

Chapter 3

TIME SERIES VERSUS INDEX NUMBER METHODS FOR SEASONAL ADJUSTMENT

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1. Introduction

This chapter argues that time series methods for the seasonal adjustment of economic price or quantity series cannot in general lead to measures of short term month-to-month measures of price or quantity change that are free of seasonal influences. This impossibility result can be seen most clearly if each seasonal commodity in an aggregate is present for only one season of each year. However, time series methods of seasonal adjustment can lead to measures of the underlying trend in an economic series and to forecasts of the underlying trend. In this context, it is important to have a well defined definition of the trend and the chapter suggests that index number techniques based on the moving or rolling year concept can provide a good target measure of the trend. The almost forgotten work of Oskar Anderson (1927) on the difficulties involved in using time series methods to identify the trend and seasonal component in a series is reviewed.

Economists and statisticians have struggled for a long time with the time series approach to seasonal adjustment. In fact, the entire topic is somewhat controversial as the following quotation indicates:

“We favor modeling series in terms of the original data, accounting for seasonality in the model, rather than using adjusted data. ... In the light of these remarks and the previous discussion, it is relevant to ask whether seasonal adjustment can be justified, and if so, how? It is important to remember that the primary consumers of seasonally adjusted data are not necessarily statisticians and economists, who could most likely use the unadjusted data, but people such as government officials, business managers, and journalists, who often have little or no statistical training. ... In general, there will be some information loss from seasonal adjustment, even when an adjustment method appropriate for the data being adjusted can be found. The situation will be worse when the seasonal adjustment is based on incorrect assumptions. If people will often be misled by using seasonally adjusted data, then their use cannot be justified.”

William R. Bell and Steven C. Hillmer (1984; 291).

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If the seasonal component in a price series is removed, then it could be argued that the resulting seasonally adjusted price series could be used as a valid indicator of short term month-to-month price change.² However, in this chapter, we will argue that there are some methodological difficulties with traditional time series methods for seasonally adjusting prices, particularly when some seasonal commodities are not present in the marketplace in all seasons. Under these circumstances, seasonally adjusted data can only represent *trends* in the movement of prices rather than an accurate measure of the change in prices going from one season to the next. Before we can adjust a price for seasonal movements, it is first necessary to measure the seasonal component. Thus as the following quotations indicate, it is first necessary to have a proper definition of the seasonal component before it can be eliminated:

“The problem of measuring—rather than eliminating—seasonal fluctuations has not been discussed. However, the problem of measurement must not be assumed necessarily divorced from that of elimination.”

Frederick R. Macaulay (1931; 121).

“This discussion points out the arbitrariness inherent in seasonal adjustment. Different methods produce different adjustments because they make different assumptions about the components and hence estimate different things. This arbitrariness applies equally to methods (such as X-11) that do not make their assumptions explicit, since they must implicitly make the same sort of assumptions as we have discussed here. ... Unfortunately, there is not enough information in the data to define the components, so these types of arbitrary choices must be made. We have tried to justify our assumptions but do not expect everyone to agree with them. If, however, anyone wants to do seasonal adjustment but does not want to make these assumptions, we urge them to make clear what assumptions they wish to make. Then the appropriateness of the various assumptions can be debated.

This debate would be more productive than the current one regarding the choice of seasonal adjustment procedures, in which no one bothers to specify what is being estimated. Thus if debate can be centered on what it is we want to estimate in doing seasonal adjustment, then there may be no dispute about how to estimate it.”

William R. Bell and Steven C. Hillmer (1984; 305).

As the above quotations indicate, it is necessary to specify very precisely what the definition of the seasonal is. The second quotation also indicates that there is no commonly accepted definition for the seasonal. In the subsequent sections of this chapter, we will spell out some of the alternative definitions of the seasonal that have appeared in the literature. Thus in sections 2 and 3 below, we spell out the very simple additive and multiplicative models of the seasonal for calendar years. In section 4, we show that these calendar year models of the seasonal are not helpful in solving the problem of determining measures of month-to-month price change free of seasonal influences. Thus in section 5, we consider moving year or rolling year models of the seasonal that are counterparts to the simple calendar year models of sections 2 and 3. These rolling year models of the seasonal are more helpful in determining month-to-month movements in prices that are free from seasonal movements. However, we argue that these seasonally adjusted measures of monthly price change are movements in an annual trend rather than true short term month-to-month movements.

² For example, consider the following quotation: “In the second place, if comparisons are required between seasons rather than between years then the estimation of the normal seasonal variation of prices appropriate to the base year forms an integral part of the calculations.” Richard Stone (1956; 77).

In section 6, we consider a few of the early time series models of the seasonal. In section 7, we consider more general time series models of the seasonal and present Anderson's (1927) critique of these unobserved components models. The time series models discussed in sections 6 and 7 differ from the earlier sections in that they add random errors, erratic components, irregular components or white noise into the earlier decomposition of a price series into trend and seasonal components. Unfortunately, this addition of error components to the earlier simpler models of the seasonal greatly complicates the study of seasonal adjustment procedures since it is now necessary to consider the tradeoff between fit and smoothness. There are also complications due to the nature of the irregular or random components. In particular, if we are dealing with micro data from a particular establishment, the irregular component of the series provided to the statistical agency can be very large due to the sporadic nature of production, orders or sales. A business economist with the Johns-Manville Corporation made the following comments on the nature of irregular fluctuations in micro data:

“Irregular fluctuations are of two general types: random and non-random. Random irregulars include all the variation in a series that cannot be otherwise identified as cyclical or seasonal or as a nonrandom irregular. Random irregulars are of short duration and of relatively small amplitude. Usually if a random irregular movement is upward one month, it will be downward the next month. This type of irregular can logically be eliminated by such a smoothing process as a fairly short term rolling average. Non-random irregulars cannot logically be identified as either cyclical or seasonal but are associated with a known cause. They are particularly apt to occur in dealing with company data. An exceptionally large order will be received in one month. A large contract may be awarded in one month but the work on it may take several months to complete. Sales in a particular month may be very large as a result of an intensive campaign or an advance announcement of a forthcoming price increase, and be followed by a month or two of unusually low sales. It takes a much longer rolling average to smooth out irregularities of this sort than random fluctuations. Even after fluctuations are smoothed out, a peak or trough may result which is not truly cyclical, or it may occur at the wrong time. Existing programs for seasonal adjustment do not, I believe, give sufficient attention to eliminating the effects of non-random irregulars.”

Harrison W. Cole (1963; 135).

Finally, in section 8, we return to the main question asked in this chapter: can price data that are seasonally adjusted by time series methods provide accurate information on the short term month-to-month movement in prices? Our answer to this question is: basically, no! Seasonally adjusted prices can only provide information on the longer term trend in prices. In view of the general lack of objectivity, reproducibility and comprehensibility of time series methods of seasonal adjustment, we suggest that a better alternative to the use of traditionally seasonally adjusted data to represent trends in prices would be the use of the centered rolling year annual indexes explained in Diewert (1983) (1996) (1999).

