now on we will not mention explicitly that $T = \frac{1}{2}$, but it should be understood that all the results below make use of this assumption.

The sign of the passive information effect is determined by

$$
\frac{1}{\tau^*+\tau_\theta}-\frac{1}{2\tau_\theta}+\frac{1}{2}\left(\theta^*-\mu_\theta\right)^2 \tag{9}
$$

Now note that since $\tau^* > \underline{\tau} > \tau_\theta$, Lemma $A.6$ implies that at $\mu_\theta = \frac{1}{2}$ the above expression is negative. On the other hand, for low and high enough μ_θ the term $\left(\theta^*-\mu_\theta\right)^2$ is large, hence the above expression is positive. Next, note that for all $\mu_\theta < \frac{1}{2}$, as μ_θ increases towards $\frac{1}{2}$, then $\theta^* - \mu_\theta$ is decreasing, while τ^* is increasing (see Lemma $A.7$). Thus, there exists a value of μ_θ, call it μ^-, such that $\mu^- < \frac{1}{2}$ and at μ^- we have

$$
\frac{1}{\tau^*\left(\mu^-\right)+\tau_\theta}-\frac{1}{2\tau_\theta}+\frac{1}{2}\left(\theta^*\left(\mu^-\right)-\mu_\theta\right)^2 = 0
$$

where we explicitly note that both the equilibrium precision level $\tau^*\left(\mu^-\right)$ and the threshold $\theta^*\left(\mu^-\right)$ are functions of μ_θ. Moreover, it follows that for all $\mu_\theta \in \left(\mu^-, \frac{1}{2}\right]$ the expression (9) is positive.

Similarly, for all $\mu_\theta > \frac{1}{2}$, as μ_θ increases from $\frac{1}{2}$, $\theta^* - \mu_\theta$ increases while τ^* decreases (see Lemma $A.7$). Thus, there exists a value of μ_θ, call it μ^+, such that $\mu^+ > \frac{1}{2}$ and at μ^+ we have

$$\frac{1}{\tau^*(\mu^+) + \tau_\theta} - \frac{1}{2\tau_\theta} + \frac{1}{2}\left(\theta^*(\mu^+) - \mu_\theta\right)^2 = 0$$

It also follows that for all $\mu_\theta \in \left[\frac{1}{2}, \mu^+\right)$ the expression 9 is positive.

Since

$$-\frac{1}{2}\frac{1}{\tau}\frac{1}{\tau+\tau_\theta}\tau^{1/2}\phi\left(\frac{x^* - \theta^*}{\tau^{-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right) < 0,$$

it follows that there exist μ^- and μ^+, with $\mu^- < \frac{1}{2} < \mu^+$, such that if $\mu_\theta \in (\mu^-, \mu^+)$ then the passive information effect is strictly positive. If $\mu_\theta \in \{\mu^-, \mu^+\}$, then the passive information effect is zero, and if $\mu_\theta \notin (\mu^-, \mu^+)$ then the passive information effect is strictly negative.

The sign of the active information effect is determined by

$$(x^* - \theta^*)\left.\frac{\partial x_i^*}{\partial \tau_\theta}\right|_{\tau_i = \tau^*} = \frac{\tau_\theta}{\tau^*}(\theta^* - \mu_\theta)^2 > 0$$

And since

$$-\frac{1}{2}\frac{1}{\tau^*}\frac{1}{\tau^*+\tau_\theta}\tau^{*^{-1.2}}\phi\left(\frac{x^* - \theta^*}{\tau^{*^{-1.2}}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right) < 0$$

It follows that the active information effect is always negative. Notice that the sign of the active information effect takes into account only the partial effect of a change in τ_θ on x_i^* (keeping θ^* constant). The effect of τ_θ on θ^* is taken into account in the expression for the coordination effect below.

Finally, consider the coordination effect. The sign of the coordination effect is given by

$$(x^* - \mu_\theta)\left.\frac{\partial \theta^*}{\partial \tau_\theta}\right|_{\tau_i = \tau^*} = -\frac{\tau + \tau_\theta}{\tau^{3/2}}\frac{(\theta^* - \mu_\theta)^2}{\frac{\tau_\theta}{\tau^{1/2}} - \frac{1}{\phi(\Phi^{-1}(\theta^*))}} > 0$$

Since $\frac{\tau^{1/2}}{\tau_\theta} > \frac{1}{\sqrt{2\pi}}$.

