Information Structure and Comparative Statics in Simple Global Games

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Very Preliminary and Incomplete

Abstract

This paper analyzes the effect of changing information structure in simple global games. I provide a complete characterization of the effect of private and public precision on the fundamental threshold. Interestingly, I find that away from the limiting case of infinitely precise signals, the threshold can be non-monotonic in both precision of public and private information. I provide intuition behind this result and argue that this has important policy consequences.

1 Introduction

Global games are a popular way of modeling coordination problems such as bank runs, currency crisis or sovereign debt crises. This popularity stems from the fact that while underlying coordination game features multiple equilibria, the global games, by distorting information structure, restores uniqueness of equilibrium. The key idea of this approach is that incomplete information impedes coordination since in this case agents have to rely on their private information when choosing their actions. As shown first by Carlson and Damme (1993), in the limit, as private information become infinitely precise, the set of equilibria becomes a singleton. Moreover, under mild conditions, this equilibrium is independent of the initial perturbation applied to the underlying complete information game (Frankel, Morris, and Pauzner (2003)).

While the limiting result is important, in reality individuals rarely have an access to extremely precise information. Thus, from an applied perspective is important to understand how the unique global game equilibrium is affected by a change in the precision of private and public information available to agents. Indeed, there is now a large literature that studies how changes in information structure in different types of coordination games with incomplete information affects equilibrium play (see for example Angeletos and Pavan (2007), Colombo, Femminis, and Pavan (2012) or Iachan and Nenov (2014)).[1] In this paper, I contribute to

[1] See for example Rochet and Vives (2004), Morris and Shin (2004) or Iachan and Nenov (2014) for global games; Angeletos and Pavan (2007) or Colombo, Femminis, and Pavan (2012) for the analysis in the quadratic-Gaussian setups (often referred to as beauty contest models).

this literature by providing a full characterizing the comparative statics result with respect to information parameters in global games away from the limit of perfectly informative signal.

I study what I call "simple global games", i.e., binary global games where the payoff difference between actions is a step function, agents hold a proper common prior, and the information structure is either "Gaussian" or "uniform". Such specifications of global games has been widely used in an applied global games literature.[2] I show that the comparative statics with respect to precision of information, away from the limit, depend on three effects which I call a "mean effect", a "dispersion effect" and an "aggregate effect". The "mean effect" captires the effect that a change in the precision of information has on the mean of the marginal agent's posterior belief, i.e. of the agent who is indifferent between taking each of the two actions. The "dispersion effect" is in turn the effect that a change in precision of public or private information has on the posterior belief's variance of the marginal agent. Finally, the "aggregate effect" is the change in proportion of agent receiving a signal less than the threshold signal.

The above three effects determine the behavior of the regime change threshold at a given precision of private and public information. Using the above decomposition I fully characterize how an incremental increase in the precision of private or public information affects the unique equilibrium away from the limit of perfectly informative signals. In particular, I provide exhaustive conditions, that depend only on the exogenous parameters of the model, under which a small increase in the precision of private or public information leads to an increase or decrease in the probability of agents coordinating on the risky action. While it is important to understand the local comparative statics with respect to parameters governing information structure, I also analyze the global behavior of the regime change threshold, i.e. the evolution of equilibrium as we continuously vary precision of information. I provide a full description of the global behavior of equilibrium and, in particular, I characterize conditions under which equilibrium threshold is a non-monotone function of information parameters. I find that when the prior belief is low it is possible that the equilibrium threshold is first increasing and then decreasing. Similarly, if the prior belief is high, it is possible that the equilibrium threshold is first decreasing and then increasing. I show that these are the only types of non-monotonicity that can arise in simple global games.

The above observations have potentially important implications for policy makers. In particular, in the case where reducing the initial uncertainty (i.e., increasing precision of the prior) is costly to policy makers, it might be optimal for the policy maker not to increase the precision of information at all since a small increase can actually have detrimental effect on the economy. Similarly, the above conditions can be used to determine when the policy maker should try to decrease initial uncertainty following an increase in the amount of information available to agents and when to increase it.

In the final part of the paper I introduce public information focusing on the setup with "Gaussian" noise. The results mentioned above are necessary building steps for analysis of the effect of the precision of an explicit public signal on equilibrium. In turns out that the presence of public information substantially complicates the analysis and renders the approach

[2] See for example Eisenbach (2013) or Rochet and Vives (2004) for applications to banking crises, Corsetti, Guimaraes, and Roubini (2006) and Morris and Shin (2006) for soveriegn debt crisis, Morris and Shin (2004) and Szkup (2013) for corporate debt crises, or Corsetti, Dasgupta, Morris, and Shin (2004) and Dasgupta (2007) for currency crises just to mentions a few.

used to characterize comparative statics in the absence of public signal inapplicable. Thus, in this part of paper I utilize a different approach. This approach is based on the observation that the equilibrium regime threshold, as a function of public signal realization, exhibits a special form of an asymmetry Using this observation I provide a complete characterization of an incremental increase in the precision of private and public information in this extended setup. Finally, I show numerically, that the non-monotonicity of the equilibrium regime change threshold carries over to the case with public information.

This paper contributes to a large and growing literature on the theory of global games and, more broadly, coordination games with incomplete information. The global games have been initially developed by Carlson and Damme (1993) and then extended by Frankel, Morris, and Pauzner (2003) to wider range of setups.[3] Since then the global games have been extended along many dimensions and the robustness of the uniqueness result have been analyzed extensively. I contribute to this literature by characterizing the comparative statics with respect to information parameters in simple global games.

I am not the first person to consider the comparative statics with respect to the precision in the simple global games. The early work on this issue includes Metz (2002) and Bannier and Heinemann (2005). Bannier and Heinemann (2005) analyze the optimal level of public and private precision in a simplified version of currency attack model developed by Morris and Shin (1998). They were the first one to report that the equilibrium threshold may not be monotone with respect to the precision of public information. My paper extends their results in several dimensions. First of all, I providing exhaustive conditions under which equilibrium regime change threshold is non-monotone in the precision of both the prior belief as well as private signal under no assumptions on parameters. Second, I provide a detailed intuition behind the forces that lead to this monotonicity that applies beyond the Gaussian noise structure. Finally, I also consider a framework with both private and public signals. More recently Iachan and Nenov (2014) provide a general analysis of comparative statics with respect to the informativeness of private signals in global games. In contrast to this paper the allow the payoffs to be functions of the underlying economic parameter. While the setup they analyze is more general than the setup in this paper, the added complexity requires them to focus either on the environment with improper prior or a limiting case where the precision of private signal tends to infinity.[4]

2 Framework

There is a continuum of agents indexed by i with $i \in [0, 1]$. The economy is characterized by a parameter θ, which represents the strength of the economy's fundamentals, that is a higher θ represent stronger fundamentals. I assume that θ in distributed according to a distribution G. Let a_i be agent i's action and denote by $A = \int a_i di$ the aggregate action in the economy. The utility function of each agent is given by $u(a_i, A, \theta)$. Before deciding which action to take each agent observes a private signal x_i. Finally, assume that the economy can be in one of the

[3] See also the recent paper by Oury (2014) for the extension to the multi-dimensional global games.

[4] The paper takes the coordination stage as given and treats changes in private and public information as comparative statics exercises. Szkup and Trevino (2009), building on the results in this paper, analyze an environment where agents' can explictly choose the precision of private information choices.

two regimes, $R \in \{0,1\}$ and initially is in regime 0. The regime changes if $f(\theta, A) \geq 0$ and stays the same if $f(\theta, A) < 0$.