2. Calendar Year Seasonal Concepts: Additive Models

In this chapter, we will restrict ourselves to considering the problems involved in seasonally adjusting a single price (and or quantity) series. Let $p_{y,m}$ and $q_{y,m}$ denote the observed price and quantity for a commodity in year y and “month” m where there are M “months” in the year. As usual, it will sometimes be convenient to switch to consecutive periods or seasons t where

$$(1) \quad t = (y-1)M + m, \quad y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.$$

Thus when it is convenient, we will sometimes relabel the price for year y and month m , $p_{y,m}$, as p_t where t is defined by (1).

We first consider the problem of defining seasonal factors for the quantity series, $q_{y,m}$.

Our reason for considering the quantity case before the price case is that a natural annual measure of quantity is simply the annual amount produced or the annual amount demanded, $\sum_{m=1}^M q_{y,m}$. Then it is natural to compare the quantity pertaining to any month, $q_{y,m}$, with the annual calendar year average quantity, Q_y , defined as:

$$(2) \quad Q_y \equiv (1/M) \sum_{m=1}^M q_{y,m}, \quad y = 1, 2, \dots, Y.$$

Note that Q_y is the arithmetic average of the “monthly” quantities $q_{y,m}$ in year y . The *additive seasonal factor* $S_{y,m}$ for month m of year y can now be defined as the difference between the actual quantity for month m of year y , $q_{y,m}$, and the calendar year annual average quantity Q_y :

$$(3) \quad S_{y,m} \equiv q_{y,m} - Q_y, \quad y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.$$

Using definitions (2) and (3), it can be verified the additive seasonal factors, $S_{y,m}$, sum to zero over the seasons in any given year; i.e., we have the following restrictions on the seasonals:

$$(4) \quad \sum_{m=1}^M S_{y,m} = 0 \quad \text{for } y = 1, 2, \dots, Y.$$

Note that the seasonal factors defined by (3) cannot be defined until the end of the calendar year y when information on the quantity for the last season in the year becomes available. The above algebra explains how additive seasonal factors can be defined. The next step is to explain how the seasonal factors may be used in a seasonal adjustment procedure. The basic hypothesis in a seasonal adjustment procedure is that seasonal factors estimated using past data will persist into the future. Thus let $S_{y,m}^*$ be an estimator for the month m seasonal factor in year y that is based on past seasonal factors, $S_{y-1,m}, S_{y-2,m}, \dots$ for month m for years prior to year y . Now rewrite equation (3) as follows:

$$(5) \quad Q_y = q_{y,m} - S_{y,m}.$$

If we now replace the actual seasonal factor $S_{y,m}$ in (5) by the estimated or forecasted seasonal factor $S_{y,m}^*$, then the right hand side of (5) becomes a forecast for the average annual quantity for year y ; i.e., we have

$$(6) \quad Q_y^* \equiv q_{y,m} - S_{y,m}^*.$$

Once an estimate for average annual output $Q_{y,m}^*$ or input is known, then annual output or input can be *forecasted* as M times $Q_{y,m}^*$. This illustrates one possible use for a seasonal adjustment procedure.

The above algebra can be repeated for prices in place of quantities. Thus define the *average level of prices* for calendar year y as:

$$(7) \quad P_y \equiv (1/M) \sum_{m=1}^M p_{y,m}, \quad y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.$$

Define the *additive seasonal price factor* $S_{y,m}$ for month m of year y as the difference between the observed month m , year y price $p_{y,m}$ and the corresponding calendar year y annual average level of prices P_y :³

$$(8) \quad S_{y,m} \equiv p_{y,m} - P_y, \quad y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.$$

Again, it can be verified using definitions (7) and (8) that the seasonal price factors, $S_{y,m}$ defined by (8), satisfy the restrictions (4), $\sum_{m=1}^M S_{y,m} = 0$, for each calendar year y .

As in the quantity case, if we have an estimator $S_{y,m}^*$ for the month m seasonal factor for year y that is based on prior year seasonals of the form defined by (8), then we can forecast the average level of prices in year y , $P_{y,m}^*$, by using the following counterpart to (6):

$$(9) \quad P_y^* \equiv p_{y,m} - S_{y,m}^*.$$

The only difference between the price and quantity cases is that usually, we are interested in forecasts of *annual total output* (or input) in the quantity case, while in the price case, we are generally interested in the *average annual level of prices*. We will now focus our attention on the price case for the remainder of this chapter. In this case, it is no longer so clear that we will always want to define the average annual level of prices for year y , P_y , by the arithmetic mean, (7); why should we not use a geometric mean or some other form of symmetric mean? Furthermore, why should the seasonal $S_{y,m}$ be additive to the annual average level of prices P_y as in (8)? Perhaps a multiplicative seasonal factors model would lead to more “stable” estimates of the seasonal factors. Thus in the following section, we consider these alternative models for the seasonal.

³ To economise on notation, we have used the same symbol for the seasonal factors in both the price and quantity contexts. However, in the remainder of this chapter, we will concentrate on the price case.

3. Calendar Year Seasonal Concepts: Multiplicative Models

We now define the calendar year y average price level p_y as the geometric mean of the “monthly” prices in that year:⁴

$$(10) \quad p_y \equiv [\prod_{m=1}^M p_{y,m}]^{1/M}, \quad y = 1, 2, \dots, Y.$$

Define the *multiplicative seasonal price factors* $s_{y,m}$ for month m of year y as the ratio of the observed month m , year y price $p_{y,m}$ to the corresponding annual average p_y defined by (10):

$$(11) \quad s_{y,m} \equiv p_{y,m} / p_y, \quad y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.$$

Using definitions (10) and (11), it can be verified that the multiplicative seasonal factors satisfy the following restrictions:

$$(12) \quad [\prod_{m=1}^M s_{y,m}]^{1/M} = 1, \quad y = 1, 2, \dots, Y.$$

If we raise both sides of (12) to the power M , then the multiplicative seasonal factors $s_{y,m}$ also satisfy the following equivalent restrictions:

$$(13) \quad \prod_{m=1}^M s_{y,m} = 1, \quad y = 1, 2, \dots, Y.$$

As in the previous section, if the multiplicative seasonal factors defined by (11) are “stable” over years, then an estimator for the year y , month m seasonal factor based on prior year seasonal factors, $s_{y,m}^*$, can be obtained and a prediction or forecast for the annual average level of prices in year y can be obtained as follows:

$$(14) \quad p_y^* \equiv p_{y,m} / s_{y,m}^*, \quad y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.$$

The multiplicative model presented in this section made two changes from the additive model considered in the previous section:

- The annual average level of prices was changed from the arithmetic mean of the monthly prices, P_y defined by (7), to the geometric mean p_y defined by (10).
- The additive model of the seasonal defined by (8) was replaced by the multiplicative model (11).