And since

$$-\frac{1}{2}\frac{1}{\tau^*}\frac{1}{\tau^*+\tau_\theta}\tau_x^{1/2}\phi\left(\frac{x^* - \theta^*}{\tau^{*^{-1/2}}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right) < 0$$

it follows that the "coordination effect" is always negative.

From the above analysis we see that the "active information effect" and the "coordination effect" always discourage information acquisition. Moreover, when $\mu_\theta = \frac{1}{2}$ both effects are zero. On the other hand, depending on μ_θ, the "passive information effect" can encourage or discourage information acquisition. When $\mu_\theta = \frac{1}{2}$ then the "passive information effect" is positive. In this case the other two effects are equal to zero, so for μ_θ in the neighborhood of $\frac{1}{2}$ an increase in the precision of public information leads to an increase in information acquisition. Finally, we see that if $\mu_\theta \leq \mu^-$ or $\mu_\theta \geq \mu^+$ then all the above effects are negative, hence an increase in the precision of private information leads to less information acquisition.

Below we show that there exists an interval of values for μ_θ, that includes $\frac{1}{2}$, such that if μ_θ takes a value in that interval then more precise public information leads to more private information acquisition, and if μ_θ takes a value outside that interval then more public information leads to less private information acquisition.

We first investigate when the cross-partial derivative $\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i=\tau^*}$ is greater than zero. Note first that the cross-partial derivative $\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i=\tau^*}$ can be re-written as

$$\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i=\tau^*} = -\frac{1}{2}\frac{1}{\tau^*}\frac{1}{\tau^*+\tau_\theta}\tau^{*1/2}\phi\left(\frac{x^*-\theta^*}{\tau^{*-1/2}}\right)\tau_\theta^{1/2}\phi\left(\frac{\theta^*-\mu_\theta}{\tau_\theta^{-1/2}}\right)$$
$$\times\left[\frac{1}{\tau^*+\tau_\theta}-\frac{1}{2\tau_\theta}+\frac{1}{2}\left(\theta^*-\mu_\theta\right)^2+\tau^*\left(x^*-\theta^*\right)\left.\frac{\partial x_i^*}{\partial \tau_\theta}\right|_{\tau_i=\tau^*}+\tau_\theta\left(x^*-\mu_\theta\right)\left.\frac{\partial \theta^*}{\partial \tau_\theta}\right|_{\tau_i=\tau^*}\right]$$

where the term pre-multiplying the square brackets is always negative.

Using the earlier observations we employ the following strategy for the proof. We show below that the term in the square brackets is increasing in μ_θ when $\mu_\theta \in \left(\mu^-, \frac{1}{2}\right)$ and it is decreasing in μ_θ when $\mu_\theta \in \left(\frac{1}{2}, \mu_\theta^+\right)$. Once we establish these two claims then it will follow immediately that there exist values of μ_θ, which we call $\widehat{\mu}^-$ and $\widehat{\mu}^+$, such that $\mu^- < \widehat{\mu}^- < \frac{1}{2} < \widehat{\mu}^+ < \mu^+$ and $\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i=\tau^*} > 0$ if and only if $\mu_\theta \in \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$ and $\left.\frac{\partial^2 U}{\partial \tau_i \partial \tau_\theta}\right|_{\tau_i=\tau^*} < 0$ otherwise.

For notational purposes define

$$\Lambda\left(\mu_\theta\right) \equiv \left[\frac{1}{\tau^*+\tau_\theta}-\frac{1}{2\tau_\theta}+\frac{1}{2}\left(\theta^*-\mu_\theta\right)^2+\tau^*\left(x^*-\theta^*\right)\left.\frac{\partial x_i^*}{\partial \tau_\theta}\right|_{\tau_i=\tau^*}+\tau_\theta\left(x^*-\mu_\theta\right)\left.\frac{\partial \theta^*}{\partial \tau_\theta}\right|_{\tau_i=\tau^*}\right]$$