2.1 Simple global game

I call a global game "simple" if the payoff and information structure satisfy the following assumptions on payoff and information structure.

Assumption 1 (Payoff Structure) *The payoff structure satisfies:*

- $a_i \in \{a_1, a_2\}$
- $u(a_2, A, \theta) - u(a_1, A, \theta) = \begin{cases} H \text{ if } f(A,\theta) \geq 0 \\ L \text{ if } f(A,\theta) < 0 \end{cases}$
- $f(\theta, A) = p_1 A + p_2 \theta - 1$ *with* $p_1, p_2 > 0$

Assumption 2 (Information Structure) *The information structure satisfies:*

- $\theta \sim N\left(\mu_\theta, \tau_\theta^{-1}\right)$ *and* $x_i = \theta + \tau_x^{-1/2} \varepsilon_i$, $\varepsilon_i \sim N(0,1)$, *or*
- $\theta \sim unif\left[-\tau_\theta, 1+\tau_\theta\right]$ *and* $x_i = \theta + \varepsilon_i$, $\varepsilon_i \sim unif\left[-\tau_x, \tau_x\right]$

Thus, I define a simple global game as a game where the actions are binary, the difference between payoffs from taking action 1 and action 2 is constant in each regime and the information structure is either "Gaussian" or "uniform" A vast majority of global games fall under this definition (e.g. see Morris and Shin (2003) or Veldkamp (2011)).

3 Equilibrium

It is well known that (under additional assumptions on the information structure) the model has unique equilibrium (Morris and Shin (1998) and Morris and Shin (2004)). The next proposition, stated without the proof, summarizes these conditions.

Proposition 1 1. *In the case of Gaussian information structure, a simple game has unique equilibrium as long as* $\frac{\tau_x^{1/2}}{\tau_\theta} > \frac{1}{\sqrt{2\pi}}$

2. *In the case of uniform information structure, a simple global game has unique equilibrium as long as* $2\tau_x > \tau_\theta$.

In both cases the equilibrium is characterized by a pair of thresholds $\{\theta^*, x^*\}$ *such that the regime changes iff* $\theta > \theta^*$ *and the agent chooses action 2 if he observes a signal* $x_i > x^*$.

The goal of the rest of this paper is to analyze how the equilibrium is affected by the particular features of the information structure such as the informativeness of the signals.

The equilibrium is determined by two equations. First, the payoff indifference condition, that says that if the regime changes if and only if $\theta \geq \theta^*$ then an agent who received a critical signal x^* is indifferent between acting, action a_2, and not acting, action a_1, i.e.

$$H \Pr\left(\theta \geq \theta^* | x^*; \tau_x, \tau_\theta\right) = L \Pr\left(\theta < \theta^* | x^*; \tau_x, \tau_\theta\right)$$

or, written more succinctly,

$$P\left(\theta^*, x^*; \tau_x, \tau_\theta\right) = 0.$$

The second equilibrium condition is the critical mass condition which says that when fundamentals are equal to θ^* then the mass of agents taking action a_2 is exactly equal to the mass of agents needed for the regime change:

$$p_1 \Pr\left(x_i \leq x^* | \theta^*; \tau_x, \tau_\theta\right) = 1 - p_2\theta^*$$

or, more succinctly as

$$M\left(\theta^*, x^*; \tau_x, \tau_\theta\right) = 0$$

In the remainder of the paper I investigate the properties of this equilibrium. In particular I investigate how the threshold x^* and θ^* changes with the precision of private and public information captured by τ_x and τ_θ, respectively.

Definition 1 *Let* $\gamma \equiv \frac{L}{H+L}$.

Parameter $\gamma \in (0,1)$, which can be interpreted as the relative attractiveness of acting, will turn out to be the key parameter when determining the comparative statics results.

3.1 Preliminaries

Let τ be a parameter describing information structure. It could be the precision of private or public information in the Gaussian information structure, or the parameter governing the error term in the uniform information structure. However, for now we abstract from the exact nature of this parameter and investigate how a change in any parameter of the information structure affects the equilibrium regime change threshold θ^*.

A change in τ affect the equilibrium by changing the equilibrium conditions. To see that recall that the equilibrium is determined by to equations:

$$\begin{aligned} M\left(x^*, \theta^*; \tau\right) &= 0 \\ P\left(x^*, \theta^*; \tau\right) &= 0 \end{aligned}$$

Totally differentiating these two equations and solving for $d\theta^*/d\tau$ we obtain:

$$\frac{d\theta^*}{d\tau} = \frac{\frac{M_\tau}{M_\theta} - \frac{M_\theta}{M_x}\frac{P_\tau}{P_x}}{1 - \frac{M_x}{M_\theta}\frac{P_\theta}{P_x}},$$

or

$$\frac{d\theta^*}{d\tau} = -\frac{\frac{\partial\theta^*}{\partial\tau} + \frac{\partial\theta^*}{\partial x^*}\frac{\partial x^*}{\partial\tau}}{1 - \left[\frac{\partial\theta^*}{\partial x^*}\right]_M \left[\frac{\partial x^*}{\partial\theta^*}\right]_P}, \tag{1}$$

where the subscript on the square brackets in the numerator indicates the equation from which the partial derivative was computed.

A change in information parameter potentially affects the regime change threshold through three channels: (1) the "mean effect" - a change in the posterior mean of the marginal agent (i.e., the agent who is indifferent between acting and not acting), (2) the "dispersion effect" - a change in the posterior variance of the marginal agent, and (3) the "aggregate effect" - a change in the mass of agents acting (or, in other word, a change in the probability that an agent receives a signal higher than the threshold signal x^* for given θ). The term $[\partial\theta^*/\partial x^*]\,[\partial x^*/\partial\tau]$ captures both the "mean effect" and the "dispersion effect", while $\partial\theta^*/\partial\tau$ captures the "aggregate effect".[5] Using this simple insight, I next determine the relative strength of these effects when varying τ_x, the parameter that affects the "precision" of private signals, and τ_θ, the parameter that affects the "precision" of the prior belief.

4 A change in the precision of private information

4.1 Gaussian Information Structure: Local Behavior

I consider first the effect of an increase in the precision of private signals for the case of Gaussian Information Structure. In this case the payoff indifference conditions is given by:

$$1-\Phi\left(\frac{\theta^*-\frac{\tau_x x^*+\tau_\theta\mu_\theta}{\tau_x+\tau_\theta}}{(\tau_x+\tau_\theta)^{-1/2}}\right)=\gamma$$

Consider an increase in τ_x. From the above equation it follows that the mean effect, i.e. the change in x^* implied by a change in the marginal agents' posterior mean keeping the posterior variance $(\tau_x+\tau_\theta)^{-1/2}$ constant, is given by

$$\left.\frac{\partial x^*}{\partial\tau_x}\right|_{Mean}=-\frac{\tau_\theta}{\tau_x^2}\left(\theta^*-\mu_\theta\right)-\frac{\tau_\theta}{\tau_x^2}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}(\gamma)$$

Thus we see that the mean effect is negative when μ_θ is low and when γ is high. The intuition behind that is simple. A change in private precision increases the weight of the threshold signal in the marginal agents' posterior belief and decreases the weight of the prior. When μ_θ is low and γ is high the threshold signal is high x^* is high and thus an increase in τ_x shifts agent's posterior belief up. Thus, agents who receive signals x^* becomes more confident that the risky action will be successful and they switch to taking risky action. As a consequence the threshold signal has to decrease.