Obviously, we do not have to make both of these changes at the same time. Thus we could combine the arithmetic mean definition for the average level of prices, P_y , with a

⁴ Obviously, this model breaks down unless *all* prices in the year are positive. We make this assumption whenever we consider multiplicative seasonal models.

multiplicative model for the seasonal factors. In this alternative model, the seasonal factors would be defined as follows:

$$(15) \quad \sigma_{y,m} \equiv p_{y,m}/p_y, \quad y = 1,2,\dots, Y \text{ and } m = 1,2,\dots, M.$$

The “mixed” seasonal factors $\sigma_{y,m}$ defined by (15) and (7) satisfy the following restrictions:

$$(16) \quad (1/M) \sum_{m=1}^M \sigma_{y,m} = 1, \quad y = 1,2,\dots, Y.$$

There is another model that would combine the geometric mean of the monthly prices pertaining to a year, p_y defined by (10), as the “right” measure of the average level of prices for a year with the following “additive” model of the seasonal factors:

$$(17) \quad \alpha_{y,m} \equiv p_{y,m} - p_y, \quad y = 1,2,\dots, Y \text{ and } m = 1,2,\dots, M.$$

The seasonal factors $\alpha_{y,m}$ defined by (17) and (10) satisfy the following somewhat messy restrictions:

$$(18) \quad \prod_{m=1}^M \{(\alpha_{y,m}/p_y) + 1\} = 1, \quad y = 1,2,\dots, Y.$$

Which of the above four models of the seasonal is the “right” one? The answer to this question depends on the purpose one has in mind. If the purpose is to *forecast* or *predict* an annual level of prices based on observing a price for one season of the year, then the determination of the “right” seasonal model becomes an empirical matter; i.e., the alternative models would have to be evaluated empirically based on how well they predicted on a case by case basis. Thus with the forecasting purpose in mind, there can be no unambiguously correct model for the seasonal factors. Of course, the actual *model evaluation problem*, if our focus is prediction, is vastly more complicated than we have indicated for at least two reasons:

- The arithmetic and geometric mean definitions for the annual average level of prices could be replaced by more general definitions of an average such as a mean of order r ,⁵ $[\sum_{m=1}^M (1/M)(p_{y,m})^r]^{1/r}$, or by a homogeneous symmetric mean⁶ of the prices pertaining to year y , say $\mu(p_{y,1}, p_{y,2}, \dots, p_{y,M})$.
- Once the “right” mean is found, then the most “stable” seasonal factors need not be of the simple additive or multiplicative type that we have considered thus far. Hence if $\mu_y \equiv \mu(p_{y,1}, p_{y,2}, \dots, p_{y,M})$ is the “right” annual mean for year y , the most stable seasonals might be defined as the following sequence of factors: $f(p_{y,1}, \mu_y), f(p_{y,2}, \mu_y), \dots, f(p_{y,M}, \mu_y)$, where f is a suitable function of two variables.

Thus corresponding to different choices for the functions μ and f , there are countless infinities of possible seasonal models that could be evaluated on the basis of their predictive powers for seasonally adjusting a specific series. However, suppose that our purpose in considering

⁵ See Hardy, Littlewood and Polyá (1934) for material on means of order r .

⁶ See Diewert (1993; 361-364) for material on homogeneous symmetric means.

seasonal adjustment procedures is to determine whether seasonally adjusted price series can provide useful information on the month-to-month movement of prices, free from seasonal influences. In the following section, we show that concepts of the seasonal that are based on calendar year concepts are useless for this purpose.

4. Calendar Year Seasonal Adjustment and Month-to-month Price Change

Suppose we use the additive calendar year method for defining seasonal factors; i.e., we use (7) and (8) in section 2 above to define the seasonal factors $S_{y,m}$ for month m of year y . Obviously, at the end of year y , we can use the additive seasonal factors $S_{y,m}$ defined by (8) to form the seasonally adjusted data for year y , $p_{y,m}^a$:

$$\begin{aligned}
 (19) \quad p_{y,m}^a &\equiv p_{y,m} - S_{y,m} \\
 &= p_{y,m} - [p_{y,m} - P_y] && \text{using definition (8) for } S_{y,m} \\
 &= P_y && y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.
 \end{aligned}$$

Similarly, if we use the multiplicative model of the seasonal defined by (10) and (11) in section 3 above, at the end of the year, we can use the multiplicative seasonal factors $s_{y,m}$ defined by (11) to form the seasonally adjusted data for year y :

$$\begin{aligned}
 (20) \quad p_{y,m}^a &\equiv p_{y,m} / s_{y,m} \\
 &= p_{y,m} / [p_{y,m} / P_y] \\
 &= P_y && y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.
 \end{aligned}$$

Thus for both the additive model and the multiplicative model, if we compare the level of the seasonally adjusted prices in months i and j in the same year y , using (19) or (20), we find that:

$$(21) \quad p_{y,i}^a / p_{y,j}^a = 1, \quad y = 1, 2, \dots, Y \text{ and } m = 1, 2, \dots, M.$$

Thus seasonally adjusted data based on calendar year models can provide absolutely no information about the month-to-month change in seasonally adjusted prices for months in the same year.

Faced with the above negative result for methods of seasonal adjustment based on calendar years, we turn to noncalendar year methods of seasonal adjustment.

5. Rolling Year Concepts for the Seasonal

The calendar year is an artificial construct that is determined by tradition. Hence instead of comparing the price of a commodity in a given season of a calendar year to an average of the calendar year prices, why not compare this price to an average of the prices in the rolling year centered around the given season?

Thus if the number of seasons M in the year is odd,⁷ then the centered rolling average of the prices in the rolling year centered around a given period $t \equiv (y-1)M + m$ is defined as

$$(22) \quad P_t \equiv (1/M) \left\{ \left[\sum_{m=1}^{(M-1)/2} p_{t-m} \right] + p_t + \left[\sum_{m=1}^{(M-1)/2} p_{t+m} \right] \right\}.$$

If M is even,⁸ then the centered rolling average of the prices in the “year” surrounding period t is conventionally defined as⁹

$$(23) \quad P_t \equiv (1/M) \left\{ (1/2)p_{t-M/2} + \left[\sum_{m=1}^{(M/2)-1} p_{t-m} \right] + p_t + \left[\sum_{m=1}^{(M/2)-1} p_{t+m} \right] + (1/2)p_{t+M/2} \right\}.$$

Note that when M is even, the centered rolling average extends over $M+1$ seasons with the two seasons furthest away from the center period t receiving only one half of the weight that the other prices receive. In words, the P_t defined by (22) or (23) are (arithmetic) average levels of prices for a year centered around the given period t . Given the centered annual average levels of prices defined by (22) or (23), we can now define the corresponding period t additive rolling year seasonal factors S_t :

$$(24) \quad S_t \equiv p_t - P_t.$$

We can also use P_t in order to define the period t multiplicative rolling year seasonal factor s_t :

$$(25) \quad s_t \equiv p_t / P_t.$$

The multiplicative model defined by (22) or (23) and (25) is known as a ratio to moving average model¹⁰ of the seasonal and it dates back to Macaulay¹¹ at least:

“A few years ago the writer was approached by the statistical department of a government bureau and asked to propose a good but simple method of discovering any seasonal fluctuations which might exist in economic time series of moderate length. He replied that, as he did not know of any simple and yet really ideal method, he would suggest graduating [smoothing] the data roughly by means of a 2 months rolling average

⁷ This will be the case for days and semesters.

⁸ This will be the case for weeks, months and quarters.

⁹ Macaulay (1931; 122) was an early pioneer in the use of this convention.

¹⁰ Joy and Thomas (1928; 241) use this terminology. Joy and Thomas (1928; 242) attributed the method to Dr. Fred R. Macaulay of the National Bureau of Economic Research.

