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MISINFORMATION*

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A candidate for political office has private information about his and his rival's qualifications. A more informative positive (negative) campaign generates a more accurate public signal about his own (his rival's) qualifications, but costs more. A high type candidate has a comparative advantage in negative campaigns if, relative to the low type, he can lower the voter's belief about his rival more effectively than he can raise her belief about himself and vice versa. In equilibrium, this comparative advantage determines whether the high type chooses a positive or negative campaign. Further, competition helps the high type separate.

1. INTRODUCTION

Much attention has been paid to the use of positive versus negative political campaigns, partly due to the explosive growth of negative campaigns in recent years.[2] Broadly speaking, a candidate uses positive campaigns to praise his own qualifications and negative campaigns to discredit his rival's qualifications. Traditional research has focused on the potential for negative campaigns to alienate voters by lowering the voters' posterior beliefs of the candidates involved and consequently depressing voter turnout (Ansolabehere et al., 1994; Skaperdas and Grofman, 1995). More recent work, however, suggests that negative campaigns provide voters with valuable information and they may not alienate voters (Kahn and Geer, 1994; Wattenberg and Brians, 1999; Lau and Rovner, 2007; Sides et al., 2010). This view also enjoys support from practitioners and political consultants.[3]

Focusing exclusively on the information provision role of political campaigns, this article addresses a hotly debated question: What drives a candidate to run a positive or a negative campaign, and how does the voter evaluate a candidate based on his chosen campaign? We show that campaign choices are determined by the qualifications of a candidate relative to those of his rival's and the voter's prior knowledge of the distribution of qualifications in the candidate pool.[4] In particular, a negative campaign can be effective for a strong candidate, and the voter's posterior belief of the candidate's qualifications may not deteriorate, and could even improve,

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[2] For instance, in the 2006 Congressional elections, 90% of ads run in the final 60 days were negative (Page, 2006). Also, using Wisconsin Advertising Project analysis of Campaign Media Analysis Group data, Wesleyan Media Project found that negative ads have steadily increased from about 27% per party in the 2004 election to about 50% per party in the 2010 Congressional elections.

[3] In U.S. News & World Report, October 6, 2008, Dick Morris pointed out: "Negative ads work and have their place. . . . Negative ads are often the only way voters can penetrate the claims of the various campaigns and get the facts. Voters always tell pollsters that they hate negative ads, but politicians continue to run them. That's because the same polls show that they work."

[4] Unfortunately, the latter is difficult to measure and observe, which may explain the seemingly conflicting empirical findings when the only available data are on the candidates' qualifications and their campaign choices.

following a negative campaign. Instead, the more informative a given campaign is, positive or negative, the stronger a candidate is perceived to be.

In the basic two-type model, a candidate has private, but imperfect, information about his own qualifications and his rival's qualifications, modeled as private beliefs about his own strength and his rival's strength. To signal his type to the voter, the candidate chooses the target of his campaign, which can be either his own qualifications (*positive campaign*) or his rival's (*negative campaign*) and the informativeness (*level*) of his campaign, which is the precision of the campaign signal the voter observes. The realized campaign signal can be favorable or unfavorable. We assume that the campaign signals are noisy but unbiased: Running a more informative campaign reduces such noise but costs more. The voter observes the candidate's campaign choices—whether the campaign is positive or negative and how informative it is—and the realized campaign signal and forms posterior beliefs about the candidates' qualifications. The candidate seeks to maximize the voter's posterior belief of his qualifications over that of his rival's, net of any campaign cost.

In this signaling game, the high type—the type with a stronger incentive to signal his type to the voter—is the one with a greater difference between his own strength and his rival's strength. That is, the high type has greater *overall strength*. Consider the scenario in which the high type candidate is stronger than the low type, but he also faces a stronger rival than the low type does. Suppose that he runs an informative positive campaign in equilibrium so that the voter can learn more about himself. Should the low type imitate? The answer is "no." The low type is more likely to get an unfavorable campaign signal than the high type because he is less likely to be qualified. In expectation, the more informative the positive campaign is, the less successful the low type is in pretending to be the high type. This smaller and decreasing benefit for the low type to imitate the high type gives rise to a least cost separating equilibrium in positive campaigns.