In the similar fashion, we can find the "dispersion effect", i.e. the effect that a change in posterior variance, caused by varying τ_x, has on x^*:

$$\left.\frac{\partial x^*}{\partial\tau_x}\right|_{Dispersion}=-\frac{1}{2}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}(\gamma)$$

We see that dispersion depends on γ and (precision levels τ_x and τ_θ) but not on μ_θ. To understand this suppose that γ is high. Then agents are particularly worried of committing a mistake and undertaking a risky action when the risky action will be unsuccessful since this will be associated with a large negative payoff. To avoid this, they sets x^* such that the threshold θ^* is smaller than the mean of his posterior belief about θ. This implies that for signals that agent does invest, the probability that investment is successful is high which compensates the

[5] Under the conditions that ensure uniqueness the denominator is always positive when determining the effect of a change in the parameters of information structure it is enough to focus on the relative strength of the mean, dispersion and aggregate effect.

agent for the possibility of the costly bad outcome. But when τ_x increases the tails of the posterior distribution shrink. In particular, holding the posterior mean constant, when the posterior mean is higher than θ^*, the probability mass below θ^* decreases implying that the possibility of making the costly error is smaller. This in turn encourages the agent to decrease his threshold signal x^*. Analogous logic holds when γ is low.

The change in x^* affects then θ^* through the critical mass condition. The effect of higher x^* on θ^* implied by the critical mass condition is captured by

$$\frac{\partial \theta^*}{\partial x^*} = \frac{\tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)}{1 + \tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)}$$

implying that a higher x^* always leads to higher regime change threshold θ^*. Therefore, the initial contribution of the mean and dispersion effect to a change in θ^* is given by

$$\frac{\partial \theta^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_x} = -\frac{\tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)}{1 + \tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)} \left[\frac{\tau_\theta}{\tau_x^2}\left(\theta^* - \mu_\theta\right) + \frac{1}{\tau_x^2}\frac{\tau_\theta + \frac{1}{2}\tau_x}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}\left(\gamma\right)\right]$$

Finally, I compute the aggregate effect captured by $\partial\theta^*/\partial\tau_x$. Using the implicit function theorem in the critical mass condition we get

$$\frac{\partial \theta^*}{\partial \tau_x} = \frac{1}{2}\frac{\tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)\frac{(x^* - \theta^*)}{\tau_x}}{1 + \tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)}$$

The sign of the distance effect depends on the relative position of x^* and θ^*. If $x^* < \theta^*$ then it follows that an increase in τ_x increases the probability that conditional on $\theta = \theta^*$ a signal received an agent is greater than x^* increasing the total proportion of agents investing at θ^*. But then it follows that at θ^* the mass of agents raking risky action is strictly greater than the mass needed for a regime to change. As a consequence, it has to be the case that θ^* decreases. The opposite is true when $x^* > \theta^*$.[6]

Substituting these expression into the numerator of equation (1) we obtain:

$$\frac{d\theta^*}{d\tau_x} = \frac{1}{2}\frac{\frac{1}{\tau_x}\left(\theta^* - \mu_\theta\right) + \frac{1}{\tau_x}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}\left(\gamma\right)}{1 - \frac{\tau_x^{1/2}}{\tau_\theta}\frac{1}{\phi(\Phi^{-1}(\theta^*))}}$$

where the denominator is negative whenever $\tau_x^{1/2}/\tau_\theta > \frac{1}{\sqrt{2\pi}}$. From the above calculations we make the following observation:

[6] Substituting the expression for x^* into the above derivative we obtain

$$\frac{\partial \theta^*}{\partial \tau_x} = \frac{1}{2}\frac{\tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)}{1 + \tau_x^{1/2} \phi\left(\frac{x^* - \theta^*}{\tau_x^{-1/2}}\right)} \left[\frac{\tau_\theta}{\tau_x^2}\left(\theta^* - \mu_\theta\right) + \frac{\sqrt{\tau_x + \tau_\theta}}{\tau_x^2}\Phi^{-1}\left(\gamma\right)\right]$$

This expression makes it clear that since x^* depends on μ_θ and γ these two parameters also determine the sign of the distance effect.

Remark 2 *The mean effect and the dispersion effects dominate distance effect. Therefore, it is a change in* x^* *that determines how* θ^* *responds to a change in* τ_x*.*

We are now ready to state the first result.[7]

Proposition 2 (The local effect of τ_x) *Consider a small change in the precision of private information. Then,*

1. *If* $\mu_\theta < \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ *then* $\frac{d\theta^*}{d\tau_x} > 0$
2. *If* $\mu_\theta = \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ *then* $\frac{d\theta^*}{d\tau_x} = 0$
3. *If* $\mu_\theta > \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ *then* $\frac{d\theta^*}{d\tau_x} < 0$

where

$$\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta) = \Phi\left(\sqrt{\frac{\tau_x}{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)\right) + \frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)$$

and

$$\frac{\partial \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)}{\partial \gamma} > 0$$

Proof. See the Appendix. ■

The above proposition specifies when it is the case that a marginal change in τ_x leads to an increase or decrease in θ^* in terms of exogenous parameters. In particular, it states that when the mean of the public belief is low, $\mu_\theta < \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$, then an increase in the private precision leads to a decrease in the threshold for regime change. The opposite is true when μ_θ is high, $\mu_\theta > \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$. This result is driven by the behavior of the mean effect. As explained above an increases in τ_x affects the mean of the posterior belief of an agent by increasing the weight the agent puts on the private signal and decreasing the weight he puts on the public signal. When $\mu_\theta < \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ then x^* is greater than μ_θ and so this shifts agent's posterior belief up. As a consequence, he decreases his threshold which, in light of remark 1, leads to a decrease in θ^*. The opposite is true when $\mu_\theta > \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$.

I need to check if x^* *is greater than* μ_θ *at* μ^{τ_x}

4.2 Gaussian Information Structure: Global Behavior

Proposition 3 provides an easily verifiable condition under which a marginal increase in private information leads to a decrease or increase in the regime change threshold. Note, however, that the condition itself depends on the current precision of private and public information. This suggests that it is possible that while an initial increase in τ_x leads to an decrease in the regime threshold a further change may actually increase the threshold, and vice versa. I now investigate whether this is possible. Indeed, for some values of parameters this is the case. Proposition 3 provides characterizations when the regime change threshold in monotone or non-monotone function of precision τ_x.

[7] A weaker version of this result has been established by Metz (2002) and Bannier and Heinemann (2005).

Proposition 3 (The path of θ^* as the function of τ_x) *Let $\underline{\tau}_x$ be the precision of private information that agents are initially endowed with.*

1. *Suppose that $\gamma > \frac{1}{2}$.*
   (a) *If $\mu_\theta \leq \gamma$ then θ^* is increasing for all $\tau_x > \underline{\tau}_x$;*
   (b) *If $\mu_\theta \in (\gamma, \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta))$ then θ^* is initially decreasing and then increasing in τ_x*
   (c) *If $\mu_\theta \geq \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta)$ then θ^* is decreasing for all $\tau_x > \underline{\tau}_x$.*
2. *Suppose that $\gamma = 1/2$.*
   (a) *if $\mu_\theta < \frac{1}{2}$ then θ^* is increasing for all $\tau_x > \underline{\tau}_x$;*
   (b) *if $\mu_\theta = \frac{1}{2}$ then θ^* is constant in τ_x;*
   (c) *if $\mu_\theta > \frac{1}{2}$ then θ^* is decreasing in τ_x;*
3. *Suppose that $\gamma < \frac{1}{2}$.*
   (a) *If $\mu_\theta < \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta)$ then θ^* is increasing for all $\tau_x > \underline{\tau}_x$.*
   (b) *If $\mu_\theta \in (\gamma, \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta))$ then θ^* is initially increasing and then decreasing in τ_x*
   (c) *$\mu_\theta \geq \gamma$ then θ^* is decreasing for all $\tau_x > \underline{\tau}_x$.*

Proof. See Appendix.