¹¹ The Federal Reserve Board (1922; 1416) used this method as a building block into its method of seasonal adjustment but the method was attributed to Mr. F. R. Macaulay of the National Bureau of Economic Research. The Board continued to use Macaulay’s method as a building block for many years; see Barton (1941; 519-520).

of a 12 months rolling average, taking the deviations of the data from this rolling average (centered), and arriving at seasonal fluctuations from these deviations. Rough as is the method, it has been widely used and favorably noticed year after year. Moreover, though the method is extremely simple, in most cases the results are quite good.”

Frederick R. Macaulay (1931; 121-122).

Macaulay’s method is known as the *ratio to moving average method*.

Obviously, we could replace the centered arithmetic average of prices defined by (22) (if the number of seasons M is odd) or by (23) (if the number of seasons in the year is even) by corresponding geometric averages (provided that all prices are positive)¹². This substitution would generate additional models for the seasonal factors. However, note that the new seasonal factors generated by these rolling year models will no longer necessarily satisfy counterparts to our calendar year consistency constraints (4), (13), (16) or (18).

As was done for our calendar year models for the seasonal factors, after half a year has passed, we can generate seasonally adjusted data for each period t . Thus after $M/2$ seasons have passed (in the case where M is even), the measure of the average level of prices centered around period t , P_t defined by (23), can be calculated and using the additive seasonal model (24), the seasonally adjusted level of price p_t^a for period t can be defined as:

$$\begin{aligned}
 (26) \quad p_t^a &\equiv p_t - S_t \\
 &= p_t - [p_t - P_t] && \text{using (24)} \\
 &= P_t.
 \end{aligned}$$

In the case of the rolling year multiplicative seasonal model (25), the seasonally adjusted level of price p_t^a can be defined as:

$$\begin{aligned}
 (27) \quad p_t^a &\equiv p_t / s_t \\
 &= p_t / [p_t / P_t] && \text{using definition (25)} \\
 &= P_t.
 \end{aligned}$$

For both the additive and multiplicative models of the seasonal, it now makes sense to compare the seasonally adjusted level of prices in period t to the seasonally adjusted level of prices in period r , even if both periods are in the same year. Thus using (26) or (27), we have:

$$(28) \quad p_t^a / p_r^a = P_t / P_r \quad \text{for all periods } t \text{ and } r.$$

However, using (28), the structure of the comparison of the seasonally adjusted prices for period t relative to period r , p_t^a / p_r^a , becomes clear: we are comparing two measures of the average

¹² Suppose there are missing prices in our data set. If we set these missing prices equal to zero, then the centered moving (arithmetic) averages of prices can still be defined and the ratio to moving average seasonal factors can still be defined. However, if any price in the rolling year is zero, then the centered moving geometric average of the prices in the rolling year is zero, which is not informative!

annual level of prices centered around the two comparison periods, P_t/P_r . Thus using seasonally adjusted data for making price comparisons between two periods leads to comparisons of two annual measures of average prices centered around the two periods being compared. Note that it is immaterial whether we use the additive or multiplicative model of the seasonal; both models lead to the same seasonally adjusted comparison given by (28). However, note that the form of the centered annual average still matters; if we replace the arithmetic means in (22) or (23) by say geometric means, then we would in general obtain different numbers on the right hand side of (28). Thus the theory of seasonal adjustment based on the rolling year concept (instead of the calendar year concept) is still not completely unambiguous. We still have to decide on what is the most appropriate functional form for the mean function, $\mu(p_{t-(m-1)/2}, \dots, p_{t-1}, p_t, p_{t+1}, \dots, p_{t+(m+1)/2})$, that aggregates the M prices (if M is odd) that are centered around period t into an annual average.

However, for our purposes, the important lesson that has been learned in this section thus far is that *seasonally adjusted data based on the calendar year or rolling year models that we have considered thus far cannot provide any information whatsoever on the short run movement of prices going from one season to the next*. In the case of rolling year models of seasonal adjustment, the seasonally adjusted number for a given period is actually an estimate of an annual average level of prices centered around the period in question; i.e., it is a measure of longer run trend rather than a true short run period to period measure of price.¹³

In the following section, we ask whether more general time series models of the seasonal can generate valid estimates of the underlying short run period to period movement in prices.

6. Early Time Series Models for the Seasonal

The rolling year model of the seasonal that was defined by (24) in the previous section can be rewritten as the following additive model of the seasonal:

$$(29) \quad p_t = P_t + S_t, \quad t = 1, 2, \dots, T$$

where p_t is the observed price in period t , P_t is a seasonal free measure of the price in period t and S_t is the period t additive seasonal factor. Similarly, the rolling year model of the seasonal defined by (25) can be rewritten as the following multiplicative model of the seasonal:

$$(30) \quad p_t = P_t s_t, \quad t = 1, 2, \dots, T$$

where p_t and P_t are still the actual and seasonally adjusted price for period t and s_t is a multiplicative seasonal factor.¹⁴ Time series models (or statistical models) of the seasonal can be obtained by appending random errors or irregular components to the right hand sides of (29) or

¹³ However, for inflation targeting purposes, central banks are very much interested in up to date measures of the trend in prices such as a rolling year average of prices.

¹⁴ Note that by taking logarithms, the multiplicative model (30) can essentially be transformed into the additive model (29).

(30). Thus let us append an unobserved additive error term to the right hand side of (29) in order to obtain the following time series model for additive seasonal factors:

$$(31) \quad p_t = P_t + S_t + E_t, \quad t = 1, 2, \dots, T.$$

Obviously, the unobserved components on the right hand side of (31) cannot be estimated without further identifying restrictions. In the remainder of this section, we shall consider a few of the early approaches to identification that have appeared in the literature.

Early approaches to identifying the components on the right hand side of (31) were made by Hart (1922) and Stone (1956; 81). Stone assumed that the trend P_t was linear and the seasonal effects S_t were constant over years; i.e., Stone assumed that the following linear regression model parameterised (31):

$$(32) \quad p_t = \alpha + \beta[t-1] + \gamma_m + E_t, \quad t = (y-1)M + m = 1, 2, \dots, T$$

where the trend P_t is defined to be the linear in time function $\alpha + \beta[t-1]$ and the seasonal factor for season m is the fixed parameter γ_m for $m = 1, 2, \dots, M$. The fixed additive seasonal effects γ_m were assumed to satisfy the following linear restriction:

$$(33) \quad \sum_{m=1}^M \gamma_m = 0.$$

In Hart's (1922) approach to the seasonal, he first fitted a linear trend to the p_t observations. He then took the arithmetic means of the deviations from the trend to represent the seasonal factors γ_m . Stone (1956; 81) showed that his formal regression model was equivalent to Hart's two stage procedure.¹⁵

Another early approach to identifying the unobserved components on the right hand side of (31) is due to Leser (1963; 1034) who added seasonal dummies to the Whittaker (1923) Henderson (1924) penalised least squares method of smoothing.¹⁶ In this method, the trend parameters P_t and the M fixed seasonal parameters γ_m are determined by minimising the following objective function with respect to P_1, P_2, \dots, P_T and $\gamma_1, \gamma_2, \dots, \gamma_M$:

$$(34) \quad \sum_{t=1}^T (p_t - P_t - \gamma_m)^2 + \lambda \sum_{t=2}^{T-1} (\Delta^2 P_t)^2,$$

where $t = (y-1)M + m$ as usual and the M seasonal fixed effects γ_m satisfy the linear constraint (33) above. $\Delta^2 P_t$ is the centered second difference of the trend time series P_t ; i.e.,

$$(35) \quad \Delta^2 P_t \equiv [P_{t+1} - P_t] - [P_t - P_{t-1}] = P_{t+1} - 2P_t + P_{t-1}.$$

The positive parameter λ , which appears in the objective function (34), is a *smoothing parameter* which trades off how well the estimated $P_t + \gamma_m$ will fit the actual data p_t (the smaller λ is, the better will be the fit) versus how smooth the trend series P_t will be (the larger

¹⁵ Stone (1956; 77) required that the number of observations be a multiple of the number of seasons in the year.