It may seem that the high type can signal his overall strength by simply running a sufficiently informative campaign, positive or negative. This, however, is not true. Suppose our high type candidate in the above scenario goes negative; then he ends up showcasing his rival's qualifications. Because the high type candidate actually faces a stronger rival than the low type does, under any negative campaign the voter will have a higher posterior belief of his rival in expectation, which hurts the candidate. Relative to the low type, then, the high type candidate is more successful in persuading the voter that he is strong than his rival is weak, and thus he should run positive campaigns. In this case, the high type candidate has a *comparative advantage in positive campaigns*. Similarly, if the high type himself is weaker than the low type but faces a weaker rival than the low type does, then he should go negative because he has a *comparative advantage in negative campaigns*.

The issue of where the comparative advantage lies is more subtle when the candidate can use either a positive or a negative campaign to separate. This occurs when the high type candidate is in the best position: He is stronger than the low type, and he faces a weaker rival than the low type does. We find an intuitive sufficient condition that ensures that, for a fixed overall strength, the high type's comparative advantage in positive campaigns increases in his own strength, leading to a monotonic characterization of the high type's comparative advantage. Under this condition, the stronger the high type candidate is, the greater is his comparative advantage in positive campaigns because it is more difficult for the low type to run positive campaigns to imitate him; and the weaker the high type's rival is, the greater is his comparative advantage in negative campaigns. Our analysis also suggests that any policy enhancing a high type candidate's comparative advantage in a kind of campaign makes it easier for him to signal his type and reduces his campaign cost and vice versa. For example, banning negative campaigns cannot help a high type candidate, but it can make him strictly worse off if his comparative advantage lies in negative campaigns.

The insight that comparative advantage determines the choice of positive versus negative campaigns is not driven by the restriction that only one campaign may be used. We show that the high type candidate may not use "contrast campaigns" in which he runs both a positive and a negative campaign even if he can. This is clear if only one kind of campaign can be used to signal his overall strength: Allowing the high type to run a kind of campaign that he avoids in the first place cannot help. It also holds when both positive and negative campaigns can be used for separation if the campaign cost is (weakly) concave in campaign levels. Intuitively, suppose the high type "substitutes" one kind of campaign for another, say by simultaneously increasing the positive campaign level and decreasing the negative campaign level to deter the low type from imitation. Such a maneuver increases the deterrence while lowering the cost. The deterrence from the more informative positive campaign becomes more effective relative to the negative campaign. In addition, due to concavity, the increase in the marginal cost of the positive campaign is smaller than the decrease in the marginal cost of the negative campaign.

The basic model is then extended to understand the candidate's campaign choices under more realistic settings. The presence of independent, third-party evidence about the candidates' qualifications is shown to hurt the high type candidate if it reduces his comparative advantage in a campaign, for instance, if the additional information is about his rival when he would rather run a positive campaign about himself. We also show that our comparative advantage characterization is robust when the candidate and his rival run competing campaigns. Further, competition lowers campaign levels because the candidates' campaigns are strategic substitutes. Finally, in a winner-take-all model we show that a high type candidate can only signal his type by running the opposite kind of campaign preferred by the low type, but separation is still driven by the high type candidate's comparative advantage.

A methodological contribution of this article is to introduce a costly signaling game in which the privately informed sender uses an information structure as a signal. The voter's belief about the qualifications of the candidate and his rival, formed after observing the candidate's campaign choices but before the realization of the campaign signal, plays a crucial role in our analysis. *Misinformation* occurs when this belief differs from the candidate's private belief, as an incorrect belief is used by the voter to interpret the realized campaign signal. The incentive of the low type to misinform the voter in this sense is what drives the separating equilibrium of this signaling game. The idea of a costly signaling game with the sender choosing an information structure is clearly applicable beyond the present context of political campaigns. A related idea already in the literature is "persuasion" in Kamenica and Gentzkow (2011).[5] They also study a model in which the sender's choice of information structure and the realized signal are observable. Unlike in this article, however, the sender in Kamenica and Gentzkow (2011) has no private information, and thus no misinformation incentives arise.