Figure 1: Non-monotonicity

■

The above proposition states that, under some conditions, θ^* can be a non-monotonic function private signals's precision. To understand why this is the case consider recall that the direction of a change in θ^* in response to a change in τ_x was driven by the sign of the mean effect. Thus, to understand the behavior of θ^* we have to understand how the mean effect changes as we keep changing τ_x.

For concreteness consider the case when $\gamma > \frac{1}{2}$ and suppose that $\gamma < \mu_\theta < \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta)$. It follows that in this case, when we consider a small increase in the precision of private information from $\underline{\tau}_x$ the mean effect is negative and hence θ^* decrease. However, a fall in θ^* increases the mean effect making it less negative. This is because a lower θ^* implies lower x^* and the mean effect is driven by the difference $x^* - \mu_\theta$. As this difference decreases the mean effect becomes weaker. As τ_x increases further and further we arrive at a x^* keeps decreasing. If μ_θ is not too low, i.e. $\mu_\theta \in (\gamma, \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta))$ then at one point a further increases in τ_x pushes x^* below μ_θ. As a result the mean effect changes sign from negative to positive. Once this happens, a further increase in τ_x will result in the positive mean effect leading to an increase in θ^*.[8] The analogous logic applies when $\gamma < \frac{1}{2}$.

[8] It is interesting to point that as mean effect increases the distance effect decrease. However, as shown above the mean and the dispersion effect always dominate the distance effect.

4.3 Uniform Information Structure

Consider now the "uniform" information structure. In this case the marginal agent's posterior belief is simply given by

$$\theta|x^* \sim unif\left[x^* - \tau_x, x^* + \tau_x\right]$$

We see that τ_x affects only the variance but not the mean of the posterior belief. Thus, in the case of the "uniform information structure" the "mean effect" is absent.

With the uniform information structure, the equilibrium conditions are given by:

$$\begin{aligned} 1 - \frac{\theta^* - x^* + \tau_x}{2\tau_x} &= \gamma \\ 1 - \frac{x^* - \theta^* + \tau_x}{2\tau_x} &= 1 - \theta^* \end{aligned}$$

In order to compute the dispersion effect compute x^* using the payoff indifference condition:

$$x^* = \theta^* - \tau_x (1 - 2\gamma)$$

Thus, the "dispersion effect" is given by

$$\frac{\partial x^*}{\partial \tau_x} = -(1 - 2\gamma)$$

Together with the effect of a change in x^* on θ^* this yields:

$$-(1 - 2\gamma) \frac{\frac{1}{2\tau_x}}{1 + \frac{1}{2\tau_x}}$$

Next, we can compute the "aggregate effect" which is given by

$$(1 - 2\gamma) \frac{\frac{1}{2\tau_x}}{1 + \frac{1}{2\tau_x}}$$

Thus, we see that in this case the "aggregate effect" and "dispersion effect" cancel out and a change in τ_x has no effect on the regime change threshold θ^*. Thus, we established the following result.

Proposition 4 *Consider a simple global game whit uniformly distributed prior belief and private signals. Then the regime change threshold θ^* does not depend on τ_x, the noise parameter of private signals. It follows that θ^* stays constant as we vary τ_x.*

The above proposition establishes that in the case of uniform information structure, a change in precision of private signal has no effect on the equilibrium threshold which stands with a contrast to the case of Gaussian information structure. Thus, we conclude that comparative statics with respect to parameters of information structure depend crucially on the information structure itself.

5 Comparative Statics with Respect to Initial Uncertainty

5.1 Gaussian Information Structure: Local Behavior

In this section I investigate the conditions under which a marginal change in τ_θ increases and decreases the regime change threshold θ^*. The mean effect associated with a change in τ_θ is given by

$$\left.\frac{\partial x^*}{\partial \tau_x}\right|_{Mean} = \frac{x^* - \mu_\theta}{\tau_x + \tau_\theta}$$

while the "dispersion effect" is given by

$$\left.\frac{\partial x^*}{\partial \tau_x}\right|_{Dispersion} = -\frac{1}{2}\frac{1}{\tau_x}\left[\theta^* - \frac{\tau_x x^* + \tau_\theta \mu_\theta}{\tau_x + \tau_\theta}\right]$$

From the above expressions we see that the "mean effect" is positive (i.e., tends to increase θ^*) when, before the change in precision of public information, the x^* is greater than μ_θ and is negative (i.e., tends to decrease x^*) when the x^* is smaller than μ_θ. The intuition behind this result is analogous to the intuition provided in section 4.1. In particular, an increase in τ_θ increases the weight given to the public belief μ_θ. When x^* is greater than μ_θ then the mean of the marginal agent's posterior belief shifts down which makes him believe that, holding θ^* constant, the regime will change with smaller probability. As a result he prefers to take action 1 rather than action 2. But this implies that the threshold signal has to increase. An analogous intuition applies when x^* is smaller than μ_θ.

Regarding the "dispersion effect", note that this effect is exactly the same as in the case of a change in precision of private information. In particular, the "dispersion effect" depends on the relative position of the regime change threshold and the posterior mean calculated the critical signal. The reason behind that is simple. An increase in τ_θ decreases the variance of the posterior belief which reduces the tails of the posterior belief. This in turn increases the probability that $\theta > \theta^*$ when θ^* is smaller then the mean of the posterior belief $\frac{\tau_x x^* + \tau_\theta \mu_\theta}{\tau_x + \tau_\theta}$ and decreases this probability when the opposite is true. In the first case, this leads to a decrease in x^* while in the second case it drives x^* up.

Finally, note that a change in the initial uncertainty τ_θ has no effect on the proportion of agents taking a risky action holding equilibrium thresholds, x^* and θ^*, constant. This is because conditional on θ, a change in τ_θ does not affect the probability that an agent receives a signal $x_i > x^*$.

Proposition 5 *Consider a small change in the precision of private information. Then,*

1. *If* $\mu_\theta < \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ *then* $\frac{d\theta^*}{d\tau_\theta} > 0$;
2. *If* $\mu_\theta = \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ *then* $\frac{d\theta^*}{d\tau_\theta} = 0$
3. *If* $\mu_\theta > \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ *then* $\frac{d\theta^*}{d\tau_\theta} < 0$

where

$$\widehat{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta) = \Phi\left(\frac{\frac{1}{2}\tau_\theta + \tau_x}{\sqrt{\tau_x(\tau_x + \tau_\theta)}}\Phi^{-1}(\gamma)\right) + \frac{1}{2}\frac{1}{\sqrt{\tau_\theta + \tau_x}}\Phi^{-1}(\gamma)$$

and

$$\frac{\partial \widehat{\mu}^{\tau_\theta}}{\partial \gamma} > 0$$

The above proposition states that if $\mu_\theta < \widehat{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ then a marginal increase in the precision of public information will lead to an increase in θ^* while the opposite is true when $\mu_\theta > \widehat{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$. The reason behind this is that when μ_θ is low then the mean effect tends to be positive since a low μ_θ implies a high signal threshold x^*. As μ_θ becomes increases the mean effect become smaller. Since the dispersion effect is constant in μ_θ and therefore, there exists a threshold μ_θ such that below this threshold an increase in τ_θ leads to an increase in θ^* while above this threshold an increase in τ_θ leads to an increase in θ^*.