¹⁶ Macaulay (1931; 89-99 and 151-156) devotes an entire chapter and appendix to this method of smoothing but he does not simultaneously estimate the P_t and the γ_m ; instead, just the P_t are estimated.

λ is, the closer P_t will be to a linear trend). In the macroeconomic literature, it is conventional to a priori choose λ to equal 1600¹⁷ but it is possible to devise more “objective” ways of choosing λ . The basic idea for trading off fit and smoothness in order to smooth a series can be traced back to the early actuarial literature¹⁸ where it was necessary to “graduate” or smooth mortality tables:

“Where, however, we have a series of observations at consecutive ages it is necessary to substitute a smooth series for the irregular one representing the ungraduated observations. The substituted series must, from the nature of things, be the result of a compromise between the two factors of smoothness and closeness to the observed facts. It is theoretically possible to assign a basis for the numerical measurement of the irregularity of a series as well as for its departure from the observed facts, and by assigning the proportion in which an increase in the one is to be taken as counterbalancing a decrease in the other, to arrive by a mathematical process at the series which best harmonizes the two factors. On any basis suggested, however, the resulting equations are numerous and unwieldy to such an extent as to render the process practically prohibitive.”¹⁹

Robert Henderson and H. N. Sheppard (1919).

Thus Henderson and Sheppard had a clear conception of the basic idea that smoothing involves a tradeoff between goodness of fit and the “smoothness” of the resulting measure of central tendency, P_t . We will return to this point later.

A problem with both the Stone (1956; 81) and Leser (1963; 1034) methods for estimating the trends P_t is that their methods seasonally adjust the entire data set of $T = YM$ observations on p_t in one step. This has the disadvantage that when another year’s data become available, the entire seasonal adjustment procedure has to be done all over again; i.e., in principle, their estimates of the seasonally adjusted data are never “final”.²⁰ Thus these procedures are not very well adapted to the needs of statistical agencies. Macaulay noted this disadvantage of the Whittaker-Henderson method for the determination of the trend:

“Professor Whittaker stresses the fact that in obtaining the graduation all observations are used. The position of each datum point affects the position of every point on the smooth curve. ... It would be highly undesirable that a change in the position of a datum point should seriously affect the position of distant parts of the smooth curve. For example, one of the great disadvantages of harmonic analysis is that the configuration of the data in one section may seriously affect the shape of the fitted curve in a far distant section.”

¹⁷ See Hodrick and Prescott (1980; 5) or Kydland and Prescott (1990; 9).

¹⁸ The early actuarial literature is responsible for other modern smoothing techniques as well. De Forest (1873; 290-292) showed how least squares rolling average smoothing functions of varying window length could be derived that were exact for cubic functions. De Forest (1873; 322-324) also showed how the weights for his exact rolling average estimators could be chosen to resemble kernel smoother weights. The concept of a spline curve (a curve made up of polynomial segments which are joined up in a continuously differentiable manner) is due to the actuary Sprague (1891; 277).

¹⁹ Both Whittaker (1923) and Henderson (1924) used the sum of squared residuals as their formal measures of fit; Whittaker used the sum of squares of third differences in place of the second differences $\Delta_2 P_t$ which appear in (34) while Henderson considered using the sum of squares of first, second and third differences as formal measures of smoothness. The general solution to the quadratic Whittaker-Henderson smoothing problem, which involves minimising (34), requires the inversion of a large matrix, which was not technologically possible in 1919 when Henderson and Sheppard wrote their study. However, Henderson (1924) showed how by strategically choosing the smoothness parameter λ , and applying the theory of difference equations, one could obtain solutions.

²⁰ Balk (1981; 77) noted that his method of seasonal adjustment suffered from this practical disadvantage.

Frederick R. Macaulay (1931; 96-97).

Thus Macaulay also noted some of the practical problems associated with using strictly periodic functions such as sines and cosines to represent trends: the resulting “smooth” curves tend to have too many wiggles that show up in the “wrong” places.²¹ Finally, Macaulay noted that all smoothing methods have difficulties in approximating the trend near the beginning and the end of the sample period:

“A Whittaker-Henderson graduation needs no extrapolation; it covers the entire range of the data. This is a distinct element of mathematical elegance and sometimes an important practical consideration. However, graduation of the ends of almost any series is necessarily extremely hypothetical unless facts outside the range covered by the graduation are used in obtaining the graduation. This is as true of the Whittaker-Henderson graduation as of any other type.”

Frederick R. Macaulay (1931; 94-95).

In view of the above difficulties with the Stone and Leser methods, we turn to a third class of methods that might be used to identify the components on the right hand side of (31), namely *moving average models*. These models have their origins in the ancient actuarial literature where the process of smoothing a mortality table was known as graduating the data.²² However, the most comprehensive study of moving average models in the context of seasonal adjustment is the monograph written by Macaulay (1931) and we now turn to his work.²³ Macaulay noted the following problem with representing the trend of an economic time series by a simple centered rolling average of the type defined by (22) or (23) in section 5 above:

“It has, however, serious drawbacks. The resulting curve is seldom very smooth and it will not give a perfect fit to data except in ranges which can be adequately described by a straight line. For example, a simple moving average, if applied to data whose underlying trend is of a second-degree parabolic type, falls always within instead of on the parabola. If applied to data whose underlying trend is of a sinusoidal type, it falls too low at maximum points and too high at minimum points.”

Frederick R. Macaulay (1931; 23).

Thus a simple, equally weighted moving average, when applied to a quadratic curve, will not exactly reproduce it; it will reproduce exactly only linear trends. Macaulay identified another potential problem with the use of an annual centered moving average to represent the trend:

“In general, if a type of smoothing be desired which shall, when applied to monthly data, eliminate seasonal and erratic fluctuations and at the same time give a smooth curve adequately describing the remaining cyclical and trend factors, something much more than a simple 12 months moving average must be used.”

Frederick R. Macaulay (1931; 23-24).

Thus Macaulay identified two problems with the use of an annual centered moving average with equal weights to represent the trend:

²¹ Higher order polynomial approximations to the trend also suffer from this defect. Cole (1963; 135) observed that the Census II program also had a tendency to put too many wiggles in the smoothed data. The Census II seasonal adjustment procedure is described in Shiskin and Eisenpress (1957). The X-11 procedure was a further refinement of Census II; see Shiskin, Young and Musgrave (1967).

²² One of the earliest moving average models was due to the actuary Woolhouse (1870). This moving average actuarial literature was reviewed by Henderson and Sheppard (1919; 23-42) except that the early work of De Forest (1873) was not discussed. Wolfenden (1925) reviewed the work of De Forest.

²³ Macaulay’s work was used as a basic building block for the seasonal adjustment methods that were pioneered by the U.S. Bureau of the Census; e.g., see Shiskin and Eisenpress (1957) and Shiskin, Young and Musgrave (1967).