This article is complementary to the existing literature on the effect of negative campaigns on voter behavior. Skaperdas and Grofman (1995) assume that negative campaigns reduce voter support for both the target and the sponsor of such campaigns without formally modeling why such a negative effect arises. In a complete information model, Harrington and Hess (1996) consider a Hotelling model in which a candidate's characteristics are known, but a candidate can, via campaign expenditures, move toward the swing voter's preferred ideology through a positive campaign or he can move the rival's ideology away from the voter through a negative campaign.[6] Our model differs from Harrington and Hess (1996) in that the candidate has private information about both his and his rival's characteristics, which are exogenously given. He can signal his characteristics through informative campaigns that produce unbiased evidence in expectation, but he cannot alter these characteristics to influence the voter.

More closely related in terms of information provision, Polborn and Yi (2006) consider a disclosure model in which a candidate knows the characteristics of both himself and his rival, but he can only verifiably disclose one dimension. They find that the higher is the value of the disclosed characteristics, the lower is the expected value of the undisclosed dimension inferred

[5] The optimal choice of information structure has also been studied in the auction design model of Bergemann and Pesendorfer (2007) and in duopoly games by Ottaviani and Moscarini (2001) and Damiano and Li (2007).

[6] Heidhues and Lagerlof (2003) show that when two candidates have correlated private information about the true state, they bias their platforms toward the voter's priors instead of revealing their private information truthfully.

by the voter in equilibrium. This implies that a candidate is more likely to choose a negative campaign when his own characteristics are bad and a positive one when his rival's characteristics are good. Although their result relies on the restriction that only one dimension can be disclosed, we have a signaling model in which the candidate is imperfectly informed. In our model, the level of campaign plays a critical role in generating the comparative advantage characterization that does not rely on this restriction. An informative negative campaign is not an attempt to hide one's own lack of qualifications, but an effective way to signal the candidate's overall strength.

2. THE BASIC MODEL

There are two political candidates, a and b. Each candidate i, $i = a, b$, is either qualified ($q_i = 1$) or unqualified ($q_i = 0$) for a political office. In the basic model, only candidate a is a player (the sender) in the signaling game described later. Candidate a may be one of two types, denoted as type (α_L, β_L) and type (α_H, β_H), respectively.[7] Each type is a pair of beliefs about the qualifications of a and b: The first component represents a's private belief that he is qualified, whereas the second component represents his private belief that his rival is qualified. These beliefs are referred to as the strength of candidate a and b, respectively. Candidate a is type (α_L, β_L) with probability $\lambda \in (0, 1)$ and type (α_H, β_H) with probability $1 - \lambda$. The candidate's type is private information, but the values of (α_L, β_L) and (α_H, β_H) as well as the type distribution are common knowledge between the voter and the candidates.

Define an *information campaign* as an observable choice of information structure—a distribution of a public signal about the qualifications of candidate a or b. An information campaign is positive if it generates a signal about candidate a's qualifications (the *target* is a) and negative if it is about b (the target is b).[8] Each information campaign generates either a favorable signal $\bar{s}$ or an unfavorable signal $\underline{s}$ about the target of the campaign. The precision of this campaign signal is $k \in [\frac{1}{2}, 1)$, which is the *level* of the campaign. More precisely, k is both the probability of the signal being $\bar{s}$ conditional on that the targeted candidate is qualified and the probability of the signal being $\underline{s}$ conditional on that the target is unqualified.