5.2 Gaussian Information Structure: Global Behavior

Proposition 3 provides an easily verifiable condition under which a marginal increase in the precision of public belief leads to a decrease or an increase in the regime change threshold. Note, however, that, as in the case of precision of private information, this condition itself depends on the current precision of private and public information. This suggests that it is possible that while an initial increase in τ_θ leads to a decrease in the regime threshold a further change may actually increase the threshold, and vice versa. In the next proposition I describe the global behavior of θ^* as a function of τ_θ.

Proposition 6 (The path of θ^* as the function of τ_θ) *Let $\underline{\tau}_\theta$ be the initial precision of public information and $\overline{\tau}_\theta$ be the highest precision of public information that satisfies the uniqueness condition.*

1. *Suppose that $\gamma > \frac{1}{2}$.*
   - (a) *If $\mu_\theta \leq \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta)$ then θ^* is increasing for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$*
   - (b) *If $\mu_\theta \in (\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta), \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta))$ then θ^* is initially increasing and then decreasing in τ_θ.*
   - (c) *If $\mu_\theta \geq \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta)$ then θ^* is decreasing for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$*
2. *Suppose that $\gamma = 1/2$.*
   - (a) *if $\mu_\theta < \frac{1}{2}$ then θ^* is increasing in τ_θ for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$*
   - (b) *if $\mu_\theta = \frac{1}{2}$ then θ^* is constant in τ_θ for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$*
   - (c) *if $\mu_\theta > \frac{1}{2}$ then θ^* is decreasing in τ_θ for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$*
3. *Suppose that $\gamma < \frac{1}{2}$.*
   - (a) *If $\mu_\theta \leq \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta)$ then θ^* is increasing in τ_θ for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$*
   - (b) *If $\mu_\theta \in (\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta), \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta))$ then θ^* is initially decreasing and then increasing in τ_θ*
   - (c) *If $\mu_\theta \geq \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta)$ then θ^* is decreasing in τ_θ for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$.*

Proof. See the Appendix. ■

5.3 Uniform Information Structure

Recall that in the case of uniform information structure we have $\theta \sim unif\left[-\tau_\theta, 1+\tau_\theta\right]$. Note that in the case of the uniform information structure the level of initial uncertainty has no effect on the equilibrium play. Therefore, it follows that the regime change threshold θ^* is constant in τ_θ.

Proposition 7 *Consider a simple global game with uniformly distributed prior belief and private signals. Then the regime change threshold does not depend on τ_θ, the noise parameter of the initial prior. It follows that θ^* stays constant as τ_θ varies.*

6 A Complete Characterization of the Behavior of θ^*

6.1 Gaussian Information Structure: Local Effects

Above I investigated under which conditions the regime change threshold is decreasing in precision of private and public information as well as characterized the dynamics of threshold as a function of these precision levels. In this section I describe how these conditions are related to each other.

In order to achieve the above goal I need to determine relation between $\widehat{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)$ and $\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right)$. This is achieved in the next lemma.

Lemma 1 *Consider $\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right)$ and $\widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)$.*

1. *If $\gamma > \frac{1}{2}$ then $\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right) > \widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)$.*
2. *If $\gamma = \frac{1}{2}$ then $\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right) = \widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)$*
3. *If $\gamma < \frac{1}{2}$ then $\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right) < \widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)$*

Moreover,

$$\frac{\partial \widehat{\mu}^{\tau_x}}{\partial \gamma} > \frac{\partial \widetilde{\mu}^{\tau_\theta}}{\partial \gamma} > 0$$

Together with the previous result, the above lemma tells us that when considering the local behavior of regime change threshold we can divide the space (γ, μ) into four regions depicted in Figure 2: (1) a region where an increase in the precision of both types of signals leads to a decrease in θ^* (when $\gamma > \frac{1}{2}$ and $\mu_\theta \in \left(\widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right), \widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right)\right)$), (2) a region where an increase in the precision of both types of signals leads to a decrease in θ^* (when $\gamma < \frac{1}{2}$ and $\mu_\theta \in \left(\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right), \widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)\right)$), (3) a region where an increase in τ_x leads to an increase in θ^* while an increase in τ_θ leads to a decrease in θ^* ($\mu_\theta > \max\left\{\left(\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right), \widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)\right)\right\}$), and, finally, (4) a region where an increase in τ_x leads to a decrease in θ^* while an increase in τ_θ leads to an increase in θ^* ($\mu_\theta > \min\left\{\left(\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right), \widetilde{\mu}^{\tau_\theta}\left(\gamma, \tau_x, \tau_\theta\right)\right)\right\}$).

Figure 2 indicates also that regions where an increase in τ_x and τ_θ have the same impact on the regime change threshold is relatively small. For the majority states $\{\gamma, \mu_\theta\}$ an increase in the precision of private and public information have the opposite effect. This is an important observation, as it implies that, depending on the state of the economy the government should encourage either provision of additional private information or try to decrease initial uncertainty but rarely do both.

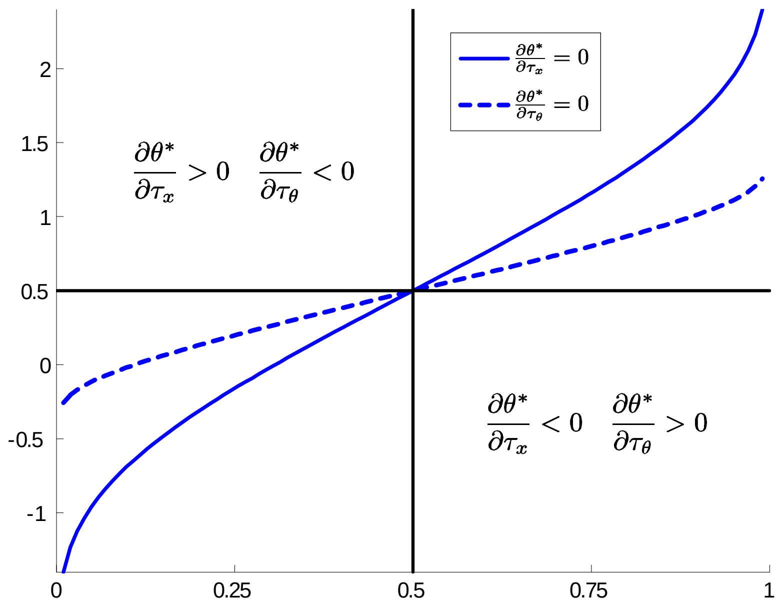

Figure 2: Local variation in θ^*

6.2 Gaussian Information Structure: Global Effects

In this section I consider the global behavior of θ^* and provide a similar characterization to the one provide for local results above. Suppose that the precision of the private information is initially fixed at the level $\underline{\tau}_x$ while the initial uncertainty is fixed at the level $\underline{\tau}_\theta$. Then, the results established in Section 4 imply that for any γ we can divide the space of μ_θ into four regions: (1) a region where θ^* decreases monotonically in τ_x and increases monotonically with τ_θ, (2) a region where θ^* is non-monotone in τ_x and τ_θ, (3) a region where θ^* in non-monotonic in τ_x but monotonically increasing in τ_θ, and, finally, (4) a region where θ^* is monotonically increasing in τ_x and monotonically decreases in τ_θ. Figure 3 depicts these regions for the case when $\gamma > \frac{1}{2}$. For $\gamma < \frac{1}{2}$ the order of the region is reversed.

As implied by Lemma 1 we can see in Figure 3 that, for any $\gamma > \frac{1}{2}$, the interval of μ_θ for which θ^* is non-monotonic in τ_x is larger than the interval for which μ_θ is non-monotonic in τ_θ implying that a non-monotonicity of the regime threshold in τ_x is a more likely phenomenon.