- If the erratic or random fluctuations E_t in the additive model (31) are very large, then the annual centered moving averages of the form defined by (22) or (23) may also have large fluctuations and hence may not be very smooth.
- The equally weighted centered rolling averages of the type defined by (22) or (23) above are exact only for linear trends; i.e., these simple moving averages will not reproduce nonlinear “smooth” trends.

Let us address the second problem first. It is possible to set up a simple linear regression model of the following type for n consecutive data points:

$$(36) \quad p_t = \alpha + \beta t + \lambda t^2 + E_t, \quad t = 1, 2, \dots, n.$$

Now assume n is odd and derive a formula for the predicted value for the p_t in the middle of the sample period. If $n = 5$, De Forest (1873; 327) showed that the resulting least squares estimator of the trend for P_3 , the middle season in the run of 5 seasons, is:

$$(37) \quad P_3 = (1/35)[-3p_1 + 12p_2 + 17p_3 + 12p_4 - 3p_5].$$

Obviously, a moving average formula of the type will exactly reproduce both linear and quadratic trends. De Forest (1873; 327) also listed the corresponding least squares moving average formulae that are exact for quadratic trends for $n = 7$ and 9 observations as well while Macaulay (1931; 46) listed the corresponding least squares based moving average formulae for $n = 13$ observations that is exact for linear and quadratic trends. Macaulay (1931; 49) called this method of generating moving average estimators for the trend, the *method of moving parabolas*. Obviously, this idea can be extended to models where the trend is a polynomial of higher order.²⁴ Recall Macaulay’s first objection to the use of an annual centered rolling average as an estimator of the trend; i.e., that this estimator will not be sufficiently smooth if the random component E_t on the right hand side of (31) is large relative to the size of the trend. In theory, this problem can be resolved by increasing the window length n ; i.e., by increasing the length of the rolling average that is exact for a least squares polynomial regression model with n observations. This will result in a smoother trend but this improvement in smoothness is achieved at a cost in terms of how closely the estimated $P_t + S_t$ will fit the actual p_t .²⁵ This is the classic conflict between fit and smoothness that we have already alluded to. As usual, Macaulay had a pretty clear understanding of the problem:

“Unless the erratic fluctuations of the data are very small as compared with the amplitude of the cyclical movements, a large number of terms will have to be used in the parabolic set of weights or the data will not be adequately ‘smoothed’. However, unless the cycles of the original data have very long periods, it will not be possible to use a large number of terms without departing too far from the underlying fundamental curve.”

Frederick R. Macaulay (1931; 49-50).

²⁴ See Sheppard (1914; 175) and Whittaker and Robinson (1926; 291-297) for models of this type.

²⁵ Once the appropriate estimate of the trend P_t has been obtained, the seasonal factors S_t can be estimated by averaging the detrended data $p_t - P_t$ over periods t that correspond to the same season of the year. However, for our purposes the exact method for the determination of the seasonal effects does not matter very much since our focus is on obtaining estimates for the trend P_t .

There is no completely unambiguous “best” way to resolve this conflict between smoothness and fit although various model selection techniques like cross validation have been developed to help solve this problem.²⁶ It is interesting to note that De Forest also had a pretty clear idea of some of the difficulties involved in estimating an unknown trend function in the face of noisy data:

“Not only is absolute accuracy unattainable, but we cannot even decide, by the method of least squares, that a certain result is the most probable of any; for the true form of the function being unknown, any particular residual error, or difference between the observed and computed values of a term, will in general be the aggregate of two errors, one of them due to the difference of form between the assumed function and the true one, and the other due to the error of observation or difference between the observed value and the true value.”

Erastus L. De Forest (1873; 301).

Macaulay had another very important objection to the use of least squares based moving average estimates for the trend when there is seasonality in the data:

“A first reason is that such a graduation [smoothing by a centered least squares moving average formula] will entirely eliminate seasonal fluctuations by only the most improbable accident. If, neglecting for the moment erratic fluctuations, the original monthly data be thought of as made up of two parts, (1) a smooth curve and (2) a seasonal fluctuation superposed on the smooth curve, the results of fitting a parabola to the smooth curve and another parabola to the seasonal fluctuations and added together, each month, the pairs of resulting ordinates. Now, if the seasonal fluctuations were constant from year to year, the smooth curve fitted to them should by the definition of seasonal fluctuations, be simply $y=0$. In general, a curve fitted to seasonals will give continuous zero values only if its weight diagram is such that equal weights to each nominal month. A simple 12 months moving average gives such equal weights to each nominal month.”

Frederick R. Macaulay (1931; 47-48).

What Macaulay seems to be saying is this: suppose that we have an additive model of the form (31) where both the trend terms P_t and error terms E_t are known to be zero. Further suppose that the seasonal terms S_t are constant in each season; i.e., $S_t = S_{(y-1)M+m} = \gamma_m$ and the γ_m satisfy (33). Now fit a polynomial trend using least squares to the $p_t = S_t$ for some window length n . In general, the resulting estimate of the trend will not be zero as it should be.²⁷ However, part of the problem is that Macaulay is following in the traditions of the literature of his day when the trend was measured first and then the detrended data were used in order to estimate seasonal factors. Macaulay’s observation shows that that this two stage procedure runs into identification problems: some of the seasonal will generally be imputed to the trend! The same problem can occur even if the trend and seasonal parameters are estimated simultaneously in a single stage procedure. In more general models of the seasonal where the seasonal factors are allowed to change over time, it becomes impossible to disentangle the effects of changing seasonal factors from the trend.²⁸ For additional material on the early history of seasonal

²⁶ See for example Craven and Wahba (1979) and Akaike (1980).

²⁷ Macaulay seemed to think that the method would work provided only that we fit a linear trend but this is not always the case. Think of a trimester model where we have data for 3 periods and the seasonal is -1 in period 1, 0 in period 2 and $+1$ in period 3. If we fit a linear trend using only $n=3$ observations, the linear trend will completely absorb the seasonal and hence the correct seasonal for trimesters 1 and 3 will not be recovered. The problem diminishes as n increases but it never completely disappears.

²⁸ Wisniewski (1934; 180) emphasised this point.

adjustment methods plus a comprehensive review of more recent methods for seasonal adjustment, see Bell and Hilmer (1984; 293-299).

The discussion in this section should alert the reader to the fact that seasonal adjustment is not as simple as it appears at first glance. Several problems have been encountered:

- In the face of noisy data, it is impossible to know what the true functional form for the “smooth” part of the price series is.²⁹
- Once the “noise” term E_t has been introduced to the right hand side of the basic equation $p_t = P_t + S_t + E_t$, we encounter the problem of trading off fit against “smoothness” and there is no unique answer to this tradeoff.³⁰
- Once the seasonal factors S_t are allowed to change over time, it becomes very difficult to disentangle the trend P_t from these changing seasonals.

In the following section, we will consider the problems associated with time series methods for seasonal adjustment more generally.