The voter, the receiver in this signaling game, first observes candidate a's campaign choices, which includes both the target and the level of the campaign, and then observes the realized campaign signal. To focus on information provision, we simplify the voter's behavior by assuming that she uses Bayes' rule to form a pair of posterior beliefs about the qualifications of both candidates. Specifically, we assume that the voter chooses two real-valued actions x_a and x_b to minimize the expected total loss $(x_a - q_a)^2 + (x_b - q_b)^2$. Then, given any posterior beliefs about the qualifications of the two candidates, π_a and π_b, the optimal actions for the voter are simply π_a and π_b, respectively.[9] The same posterior beliefs π_a and π_b, together with a campaign cost function $C(k)$, determine the payoff to candidate a. In the basic model, candidate a maximizes the difference of the voter's posterior belief about himself over b, net of any campaign cost. The payoff to a is

$$\pi_a - \pi_b - C(k),$$

where C is continuous and strictly increasing, with $C(\frac{1}{2}) = 0$.[10]

[7] We use the subscripts H and L here to economize on notation. No restriction or ordering is placed on parameter values α_L, α_H and β_L, β_H to allow for a full characterization. In the analysis, we show explicitly the condition that identifies a candidate as the high type or low type.

[8] The word "negative" is commonly used to mean that a campaign attacks the rival and attempts to lower the voter's opinion of the rival. Here it simply means that the target of the information campaign is the rival, as the candidate cannot bias the realization of the campaign signal.

[9] We use the voter's posterior beliefs as a reduced-form way of modeling the candidates' value to the voter in the future. In general, however, the value of a candidate is determined by the voter's future decision problem and is often convex in her posterior beliefs. See, for example, Morris (2001) and Li (2007).

[10] The assumptions of strict monotonicity and no fixed cost on the function C ensure the existence of a least cost separating equilibrium. They are made to simplify the analysis and are not crucial to our results.

The payoff specification above has natural interpretations in political campaigns: It is appropriate in a parliamentary system where the number of seats is proportional to the voters' support or when the political mandate as measured by the winning margin is important for the candidates. In comparison, we use a winner-take-all model in Section 5 to study how a plurality electoral system affects the candidate's campaign choices. These two payoff specifications can also be used to model marketing and advertising campaigns, for instance, when a firm aims to expand its market shares, which are a function of its product quality, or when it aims to capture a market completely.

For simplicity, candidate a's private type is modeled directly as a pair of beliefs about whether he and his rival are qualified. Instead, we can explicitly model how candidate a forms his beliefs—(α_L, β_L) and (α_H, β_H)—after observing a private, imperfect signal. To do so, we need to specify a signal structure conditional on the four underlying states, which are the candidates' true qualifications. This is done in Section 4 to study electoral competition between the candidates.[11]

In this model, candidate a cannot directly control the realization of the campaign signal, which is consistent with the idea of information provision. Although we assume that the voter can perfectly observe the informativeness of a campaign for simplicity, the results hold qualitatively if the voter only observes a noisy measure of campaign levels. The assumption that voters can judge the relative informativeness of a campaign is supported by empirical research in marketing and media studies. For instance, using survey and advertising data from the 2000 presidential campaign and two 1998 gubernatorial races, Sides et al. (2010) show that voters separate judgments about the tone of a campaign (positive or negative) from judgments about the quality of information received.[12] Further, we have implicitly assumed that candidate a cannot simultaneously run both a positive and a negative campaign to focus on his choice of campaign target. Section 3.3 extends the analysis to "contrast campaigns" in which a can run both kinds of campaigns and shows that he prefers to run only one campaign under reasonable assumptions.

Information campaigns are assumed to be costly, and a higher level of campaign, whether positive or negative, costs more than a lower level one. The idea is that it costs little for the candidate to gloat about himself, but much more is required to establish or to refute detailed claims based on the biographical, legal, educational, financial, or the voting records of a candidate. Such research cost, which depends on the informativeness of the campaign, is a nonnegligible part of campaign expenditures.[13] To focus on how the candidate's campaign choices depend on his characteristics, we assume that there is one continuous and strictly increasing cost function in both kinds of campaigns. Our analysis extends easily to allow different cost for positive and negative campaigns, capturing any adverse social effect of negative campaigns.