Differentiating $\Lambda\left(\mu_\theta\right)$ with respect to μ_θ we obtain

$$\frac{d\Lambda\left(\mu_\theta\right)}{d\mu_\theta} = \left\{\left(\frac{\partial \theta^*}{\partial \mu_\theta}-1\right)\left(\theta^*-\mu_\theta\right)+2\frac{\tau_\theta}{\tau^*}\left(\frac{\partial \theta^*}{\partial \mu_\theta}-1\right)\left(\theta^*-\mu_\theta\right)\right.$$
$$\left.+\frac{\tau_\theta\left(\tau^*+\tau_\theta\right)}{\tau^*}\left[\left(\frac{\partial \theta^*}{\partial \mu_\theta}-1\right)\frac{\partial \theta^*}{\partial \tau_\theta}+\left(\theta^*-\mu_\theta\right)\frac{\partial^2 \theta^*}{\partial \tau_\theta \partial \mu_\theta}\right]\right\}$$
$$\left\{-\frac{1}{\left(\tau^*+\tau_\theta\right)^2}+\left(\theta^*-\mu_\theta\right)\frac{\partial \theta^*}{\partial \tau^*}-\frac{\tau_\theta}{\tau^{*2}}\left(\theta^*-\mu_\theta\right)^2+\frac{\tau_\theta}{\tau^*}\left(\theta^*-\mu_\theta\right)\frac{\partial \theta^*}{\partial \tau^*}-\frac{\tau_\theta^2}{\tau^{*2}}\left(\theta^*-\mu_\theta\right)\frac{\partial \theta^*}{\partial \tau_\theta}\right.$$
$$\left.+\frac{\tau_\theta\left(\tau^*+\tau_\theta\right)}{\tau^*}\left[\frac{\partial \theta^*}{\partial \tau^*}\frac{\partial \theta^*}{\partial \tau_\theta}+\left(\theta^*-\mu_\theta\right)\frac{\partial^2 \theta^*}{\partial \tau_\theta \partial \tau^*}\right]\right\}\frac{d\tau^*}{d\mu_\theta}$$

Note that $\frac{\partial \theta^*}{\partial \mu_\theta}$ is always less than zero. Moreover, when $\mu_\theta < \frac{1}{2}$ then: (1) $\left(\theta^*-\mu_\theta\right) > 0$, (2) $\frac{\partial \theta^*}{\partial \tau_\theta} > 0$, and (3) $\frac{\partial \theta^*}{\partial \tau^*} < 0$. Lemmas 14 and 15 in the Online Appendix show that when

$\mu_\theta < \frac{1}{2}$ we also have: (4) $\frac{\partial^2\theta^*}{\partial\tau_\theta\partial\mu_\theta} < 0$, (5) $\frac{\partial^2\theta^*}{\partial\tau_\theta\partial\tau^*} < 0$, and (6) $\frac{\partial\tau^*}{\partial\mu_\theta} > 0$. Note, however, that (1), (2) and (4) imply that the term in the first brackets in the expression for $\frac{d\Lambda(\mu_\theta)}{d\mu_\theta}$ is negative. Similarly, (1), (2), (3), (5) and (6) imply that the remaining term is also negative. Therefore, we conclude that when $\mu_\theta \in \left(\mu^-, \frac{1}{2}\right)$ then $\Lambda(\mu_\theta)$ is continuously decreasing. This proves the existence of $\widehat{\mu}^-$ such that for all $\mu_\theta \in \left(\widehat{\mu}^-, \frac{1}{2}\right)$ we have $\left.\frac{\partial^2 U}{\partial\tau_i\partial\tau_\theta}\right|_{\tau_i=\tau^*} > 0$ and for all $\mu_\theta < \widehat{\mu}^-$, $\left.\frac{\partial^2 U}{\partial\tau_i\partial\tau_\theta}\right|_{\tau_i=\tau^*} < 0$.