7 Model with Public Information

Above we considered a situation where agents have a common proper prior and have access to private information. We argued that we can interpret the prior both as an initial uncertainty level in the economy or the realization of public information. However, in many applications we might allow both for a common prior, which captures history of previous play and an explicit public signal. An additional advantage of such extension is that it allows us to perform analysis ex-ante, unconditional on the realization of the public signal. Finally, this is the typical

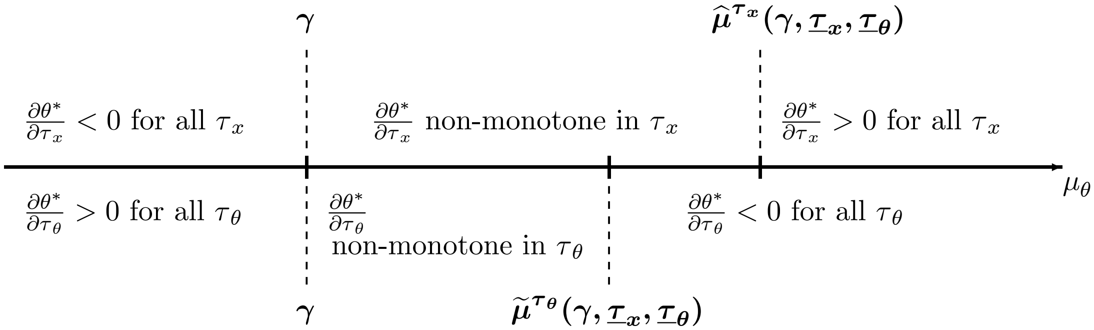

Figure 3: Characterization of dynamics when $\gamma > \frac{1}{2}$

specification used in the closely related quadratic-gaussian setup introduced by Morris and Shin (2002).

Suppose that as before, agents share a common prior $\theta \sim N\left(\mu_\theta, \tau_\theta^{-1}\right)$ and each of them obtains a private signal $x_i \sim N\left(\theta, \tau_x^{-1}\right)$. In addition, agents have also access to a public signal $y \sim N\left(\theta, \tau_y^{-1}\right)$, which is observed by all of them. Agents decide on their actions after observing their signals.

Given that agents observe a public signal before they decided on their actions, the equilibrium regime change threshold will depend on the realization of y. In particular, define a public belief as

$$\theta | y \sim N\left(z, \tau_z^{-1}\right)$$

where

$$z = \frac{\tau_y y + \tau_\theta \mu_\theta}{\tau_y + \tau_\theta} \text{ and } \tau_z = \tau_\theta + \tau_y$$

Then, the equilibrium regime threshold is defined by the critical mass condition, now given by:

$$\frac{\tau_z}{\tau_x^{1/2}}\left(\theta^* - z\right) + \sqrt{\frac{\tau_z + \tau_x}{\tau_x}}\Phi^{-1}\left(\gamma\right) - \Phi^{-1}\left(\theta^*\right) = 0$$

Thus, we see that with the public signal the mean of the common belief, z, takes the role of the mean of the prior emphasized in Section 4. In particular, the uniqueness conditions becomes now $\tau_x^{1/2}/t_z > 1/\sqrt{2\pi}$. Under this assumption all the result established earlier hold, conditional on z. However, now we can also ask questions how the change in precision of private or public information affects the equilibrium ex-ante, unconditional on z, rather than ex-post, conditional on z (which I considered in section 4). That is, we are interested in determining the sign of

$$\frac{\partial}{\partial \tau_x} E_z\left[\theta^*\left(z\right)\right] \text{ and } \frac{\partial}{\partial \tau_z} E_z\left[\theta^*\left(z\right)\right]$$

where $E_z\left(\cdot\right)$ denotes expectations with respect to the public belief z. The main goal of the reminder of this section is to establish conditions under which an increase in the precision of private (or public) information leads to an increase or decrease in θ^*.

7.1 Technical Preliminaries

Determining how an increase in the precision of public or private information affect the equilibrium in the presense of an explicit public signal is a challenging task. This is because $\theta^*(z)$ defined only implicitly and the integrals such as $\partial E_z[\theta^*(z)]/\partial\tau_x$ that contain derivative of normal densities are notoriously hard to compute. In this section I establish useful symmetry and asymmetry properties of both $\partial\theta/\partial\tau_x$ and $\partial\theta^*/\partial\tau_z$ that will prove useful in the analysis below. More importantly, this approach is not tied to the explicit form of $\partial\theta/\partial\tau_x$ and $\partial\theta^*/\partial\tau_z$ and hence can be of interest to applied theorists working on similar models as the integrals we are dealing here commonly arises in the setting with normally distributed random variables. For the details we invite an interested reader in these technique to the appendix B. Here I only state the results concerning $\partial\theta^*/\partial\tau_x$ and $\partial\theta^*/\partial\tau_z$.

Definition 3 *A function $f:\mathbb{R}\to\mathbb{R}$ is asymmetric with respect to point (a,b) if for all $\varepsilon>0$:*

$$-[f(a+\varepsilon)-b]\neq f(a-\varepsilon)-b$$

A function is positively asymmetric with respect to (a,b) if

$$-[f(a+\varepsilon)-b]<f(a-\varepsilon)-b$$

and negatively asymmetric with respect to (a,b) if

$$-[f(a+\varepsilon)-b]>f(a-\varepsilon)-b$$

The above definition generalizes symmetry of the function with respect to the origin. As shown in the appendix, this property is extremely useful when computing the integrals where an integrand in a product of a normal density and some function f. Next, we show that $\partial\theta/\partial\tau_x$ and $\partial\theta^*/\partial\tau_z$ are asymmetric with respect to point $(\mu^{\tau_x},0)$ and $(\mu^{\tau_\theta},0)$, respectively. The next lemma states this result formally.

Lemma 2 *Consider $\partial\theta^*/\partial\tau_x$.*

1. *If $\gamma>\frac{1}{2}$ then $\partial\theta^*/\partial\tau_x$ is positively asymmetric with respect to $(\mu^{\tau_x},0)$ as function of z*
2. *If $\gamma=\frac{1}{2}$ then $\partial\theta^*/\partial\tau_x$ is symmetric with respect to $(\mu^{\tau_x},0)$ as function of z*
3. *If $\gamma<\frac{1}{2}$ then $\partial\theta^*/\partial\tau_x$ is negatively asymmetric with respect to $(\mu^{\tau_x},0)$ as function of z*

We have a similar result for $\partial\theta^*/\partial\tau_z$.

Lemma 3 *Consider $\partial\theta^*/\partial\tau_z$.*

1. *If $\gamma>\frac{1}{2}$ then $\partial\theta^*/\partial\tau_z$ is positively asymmetric with respect to $(\mu^{\tau_z},0)$ as function of z*
2. *If $\gamma=\frac{1}{2}$ then $\partial\theta^*/\partial\tau_x$ is symmetric with respect to $(\mu^{\tau_z},0)$ as function of z*
3. *If $\gamma<\frac{1}{2}$ then $\partial\theta^*/\partial\tau_x$ is negatively asymmetric with respect to $(\mu^{\tau_z},0)$ as function of z*

The natural reference for the derivatives with respect to τ_x and τ_z is the point in which the respective derivative intersects the zero line. The above lemmas indicate that when $\gamma > \frac{1}{2}$ then the change in in regime change threshold is higher, in the absolute terms, for realization of z above that point than those below. The opposite is true for the case of $\gamma < \frac{1}{2}$. Finally, when $\gamma = \frac{1}{2}$ then the derivatives are symmetric with respect to these points.