The idea that a sender can potentially misinform the receiver by manipulating the way the latter interprets the realization of an information structure features naturally in other settings. We briefly mention two of them before turning to the analysis. First, the sender is a producer attempting to increase its market share at the expense of its rival and the receiver is the consumer. The producer has private information about the quality of its own product and can let the consumer observe a signal about the quality of his own product through advertising, free trials, and other promotions. It can adjust the informativeness of its signal by, for instance, varying the frequency of its advertisements or the number of features available in the free trials, but cannot bias the realization to its favor. Nonetheless, the consumer may be misled by the producer's choice of informativeness of the signal in evaluating the realization. In the second setting, the sender is a defendant charged with a crime and the receiver is the jury. The defendant has private information about the strength of the testimony of potential expert witnesses and can choose how much testimony to present to the jury. Although the defendant

[11] In the basic model, however, such a structure is unnecessary and merely clutters the notation.

[12] In particular, the voters can judge whether a political campaign "gave voters a great deal of useful information, some, not too much, or no useful information at all."

[13] For instance, using Federal Election Commission data, the Center for Responsive Politics shows that in the 2008 presidential campaign, such research cost and consultant fee amounted to approximately $7 million.

cannot bias the outcome of any testimony in his favor, he can potentially manipulate the jury's interpretation of the outcomes.

3. EQUILIBRIUM INFORMATION CAMPAIGNS

We look for perfect Bayesian equilibria (PBE) of this game in which the candidate makes campaign choices to maximize his expected payoff and the voter updates her beliefs according to Bayes' rule on the equilibrium path. The innovation of our model is that the signal is an information structure, not an action, as in a typical signaling game. Therefore, how the voter evaluates a chosen information structure, or her *interim beliefs* about the qualifications of the candidates, is an important component of our analysis. These interim beliefs are formed after observing candidate a's campaign choices but before observing the realized campaign signal.

Because the campaign signal is unbiased, the voter's posterior beliefs are pinned down by Bayes' rule for any given interim beliefs and that the same realized signal is evaluated differently if the voter has different interim beliefs. We begin by investigating how campaign choices depend on the voter's interim beliefs. Suppose candidate a runs an informative positive campaign of level k when the candidate's private belief of his qualifications is α and the voter's interim belief about him is $\tilde{\alpha}$. The voter's expected posterior belief about candidate a, $\Pi(\alpha, \tilde{\alpha}; k)$, is given by

$$\begin{aligned}\Pi(\alpha,\tilde{\alpha};k) &= (\alpha k+(1-\alpha)(1-k))\frac{\tilde{\alpha}k}{\tilde{\alpha}k+(1-\tilde{\alpha})(1-k)}\\ &\quad +(\alpha(1-k)+(1-\alpha)k)\frac{\tilde{\alpha}(1-k)}{\tilde{\alpha}(1-k)+(1-\tilde{\alpha})k},\end{aligned} \tag{1}$$

where the first fraction shows how the voter upgrades her belief about a's qualifications if she observes a favorable signal $\bar{s}$, and the second fraction shows how she downgrades her belief if she observes an unfavorable signal $\underline{s}$. Clearly, Π increases in the candidate's own strength: The higher is α, the more likely the voter observes a favorable signal. The function Π also increases in the voter's interim belief $\tilde{\alpha}$: The stronger she thinks the candidate is, the more favorably she interprets each realized campaign signal. Moreover, because a more informative signal is more convincing, the voter's posterior belief after a favorable signal increases in k whereas her posterior belief after an unfavorable signal decreases in it.