7. Anderson’s Critique of Time Series Models

The basic problem with all of the above time series methods for seasonal adjustment is that each method is more or less arbitrary. For example, let us start with Person’s (1919; 8) decomposition of a time series into unobserved components. Using his classification, our representative price series p_t is assumed to have the following decomposition:

$$(38) \quad p_t = T_t + C_t + S_t + E_t, \quad t = 1, 2, \dots, T,$$

where T_t is the long term trend portion of p_t at period t , C_t is the business cycle component of the series at time t , S_t is the seasonal component and E_t is an “error” or “erratic” component for period t . Thus comparing (38) with our earlier additive decomposition (31), it can be seen that our earlier trend term P_t is now decomposed into a longer term trend T_t plus a shorter term trend C_t that represents trends over the course of a normal business cycle.³¹ Recall our earlier discussion in section 3 above where we noted that there were other alternatives to the additive model of the seasonal. The same discussion is relevant to our present more complex additive model of the seasonal defined by (38) above. Thus, after further reflection on the adequacy of the

²⁹ Note that it is also not a trivial matter to define exactly what “smooth” means.

³⁰ In every nonparametric method for smoothing, a *smoothing parameter* determines the tradeoff between fit and smoothness. See Buja, Hastie and Tibshirani (1989) for a catalogue of these smoothing parameters.

³¹ This type of decomposition can be traced back to Cournot. Cournot (1838; 25) initially distinguished “secular variations” and “periodic variations” (which correspond to the T_t and C_t parts of Person’s decomposition) and later, Cournot (1863; 149) added “transitory” or “accidental” perturbations (which correspond to the E_t part of Person’s decomposition).

additive model, we may decide that the following multiplicative model of the seasonal is more plausible:

$$(39) \quad p_t = T_t C_t S_t E_t, \quad t = 1, 2, \dots, T.$$

Upon even more reflection, we might decide that both the additive and the multiplicative models of the seasonal, (38) and (39) respectively, are too restrictive and so we postulate the existence of a function F such that

$$(40) \quad p_t = F(T_t, C_t, S_t, E_t), \quad t = 1, 2, \dots, T.$$

It is very obvious that it will be necessary to:

- Make some arbitrary assumptions in order to determine the functional form for F .
- Even if F is determined as in (38) or (39), it will be necessary to make further arbitrary assumptions in order to identify the components, T_t , C_t , S_t and E_t .

The above *fundamental functional form determination and unobserved component identification problem* has been noticed in the literature but the most complete statement of it by Anderson has been largely forgotten:

“We must either obtain the missing ($mN-N$) equations from other sources, which can happen only in very exceptional cases, or introduce some preliminary assumptions, some hypotheses concerning the construction of the aggregates V , which would replace the missing equations. Thus, in most cases with which the social investigator has to deal in practice, in the decomposition of series into components, neither the definition of the function F nor the finding of the numerical meanings of the effects caused by the aggregates of cases V' , V'' , V''' , V'''' [T_t , C_t , S_t , E_t] is possible without the introduction of different hypotheses which are more or less arbitrary.”

Oskar Anderson (1927; 552-553).

Assuming that we have solved the functional form problem and say have chosen the additive model (38), Anderson went on to explain how the various components on the right hand side of (38) might be identified:

“Further, the investigator again limits arbitrarily the circle of his possibilities. For example:

- (a) assuming that the secular component [T_t] represents a polynomial function of the argument t (time or ordinal number) ...
- (b) assuming that the cyclical component [C_t] can be represented as a more or less complex trigonometrical function;
- (c) assuming that the residual component e [E_t] is a random series.”

Oskar Anderson (1927; 554).

From the above quotations, it can be seen that Anderson had a very clear conception of the difficulties involved in finding the “right” functional form for a time series seasonal model and in determining the unobserved components in such a model. Similar criticisms of time series models of the seasonal have been expressed in more recent times:

“It is necessary, in these situations, to restrict the class (20) of models so that the seasonal component of a series can be determined, theoretically and empirically. Often, restrictions are provided by the nature of the problem or by specific information ... The problem here, as elsewhere, is that a consensus on this theory is lacking. One person prefers to define trend or cyclic effects in one way, another differently. In multivariate approaches, there are probably as

many varieties of plausible specifications of relationships among and between their components, all essentially compatible with the data, as there are social scientists (economists, statisticians, etc.) to specify these variables and relationships. This situation is evidently a general one in econometric modelling, where a variety of specifications, including a purely autoregressive equation, are all compatible with the data and all have comparable predictive power.”

David A. Pierce (1978; 245-246).

The above Anderson critique of time series models indicates that these models generate a huge range of plausible seasonal adjustment factors. How could this range be reduced? One possible solution would be to take an *axiomatic* or *test approach* to the determination of the unknown function F and the unobserved components in (40).

In this approach, alternative seasonal adjustment procedures would be judged by their axiomatic properties. This test approach to seasonal adjustment procedures has in fact been formally and informally pursued by Hart (1922; 342-347), Macaulay (1931; 100-104), Lovell (1963) (1966), Grether and Nerlove (1970), Fase, Koning and Volgenant (1973) and Pierce (1978; 246-247) among others³² but no consensus has been reached on what the appropriate set of axioms should be. Perhaps part of the problem has been that it is too difficult to work with the very general seasonal model defined by (40). Perhaps, it would be better to start with the very simple seasonal model defined by

$$(41) \quad p_t = f(P_t, S_t)$$

where p_t is the series to be seasonally adjusted, P_t is the trend and S_t is the seasonal component; i.e., we have combined the trend and cycle terms T_t and C_t in (41) into a single trend term P_t and we have dropped the irregular term E_t in the simplified model defined by (41). P_t and S_t would be functions of the price data surrounding p_t for some window length n ; i.e., we would also have:

$$(42) \quad P_t = g(p_{t-n}, \dots, p_{t-1}, p_t, p_{t+1}, \dots, p_{t+n}); S_t = h(p_{t-n}, \dots, p_{t-1}, p_t, p_{t+1}, \dots, p_{t+n})$$

for some functions g and h and for some window length n . Thus the functions f , g and h would have to be determined (perhaps along with the window length n)³³ by this simplified axiomatic approach. The axiomatic framework generated by (41) and (42) would appear to be a closer counterpart to the test or axiomatic approach to index number theory, which also abstracts from stochastic elements. If a consensus set of axioms for the model (41)-(42) were to lead to a definite seasonal adjustment procedure, then perhaps, stochastic considerations could be introduced at a later stage, as is the case with index number theory based on the test approach. However, until economists and statisticians can agree on a “reasonable” axiomatic framework for the test approach to seasonal adjustment procedures, this approach will not be of much help to statistical agencies.

³² Cole (1963; 136) informally introduced a “test” in the following quotation: “Theoretically, a twelve month rolling average of a seasonally adjusted series should be the same as the twelve month rolling average of the original series. In the exhibit I have given you, you can see that in certain cases differences were almost as much as 10%. When differences as great as these occur, we have reason to wonder if the other results obtained are reliable.”

³³ As we have seen earlier, choosing the “right” window length is not a trivial problem.