Finally, in order to establish the change in the expected threshold θ^* we need the unconditional distribution of z. Since both the prior belief and the public signal are distributed according to normal distribution it is given by:

$$z \sim N\left(\mu_\theta, \frac{\tau_z}{\tau_y\left(\tau_z + \tau_y\right)}\right)$$

7.2 Change in the precision of private information

Having established the results regarding asymmetry of $\partial\theta^*/\partial\tau_x$ we can now characterize the local behavior of a change in the expected regime change threshold, $E\left[\theta^*\right]$. According to the analysis in Section 4 we know that $\partial\theta^*/\partial\tau_x$ is positive when μ_θ is high while it is negative when μ_θ is low. Therefore, we should expect that if μ_θ is high then the change in expected θ^* should be positive when μ_θ is high and is negative when μ_θ is low. The next lemma shows that this intuition is indeed correct.

Proposition 8 *There exists a unique μ_x^* such that:*

1. *If $\mu_\theta > \mu_x^*$ then $\partial E\left[\theta^*\right]/\partial\tau_x > 0$*
2. *If $\mu_\theta = \mu_x^*$ then $\partial E\left[\theta^*\right]/\partial\tau_x = 0$*
3. *If $\mu_\theta < \mu_x^*$ then $\partial E\left[\theta^*\right]/\partial\tau_x < 0$*

Proof. See Appendix. ■

The above result establishes that the local behavior of the regime change threshold in τ_x carry over to the case with public information.

7.3 Change in the precision of public information

In this section we turn our attention to the effect of an increase in the precision of public information.

Proposition 9 *There exists a unique μ_z^* such that:*

1. *If $\mu_\theta > \mu_z^*$ then $\partial E\left[\theta^*\right]/\partial\tau_z < 0$*
2. *If $\mu_\theta = \mu_z^*$ then $\partial E\left[\theta^*\right]/\partial\tau_z = 0$*
3. *If $\mu_\theta < \mu_z^*$ then $\partial E\left[\theta^*\right]/\partial\tau_z > 0$*

Proof. See Appendix. ■

The above result establishes that the local behavior of the regime change threshold in τ_x carry over to the case with public information.

8 Appendix

This Appendix contains all the proofs skipped in the main section of the paper.

Proof of Proposition 2. Recall from section 2 that

$$\frac{d\theta^*}{d\tau_x} = -\frac{\frac{\partial\theta^*}{\partial\tau_x} + \frac{\partial\theta^*}{\partial x^*}\frac{\partial x^*}{\partial\tau_x}}{1 - \left[\frac{\partial\theta^*}{\partial x^*}\right]_M \left[\frac{\partial x^*}{\partial\theta^*}\right]_P}$$

Computing the respective partial effects we obtain the following expression for the total effect of an increase in the precision of private information on θ^*:

$$\frac{d\theta^*}{d\tau_x} = \frac{1}{2}\frac{\frac{1}{\tau_x}\left(\theta^* - \mu_\theta\right) + \frac{1}{\tau_x}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)}{1 - \frac{\tau_x^{1/2}}{\tau_\theta}\frac{1}{\phi(\Phi^{-1}(\theta^*))}},$$

where the sign of $d\theta^*/d\tau_x$ depends on the sign of the numerator:

$$\frac{1}{\tau_x}\left(\theta^* - \mu_\theta\right) + \frac{1}{\tau_x}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)$$

We first find the condition on μ_θ such that $d\theta^*/d\tau_x = 0$. Keep $\gamma, \tau_x, \tau_\theta$ constant. Then $d\theta^*/d\tau_x = 0$ if and only if

$$\theta^* = \mu_\theta - \frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)$$

Since θ^* is determined by the critical mass conditin, we know $\theta^* = \mu_\theta - \frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)$ if and only if the ciritical mass condition evaluated at this value of θ^* is equal to zero. i.e.:

$$\frac{\tau_\theta}{\tau_x^{1/2}}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right) + \sqrt{\frac{\tau_x+\tau_\theta}{\tau_x}}\Phi^{-1}\left(\gamma\right) - \Phi^{-1}\left(\mu_\theta - \frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)\right) = 0$$

or, if

$$\mu_\theta = \Phi\left(\sqrt{\frac{\tau_x}{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)\right) + \frac{1}{\sqrt{\tau_x+\tau_\theta}}$$

It follows that

$$\frac{d\theta^*}{d\tau_x} = 0 \text{ iff } \mu_\theta = \Phi\left(\sqrt{\frac{\tau_x}{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)\right) + \frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)$$

Let

$$\widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right) \equiv \Phi\left(\sqrt{\frac{\tau_x}{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)\right) + \frac{1}{\sqrt{\tau_x+\tau_\theta}}$$

and note that the critical mass condition is decreasing in μ_θ and in θ^*. Thus, if $\mu_\theta > \widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right)$ then $\theta^* < \mu_\theta - \frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)$ while if $\mu_\theta < \widehat{\mu}^{\tau_x}\left(\gamma, \tau_x, \tau_\theta\right)$ then $\theta^* > \mu_\theta - \frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}\left(\gamma\right)$. This establishes the second part of the proposition. ∎

Proof of Proposition 3. Differentiating $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ with respect to τ_x and simplifying we obtain:

$$\frac{\partial \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)}{\partial \tau_x} = \frac{1}{2}\Phi^{-1}(\gamma)\frac{1}{(\tau_x + \tau_\theta)^{3/2}}\left[\frac{\tau_\theta}{\tau_x^{1/2}}\phi\left(\sqrt{\frac{\tau_x}{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)\right) - 1\right]$$

Under the uniqueness condition ($\tau_x^{1/2}/\tau_\theta > 1/\sqrt{2\pi}$) the term in the square brackets is negative and hence

$$\begin{aligned}
\frac{\partial \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)}{\partial \tau_x} &< 0 \text{ if } \gamma > \frac{1}{2} \\
\frac{\partial \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)}{\partial \tau_x} &= 0 \text{ if } \gamma = \frac{1}{2} \\
\frac{\partial \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)}{\partial \tau_x} &> 0 \text{ if } \gamma < \frac{1}{2}
\end{aligned}$$

Suppose that $\gamma > 1/2$. We know that as $\tau_x \to \infty$ then $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta) \to \gamma$ and that $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ is strictly decreasing in τ_x. Therefore, it follows that $\forall \tau_x \in [\underline{\tau}_x, \infty)$ $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta) \geq \gamma$ and that for all $\tau_x > \underline{\tau}_x$ we have $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta) < \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta)$. From these two observations we note that if $\mu_\theta < \gamma$ then $\mu_\theta < \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ for all $\tau_x \in [\underline{\tau}_x, \infty)$ and hence, by Proposition 1 θ^* is decreasing in τ_x for all $\tau_x \in [\underline{\tau}_x, \infty)$. Similarly, if $\mu_\theta \geq \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta)$ then we know that $\mu_\theta \geq \widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ for any $\tau_x \in [\underline{\tau}_x, \infty)$ with strict inequality if $\tau_x > \underline{\tau}_x$. From Proposition 1 it follows then that θ^* is increasing for all $\tau_x > \underline{\tau}_x$. Finally, consider $\mu_\theta \in (\gamma, \widehat{\mu}^{\tau_x}(\gamma, \underline{\tau}_x, \tau_\theta))$. In that case we know that μ_θ is initially lower than $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ but as τ_x increases $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ decreases monotonically towards γ. As a consequence, there exists a precision level $\widehat{\tau}_x$ such that if $\tau_x < \widehat{\tau}_x$ then θ^* is a decreasing function of τ_x and if $\tau_x > \widehat{\tau}_x$ then θ^* is an increasing function of τ_x. This establishes the first part of the proposition.The argument for the case when $\gamma < 1/2$ is analogous.