Inspection of expression (1) leads to the following result for any given α, $\tilde{\alpha}$, and k.[14]

LEMMA 1. *(i)* $\Pi(\alpha, \tilde{\alpha}; k) = \alpha$ *if* $\tilde{\alpha} = \alpha$*; (ii)* $\Pi(\alpha, \tilde{\alpha}; k)$ *decreases in k if* $\alpha < \tilde{\alpha}$*; and (iii)* $\Pi(\alpha, \tilde{\alpha}; k)$ *increases in k if* $\alpha > \tilde{\alpha}$.

Albeit simple, Lemma 1 is important in understanding the direction of a candidate's attempt to influence the voter.[15] Part (i) shows that if the voter has the correct interim belief of the candidate's strength, there is no value to an information campaign.[16] No campaign can change the voter's expected posterior belief by the law of iterated expectations, as the expected upgrade of the voter's posterior belief is canceled by the downgrade. If instead, the voter's interim belief differs from the candidate's private belief, candidate a can influence the voter's perception by adjusting how informative his campaign is. In this case, we say that *misinformation* occurs. If candidate a is privately less confident about his own qualifications than the voter, part (ii) of Lemma 1 shows that he would like to "obscure" the bad news by reducing the informativeness of his campaign signal. Intuitively, if the voter overestimates the candidate given his campaign

[14] The voter's interim beliefs are formed endogenously in equilibrium. Lemma 1 examines if and how a candidate can misinform the voter for given parameter values.

[15] Lemma 1 also holds if the voter can only observe a noisy, but unbiased, signal of campaign level k. The noisier is the voter's observed signal, however, the fewer incentives the candidate has to run informative campaigns.

[16] The marginal value of information to the voter, however, is always positive.

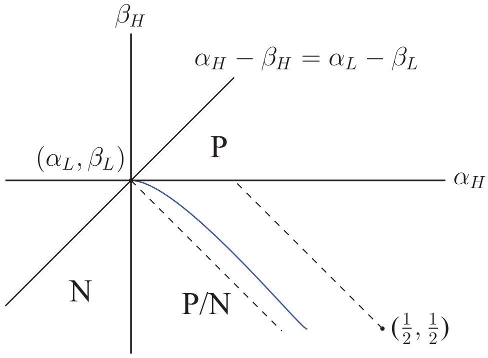

FIGURE 1

EQUILIBRIUM CAMPAIGN CHOICES OF TYPE (α_H, β_H)

choices, the later observed informative campaign signal can only lower her posterior belief of the candidate on average. But a less informative campaign signal, favorable or unfavorable, is less damaging. Part (iii) shows that if candidate *a* is privately more confident about his qualifications than the voter, then he would like to choose a more informative campaign to "highlight" the good news about himself.

The voter's expected posterior belief $\Pi(\beta, \tilde{\beta}; k)$ for candidate *b* after a negative campaign of level *k*, given private belief β and interim belief $\tilde{\beta}$, is similar. Naturally, by running an informative negative campaign, candidate *a* lowers the voter's posterior belief about his rival if he believes that *b* is worse than the voter thinks, but raises her posterior belief if he believes that *b* is better.

3.1. *Least Cost Separating Equilibrium.* Because campaign choices are a signal of the candidates' qualifications, we focus on separating equilibria in which the voter learns the candidate's private type. Unlike the assumption implicit in some of the political science literature, in this model, candidate *a*'s incentives to separate are not determined by whether he is stronger or weaker than his rival candidate *b*, for instance, whether α_H is larger or smaller than β_H. Rather, they depend on the comparison of the payoffs that the two types receive under complete information. If the candidate's type was known, the high type is the one that has a greater difference in strength between himself and the rival, or

$$\alpha_H - \beta_H > \alpha_L - \beta_L.$$

We refer to the difference $\alpha_H - \beta_H$ as the *overall strength* of the high type.