8. The Interpretation of Seasonally Adjusted Data

What are we to make of the above catalogue of problems associated with time series methods for seasonally adjusting a price series? A number of tentative conclusions can be drawn from the above discussion:

- For each of the methods of time series seasonal adjustment that we have considered above, in every case, the seasonally adjusted period t price P_t has the interpretation as a measure of longer term trend in the unadjusted series p_t . Thus month-to-month comparisons of the seasonally adjusted prices are best interpreted as month-to-month comparisons in the *trend* of the price series rather than a true short run month-to-month comparison of prices.
- In all of the time series methods for seasonal adjustment that we have considered, either: (a) a final estimate for the trend P_t for a given period t is not available until at least an additional half year of data on p_t have been collected³⁴ or (b) as new data become available, new estimates for the trend factors P_t have to be *recomputed*, and thus in principle, estimates are never final. Moving average methods of seasonal adjustment like the Census II method³⁵ or the X-11 method³⁶ fall into category (a) while the Stone and Leser methods fall into category (b). This point indicates that seasonally adjusted price series that use time series methods cannot provide timely and accurate information on the short term movement of prices.

Information provided by a statistical agency should be *objective* and *reproducible*. Objective means that the methods used to generate data should be based on definite criteria that can be explained to the informed public. Moreover, there should be some consensus among

³⁴ Again recall Macaulay's (1931; 26) point that smoothing methods are inherently inaccurate at the endpoints of the data set in their domain of definition: "The tail end of any curve has necessarily a large probable error, and thoroughly inadequate results which would be likely to check with later data, when received are generally quite improbable. This is just as true of graduations such as the Whittaker-Henderson, which need no extrapolation, as of graduations which require extrapolation. Moreover, mathematical extrapolation does not solve this difficulty."

³⁵ The length of time it takes to determine final estimates depends on the length of the rolling average formula selected to represent the trend. Shiskin and Eisenpress (1957; 419-420), while discussing the Census II method, made the following observations on how the length of the moving average should be determined: "Graduation formulas are available which provide smooth and flexible curves and also eliminate seasonal fluctuations; for example, Macaulay's 43 term formula. But such formulas involve the loss of a relatively large number of points at the beginnings and ends of series. Graduation formulas which provide similarly smooth and flexible curves and involve the loss of relatively few points do not also eliminate seasonal variations. The computation for a preliminary seasonally adjusted series is now easy mechanically; on the other hand, the replacement of missing points is difficult conceptually. We, therefore, choose one of the formulas that requires a preliminary seasonally adjusted series, but also minimizes the loss of points, the Spencer fifteen-month weighted rolling average." Thus using the Census II method for seasonal adjustment, it would be necessary to wait seven months after the collection of the unadjusted p_t in order to obtain a final estimate for the corresponding seasonally adjusted P_t . Spencer's (1904) 15 term formula is described in Macaulay (1931; 55).

³⁶ Bell and Hillmer (1984; 308) make the following observation: "For X-11 with standard options, the final adjustment is effectively obtained three years after the initial adjustment..." Bell and Hilmer (1984; 296) also note that: "Eventually (typically after three years) the X-11 ARIMA adjustments converge to the X-11 adjustments..." Thus for these methods, one has to wait approximately three years to obtain final estimates.

experts that the criteria used are the “best” that are currently available. Reproducible means that if a competent statistician or economist were brought into the statistical agency and given the relevant criteria and methodology and told that he or she should produce a series, then these different statisticians and economists when given these instructions and the same data set would in fact produce roughly the same series. Given Anderson’s (1927) fundamental critique of the general impossibility of unambiguous identification of the components in a time series model of the seasonal, *it seems doubtful that seasonally adjusted data can be completely objective since different analysts will generally make different functional form assumptions and place different identifying restrictions on the model in order to identify the unobserved components*. It also seems doubtful that seasonal adjustment procedures like X-11 and its successors are reproducible since different operators of these adjustment procedures will generally make different choices as they go through the menu of options that are available.

One could also argue that current times series methods for seasonal adjustment are not *comprehensible*; i.e., they are so complex that they cannot be readily explained to the informed public:

“Even though the public appears for the most part to be comfortable with seasonally adjusted data, we doubt that many users understand the methods by which the data are produced. It may be too much to expect the statistically unsophisticated person to understand the procedures underlying seasonal adjustment, but even statistical experts are often mystified by these procedures, including the most widely used method, Census X-11. This method uses a set of moving averages to produce seasonally adjusted data; and although the basic idea of moving averages is simple enough, the method in which they are applied in the X-11 program is extremely complex. Moreover, the theoretical statistical underpinnings of X-11 and many other seasonal adjustment methods are not understood by many users. Thus many users of adjusted data merely trust that the adjustment procedure is providing useful data, and critics have advocated the abolishment of seasonal adjustment.”

William R. Bell and Steven C. Hillmer (1984; 291)

Finally, we have not stressed the difficult problems involved in seasonally adjusting sporadic or intermittently available data; i.e., for many micro international price series, the corresponding commodity is simply not available for one or more seasons of the year. This problem has received very little attention in the time series seasonal adjustment literature (and in the index number literature as well, although Zarnowitz³⁷ (1961; 243-246), Turvey (1979) and Balk (1980) (1981) are notable exceptions), even though, in many data sets, the problem is pervasive. If we attempt to seasonally adjust price series of this type (which can be done using additive models for the seasonal), then *it is clear that comparisons of the resulting seasonally adjusted prices from one season to the next cannot give any information about the actual short run changes in price for comparison periods when the commodity is not available*. This point just reinforces our earlier conclusion that seasonally adjusted data cannot adequately represent the short term season to season change in prices; they can only represent movements in the longer term trend in prices. In principle, seasonally adjusted data could provide valuable information on the longer run trend in prices. However, major problem with existing time series methods for seasonal adjustment is that these methods decompose a price or quantity series into trend, seasonal and irregular components but the method of decomposition is far from being

³⁷ “There is simply no escape from the truism that any comparison of two magnitudes such as p_i and p_j requires that both of them be actually given.” Zarnowitz (1961; 244).

unique, as Anderson's critique shows. Thus it may be helpful to use some ideas from the index number literature to unambiguously determine the trend.

This can be done using the rolling or rolling year indexes suggested by Diewert (1983) (1996) (1999). Once the trend component has been determined in a unique fashion using index numbers, econometric methods could be utilized in order to use current information to forecast this unambiguous trend component.³⁸ The major advantage of the rolling year index number method for finding the trend in a price or quantity series is that it is perfectly *reproducible* once a consensus has been reached on the choice of the index number formula to be used. On the other hand, when using econometric methods for finding the trend that are based on moving average methods, one has to decide on the structure of the seasonal (Anderson's identification problem), the length of the moving average window and the tradeoff between fit and smoothness. The rolling year index number method is much more "objective".³⁹

However, it should be noted that the rolling year index for say February of this year is a measure of annual price change that is centered around a rolling year that lags the current rolling year ending in February by six months. Thus to obtain the rolling year measure of the seasonally adjusted trend in prices that is centered around this February, we would have to wait seven months. Hence although the production of rolling year indexes might lead to unemployment for the seasonal adjusters in a statistical agency, they could readily find new employment in the *forecasting* branch of the agency, since there would still be a demand on the part of users for forecasts of the annual rolling year estimate of price change that is centered around the current month.

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³⁸ Diewert (1983) (1996; 52-54) (1999), building on the pioneering ideas of Mudgett (1955) and Stone (1956), argued for the use of rolling year indexes as a substitute for seasonally adjusted data.

³⁹ Moreover the index number method for finding price and quantity trends in expenditure aggregates can easily deal with strongly seasonal commodities whereas econometric methods will either fail or be highly complex.

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