Using an analogous reasoning we consider the case when $\mu_\theta \in \left(\frac{1}{2}, \mu_\theta^+\right)$. Recall that $\frac{\partial\theta^*}{\partial\mu_\theta} < 0$. Moreover, when $\mu_\theta > \frac{1}{2}$ then: (1) $(\theta^* - \mu_\theta) < 0$, (2) $\frac{\partial\theta^*}{\partial\tau_\theta} > 0$, and (3) $\frac{\partial\theta^*}{\partial\tau^*} > 0$. Again, in the online appendix we also show that when $\mu_\theta > \frac{1}{2}$ then we also have: (4) $\frac{\partial^2\theta^*}{\partial\tau_\theta\partial\mu_\theta} < 0$, (5) $\frac{\partial^2\theta^*}{\partial\tau_\theta\partial\tau^*} > 0$, and (6) $\frac{\partial\tau^*}{\partial\mu_\theta} < 0$. Comparing these observations when $\mu_\theta > \frac{1}{2}$ with those when $\mu_\theta < \frac{1}{2}$, we see that the sign of most of these terms is now reversed compared to the case when $\mu_\theta < \frac{1}{2}$. Thus, we find that when $\mu_\theta \in \left(\frac{1}{2}, \mu_\theta^+\right)$ then $\Lambda(\mu_\theta)$ is continuously increasing. It follows that there exists $\widehat{\mu}^+$ such that for all $\mu_\theta \in \left(\frac{1}{2}, \widehat{\mu}^+\right)$ we have $\left.\frac{\partial^2 U}{\partial\tau_i\partial\tau_\theta}\right|_{\tau_i=\tau^*} > 0$ and for all $\mu_\theta > \widehat{\mu}^+$, $\left.\frac{\partial^2 U}{\partial\tau_i\partial\tau_\theta}\right|_{\tau_i=\tau^*} < 0$. ■

A.6.2 Effects of Increasing Public Information on Coordination

Proposition 6 *Let $T = \frac{1}{2}$ and suppose that the precision of public information increases.*

1. *If $\mu_\theta < \frac{1}{2}$ then the ex-ante probability of a successful investment decreases.*
2. *If $\mu_\theta = \frac{1}{2}$ then the ex-ante probability of a successful investment is unchanged.*
3. *If $\mu_\theta > \frac{1}{2}$ then the ex-ante probability of a successful investment increases.*

Proof. The change in the probability of a successful investment due to a change in τ_θ is equal to

$$\begin{aligned}\frac{d\Pr(\theta > \theta^*)}{d\tau_\theta} &= \frac{d}{d\tau_\theta}\left[1 - \Phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right)\right] \\ &= -\tau_\theta^{1/2}\phi\left(\frac{\theta^* - \mu_\theta}{\tau_\theta^{-1/2}}\right)\left[\frac{d\theta^*}{d\tau_\theta} + \frac{1}{2\tau_\theta}(\theta^* - \mu_\theta)\right]\end{aligned}$$

where

$$\frac{d\theta^*}{d\tau_\theta} = \frac{\partial\theta^*}{\partial\tau_\theta} + \frac{\partial\theta^*}{\partial\tau^*} \times \frac{d\tau^*}{d\tau_\theta}$$

That is, the total change in θ^* in response to a change in τ_θ is equal to the partial change in θ^* due to a change in τ_θ, holding investors' information precision choices constant (captured by $\frac{\partial\theta^*}{\partial\tau_\theta}$), and the change in θ^* due to changes in precision choices caused by an increase in τ_θ (captured by $\frac{\partial\theta^*}{\partial\tau^*} \times \frac{d\tau^*}{d\tau_\theta}$).

We now analyze $\frac{d\theta^*}{d\tau_\theta}$ in more detail. Note that

$$\frac{\partial\theta^*}{\partial\tau_\theta} = -2\frac{\tau}{\tau_\theta}\frac{\partial\theta^*}{\partial\tau^*}$$

Thus, $\frac{\partial\theta^*}{\partial\tau_\theta}$ and $\frac{\partial\theta^*}{\partial\tau^*}$ are always of opposite signs. Moreover, from Proposition 5 we know that for all $\mu_\theta \notin \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$, $\frac{d\tau^*}{d\tau_\theta} < 0$. Therefore, we conclude that as long as $\mu_\theta \notin \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$ then

$$sgn\left(\frac{d\theta^*}{d\tau_\theta}\right) = sgn\left(\frac{\partial\theta^*}{\partial\tau_\theta}\right)$$