Finally, when $\gamma = 1/2$ then $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta) = 1/2$ and hence it $\widehat{\mu}^{\tau_x}(\gamma, \tau_x, \tau_\theta)$ constant in τ_x. ■

Proof of Proposition 4. Recall from section 2 that

$$\frac{d\theta^*}{d\tau_\theta} = -\frac{\frac{\partial \theta^*}{\partial \tau_\theta} + \frac{\partial \theta^*}{\partial x^*}\frac{\partial x^*}{\partial \tau_\theta}}{1 - \left[\frac{\partial \theta^*}{\partial x^*}\right]_M \left[\frac{\partial x^*}{\partial \theta^*}\right]_P}$$

As noted in Section 5 $\partial \theta^* / \partial \tau_\theta$. Computing the remaining effects and simplifying we obtain:

$$\frac{d\theta^*}{d\tau_\theta} = \tau_x^{1/2}\frac{\frac{1}{\tau_x}(\theta^* - \mu_\theta) + \frac{1}{2}\frac{1}{\tau_x}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)}{\frac{1}{\phi(\Phi^{-1}(\theta^*))} - \frac{\tau_\theta}{\tau_x^{1/2}}}$$

where the denominator is always positive. Therefore, we see that the sign of $d\theta^*/d\tau_\theta$ depends on the sign of the numerator:

$$\frac{1}{\tau_x}(\theta^* - \mu_\theta) + \frac{1}{2}\frac{1}{\tau_x}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)$$

We first find the condition on μ_θ such that $d\theta^*/d\tau_\theta = 0$. Fix $\gamma, \tau_x, \tau_\theta$. Then $d\theta^*/d\tau_\theta = 0$ if and only if

$$\theta^* = \mu_\theta - \frac{1}{2}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)$$

But we know that θ^* solves the following equation:

$$\frac{\tau_\theta}{\tau_x^{1/2}}(\theta^* - \mu_\theta) + \sqrt{\frac{\tau_x + \tau_\theta}{\tau_x}}\Phi^{-1}(\gamma) - \Phi^{-1}(\theta^*) = 0$$

Thus $\theta^* = \mu_\theta - \frac{1}{2}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}(\gamma)$ if and only if:

$$\frac{\tau_\theta}{\tau_x^{1/2}}\left(-\frac{1}{2}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)\right) + \sqrt{\frac{\tau_x + \tau_\theta}{\tau_x}}\Phi^{-1}(\gamma) - \Phi^{-1}\left(\mu_\theta - \frac{1}{2}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)\right) = 0$$

or

$$\mu_\theta = \Phi\left(\sqrt{\frac{\tau_x}{\tau_\theta + \tau_x}}\frac{\frac{1}{2}\tau_\theta + \tau_x}{\tau_x}\Phi^{-1}(\gamma)\right) + \frac{1}{2}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)$$

Therefore, it follows that

$$\frac{d\theta^*}{d\tau_\theta} = 0 \text{ iff } \mu_\theta = \Phi\left(\sqrt{\frac{\tau_x}{\tau_\theta + \tau_x}}\frac{\frac{1}{2}\tau_\theta + \tau_x}{\tau_x}\Phi^{-1}(\gamma)\right) + \frac{1}{2}\frac{1}{\sqrt{\tau_x + \tau_\theta}}\Phi^{-1}(\gamma)$$

Next, note that the critical mass condition to decreasing in μ_θ and θ^*. It follows that if $\mu_\theta > \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ then $\theta^* < \mu_\theta - \frac{1}{2}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}(\gamma)$ and hence $d\theta^*/d\tau_\theta > 0$ while if $\mu_\theta < \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ then $\theta^*?\mu_\theta - \frac{1}{2}\frac{1}{\sqrt{\tau_x+\tau_\theta}}\Phi^{-1}(\gamma)$ and hence $d\theta^*/d\tau_\theta < 0$. This proves the proposition. ■

Proof of Proposition 6. Differentiating $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ with respect to τ_x and simplifying we obtain:

$$\frac{\partial\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)}{\partial\tau_\theta} = \frac{1}{4}\frac{1}{(\tau_x + \tau_\theta)^{3/2}}\Phi^{-1}(\gamma)\left[\phi\left(\frac{\frac{1}{2}\tau_\theta + \tau_x}{\sqrt{(\tau_x + \tau_\theta)\tau_x}}\Phi^{-1}(\gamma)\right) - \frac{\tau_x^{1/2}}{\tau_\theta}\right]\frac{\tau_\theta}{\tau_x^{1/2}}$$

Under the uniqueness condition the term in the square brackets is negative and hence it follows that:

$$\begin{aligned}
\frac{\partial\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)}{\partial\tau_\theta} &< 0 \text{ if } \gamma > \frac{1}{2} \\
\frac{\partial\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)}{\partial\tau_\theta} &= 0 \text{ if } \gamma = \frac{1}{2} \\
\frac{\partial\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)}{\partial\tau_\theta} &> 0 \text{ if } \gamma < \frac{1}{2}
\end{aligned}$$

Finally, note that uniqueness condition puts a constraint on how high the public information precision can be. Since $\tau_\theta \leq \frac{1}{\sqrt{2\pi}}$ we let the highest τ_θ available to be denoted $\overline{\tau}_\theta = \frac{1}{\sqrt{2\pi}}$.

Suppose that $\gamma > 1/2$. We know that as $\tau_\theta \to \overline{\tau}_\theta$ then $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta) \to \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta)$ and that $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ is strictly decreasing in τ_θ. It follows that for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta)$ $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta) \geq \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta)$ and that for all $\tau_\theta \in (\underline{\tau}_\theta, \overline{\tau}_\theta]$ we have $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta) > \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta)$. From these two observations we know that if $\gamma > \frac{1}{2}$ and $\mu_\theta < \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta)$ then $\mu_\theta < \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$ and hence, by Proposition 5, θ^* is increasing in the precision of public information for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$. Similarly, if $\mu_\theta > \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta)$ then $\mu_\theta > \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$ and hence θ^* is decreasing in the precision of public information for all $\tau_\theta \in [\underline{\tau}_\theta, \overline{\tau}_\theta]$. The argument for the case when $\gamma < \frac{1}{2}$ is analogous. Finally, consider $\mu_\theta \in (\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta), \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta))$. In that case we know that for τ_θ close to $\underline{\tau}_\theta$, μ_θ is lower than $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ but as τ_θ increases $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta)$ decreases monotonically towards $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta)$. As a consequence, there exists a precision level $\widehat{\tau}_\theta$ such that if $\tau_\theta < \widehat{\tau}_\theta$ then θ^* is a decreasing function of τ_θ and if $\tau_\theta > \widehat{\tau}_\theta$ then θ^* is an increasing function of τ_θ. This establishes that when $\mu_\theta \in (\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \overline{\tau}_\theta), \widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \underline{\tau}_\theta))$ then θ^* is a non-mnotone function of τ_θ.The argument for the case when $\gamma < 1/2$ is analogous.

Finally, for the case when $\gamma = \frac{1}{2}$ note that in that case $\widetilde{\mu}^{\tau_\theta}(\gamma, \tau_x, \tau_\theta) = \frac{1}{2}$ and is constant in τ_x. ■

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