Next, suppose that $\mu_\theta \in \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$. In this case, according to Proposition 5, $\frac{d\tau^*}{d\tau_\theta} > 0$ and hence $\frac{\partial\theta^*}{\partial\tau_\theta}$ and $\frac{\partial\theta^*}{\partial\tau^*} \times \frac{d\tau^*}{d\tau_\theta}$ are of opposite signs. Therefore, in this case we have to compare the magnitudes of these derivatives to determine whether and when $\frac{\partial\theta^*}{\partial\tau_\theta}$ is positive or negative. We start by noting that

$$\frac{d\tau^*}{d\tau_\theta} = \frac{d\tau_i^*}{d\tau_\theta} = \frac{1}{1 - \left.\frac{\partial\tau_i^*}{\partial\tau}\right|_{\tau=\tau^*}} \left.\frac{\partial\tau_i^*}{\partial\tau_\theta}\right|_{\tau_i=\tau^*}. \tag{10}$$

More precisely, the total effect of a change in τ_θ on the unique equilibrium precision choice τ^* is equal to the change in investor i's precision choice τ_i^* holding other investors' precision choices constant $\left(\frac{\partial\tau_i^*}{\partial\tau_\theta}\right)$ and evaluated at the equilibrium precision level τ^*, times a multiplier effect due to the adjustment in precision choices of other investors. Now,

$$\begin{aligned}
\left.\frac{\partial\tau_i^*}{\partial\tau_\theta}\right|_{\tau_i=\tau^*} &= \frac{-\left[\frac{1}{\tau^*+\tau_\theta} - \frac{1}{2\tau_\theta} + \frac{1}{2}\left(\theta^*-\mu_\theta\right)^2 + \frac{\tau_\theta}{\tau^*}\left(\theta^*-\mu_\theta\right)^2 + \tau_\theta\left(x^*-\mu_\theta\right)\frac{\partial\theta^*}{\partial\tau_\theta}\right]}{\left[\frac{1}{2\tau^*} + \frac{1}{\tau^*+\tau_\theta} - \frac{1}{2}\left(\frac{\tau_\theta}{\tau^*}\right)^2\left(\theta^*-\mu_\theta\right)^2\right] + C''\left(\tau^*\right)} \\
&< \frac{-\left[\frac{1}{\tau^*+\tau_\theta} - \frac{1}{2\tau_\theta}\right]}{\left[\frac{1}{2\tau^*} + \frac{1}{\tau^*+\tau_\theta} - \frac{1}{2}\left(\frac{\tau_\theta}{\tau^*}\right)^2\left(\theta^*-\mu_\theta\right)^2\right]}
\end{aligned}$$

since the numerator is maximized when $\theta^* = \mu_\theta$ and $C''(\tau) > 0$. Since $|\theta^* - \mu_\theta|$ is increasing as μ_θ moves away from $\frac{1}{2}$ and μ_θ is restricted to belong to $\left(\widehat{\mu}^-, \widehat{\mu}^+\right)$, we can show that

$$\left.\frac{\partial\tau^*}{\partial\tau_\theta}\right|_{\tau_i=\tau^*} < \frac{{\tau^*}^2\left(\tau^*-\tau_\theta\right)}{\tau_\theta\left(3{\tau^*}^2+\tau_\theta^2\right)}$$

where we used the fact that $\left(\theta^* - \widehat{\mu}^-\right)^2 < \frac{(\tau^*-\tau_\theta)}{\tau_\theta((\tau^*-\tau_\theta))}$ (see the proof of Proposition 5)

Now, recall that we assumed that the lower bound for the precision choice of players, $\underline{\tau}$, is such that the multiplier effect is less than 6.[36] This implies that

$$\frac{d\tau^*}{d\tau_\theta} < 6 \times \frac{\tau^{*^2}\left(\tau^* - \tau_\theta\right)}{\tau_\theta\left(3\tau^{*^2} + \tau_\theta^2\right)}$$

With the above observations we are ready to determine the sign of $\frac{d\theta^*}{d\tau_\theta}$ when $\mu_\theta \in \left(\widehat{\mu}^-, \widehat{\mu}^+\right)$. We will consider separately two cases: (1) $\mu_\theta \in \left(\widehat{\mu}^-, \frac{1}{2}\right)$, and (2) $\mu_\theta \in \left(\frac{1}{2}, \widehat{\mu}^+\right)$. Recall that when $\mu_\theta < \frac{1}{2}$ then $\frac{\partial\theta^*}{\partial\tau^*} < 0$, hence it follows that

$$\begin{aligned}
\frac{\partial\theta^*}{\partial\tau_\theta} + \frac{\partial\theta^*}{\partial\tau^*}\frac{d\tau^*}{d\tau_\theta} &> -2\frac{\tau^*}{\tau_\theta}\frac{\partial\theta^*}{\partial\tau^*} + 6\frac{\tau^{*^2}\left(\tau^* - \tau_\theta\right)}{\tau_\theta\left(3\tau^{*^2} + \tau_\theta^2\right)}\frac{\partial\theta^*}{\partial\tau^*} \\
&= -2\frac{\tau^*}{\tau_\theta}\frac{\partial\theta^*}{\partial\tau^*}\left[1 - 6\frac{\tau^*\left(\tau^* - \tau_\theta\right)}{2\left(3\tau^{*^2} + \tau_\theta^2\right)}\right] \\
&> -2\frac{\tau^*}{\tau_\theta}\frac{\partial\theta^*}{\partial\tau^*}\left[1 - 6\frac{1}{6}\right] = 0
\end{aligned}$$

Similarly, when $\mu_\theta > \frac{1}{2}$ then $\frac{\partial\theta^*}{\partial\tau^*} > 0$ and so it can be shown that

$$\frac{\partial\theta^*}{\partial\tau_\theta} + \frac{\partial\theta^*}{\partial\tau^*}\frac{d\tau^*}{d\tau_\theta} < 0$$

The above inequalities in turn imply that, under our assumptions on parameters,

$$sgn\left(\frac{d\theta^*}{d\tau_\theta}\right) = sgn\left(\frac{\partial\theta^*}{\partial\tau_\theta}\right)$$

Going back to the expression for $\frac{d\Pr(\theta \geq \theta^*)}{d\tau_\theta}$, note that from lemma $A.6$ we know that

$$sgn\left(\frac{\partial\theta^*}{\partial\tau_\theta}\right) = sgn\left(\theta^* - \mu_\theta\right)$$

The result follows immediately then from the fact that

$$sgn\left(\frac{d\theta^*}{d\tau_\theta}\right) = sgn\left(\frac{\partial\theta^*}{\partial\tau_\theta}\right) = sgn\left(\theta^* - \mu_\theta\right)$$

■

[36] The magnitude of the multiplier effect depends on the slope of the best response function $\tau_i^*(\tau)$ and potentially can take any value in $(0, \infty)$. However, by choosing appropriately high $\underline{\tau}$, one can ensure that the multiplier effect is not only finite but also control its absolute magnitude. Our maintained assumption (made in Section 3.2) is that $\underline{\tau}$ is high enough so that the multiplier effect is smaller than 6. See also Footnote 14.

Derivation of Equation (6) Differentiate the ex-ante utility with respect to τ_θ and note that

$$\frac{dx^*}{d\tau_\theta} f_\tau\left(x^*\right)\left[T \Pr\left(\theta < \theta^* | x^*\right) - (1-T) \Pr\left(\theta > \theta^* | x^*\right)\right] = 0$$

and

$$-\int_{-\infty}^{\theta^*} \int_{x^*}^{\infty} \frac{\partial}{\partial \tau} T f_\tau\left(x|\theta\right) g_{\tau_\theta}\left(\theta\right) dx d\theta - \int_{\theta^*}^{\infty} \int_{-\infty}^{x^*} \frac{\partial}{\partial \tau}\left(1-T\right) f_\tau\left(x|\theta\right) g_{\tau_\theta}\left(\theta\right) dx d\theta - C'\left(\tau\right) = 0.$$

Using the above observations and simplifying the terms that include $\frac{d\theta^*}{d\tau_\theta}$ we obtain

$$\begin{aligned}\frac{dU}{d\tau_\theta} = & -\int_{-\infty}^{\theta^*} \frac{\partial}{\partial \tau_\theta} T\left[1 - F_{\tau^*}\left(x^*|\theta\right)\right] g_{\tau_\theta}\left(\theta\right) dx d\theta + \int_{\theta^*}^{\infty} \frac{\partial}{\partial \tau_\theta}\left(1-T\right)\left[1 - F_{\tau^*}\left(x^*|\theta\right)\right] g_{\tau_\theta}\left(\theta\right) dx d\theta \\ & -\frac{d\theta^*}{d\tau_\theta}\left(1 - F\left(x^*|\theta^*\right)\right) g_{\tau_\theta}\left(\theta^*\right)\end{aligned}$$

which is the equation in the text.