As illustrated in Figure 1, there are three cases regarding the location of type (α_H, β_H) in the α_H–β_H parameter space, holding type (α_L, β_L) fixed at the origin. In the first case, referred to as the P-region, we have $\alpha_H > \alpha_L$ and $\beta_H > \beta_L$. Here the high type candidate is stronger than the low type but also faces a stronger rival than the low type does. In Figure 1, the P-region is located below the line $\alpha_H - \beta_H = \alpha_L - \beta_L$ and above the horizontal axis. In the second case, referred to as the N-region, we have the opposite scenario of $\alpha_H < \alpha_L$ and $\beta_H < \beta_L$: The high type candidate is weaker than the low type, but also faces a weaker rival. The N-region is located below $\alpha_H - \beta_H = \alpha_L - \beta_L$ and to the left of the vertical axis. In the third case, referred to as P/N-region, both $\beta_H \leq \beta_L$ and $\alpha_H \geq \alpha_L$ hold, with at least one strictly. The P/N-region is simply the third quadrant of Figure 1, where the high type candidate is stronger than the low type and faces a weaker rival than the low type does.

Regardless of the location of the high type candidate, there is always a separating equilibrium in which the low type runs an uninformative, costless campaign, whereas the high type uses an informative campaign to signal his overall strength. As is standard in the signaling literature, we focus on the least cost separating equilibrium. The existence and uniqueness of the least cost separating equilibrium is a direct consequence of Lemma 1.[17]

PROPOSITION 1. *In any separating equilibrium, the high type candidate runs a positive campaign in the P-region and a negative campaign in the N-region, and he may run either kind of campaign in the P/N-region. Further, a least cost separating equilibrium exists and is generically unique.*

Should the high type candidate, the type with greater overall strength, go positive or negative? The answer depends on his location. To begin with, suppose that the high type is in the P-region and that there exists a separating equilibrium of level k^p. Then in equilibrium, the high type candidate receives $\Pi(\alpha_H, \alpha_H; k^p)$, which is simply α_H by part (i) of Lemma 1. If the low type candidate runs the same campaign to imitate the high type, he only gets $\Pi(\alpha_L, \alpha_H; k^p)$. Because $\alpha_H > \alpha_L$, the low type's deviation payoff is strictly smaller than α_H by part (ii) of Lemma 1. Intuitively, the low type candidate is less successful in raising the voter's posterior belief of his own strength than the high type in any informative positive campaign. Moreover, the greater is k^p, the smaller is the low type's gain from imitating the high type. Thus there exists a unique campaign level k^p_H, given by

$$\alpha_L - \beta_L = \Pi\big(\alpha_L, \alpha_H; k^p_H\big) - \beta_H - C\big(k^p_H\big). \tag{2}$$

At this level, the low type is indifferent between running an uninformative campaign and imitating the high type. In essence, the high type can signal his overall strength in a positive campaign because he is informing the voter in the dimension in which he is stronger than the low type. Similarly, in the N-region, the high type should signal his overall strength in a negative campaign because $\beta_H < \beta_L$. Facing a worse rival than the low type, the high type candidate is better at lowering the voter's posterior belief of candidate b in any informative negative campaign than the low type. The least cost separating level of negative campaign, k^n_H, is given by

$$\alpha_L - \beta_L = \alpha_H - \Pi\big(\beta_L, \beta_H; k^n_H\big) - C\big(k^n_H\big). \tag{3}$$

Separation of the types is not just a matter of the high type candidate running a sufficiently informative campaign. In the P-region, for instance, there exists no equilibrium in which the high type can separate from the low type by running a negative campaign. Suppose separation is possible in a negative campaign of level $k^n > \frac{1}{2}$. Then in this putative equilibrium, the high type candidate gets $\alpha_H - \beta_H$. If the low type imitates the high type by running the same negative campaign, he gets $\alpha_H - \Pi(\beta_L, \beta_H; k^n_H)$, which is strictly larger than the high type's payoff because $\beta_L < \beta_H$. The reason is simple: The low type faces a worse rival and can lower the voter's posterior belief about his rival more successfully than the high type. Hence, whenever the high type prefers to run a negative campaign, the low type prefers to do the same, which is a contradiction. Similarly, in the N-region, the high type cannot separate by running a positive campaign.

Finally, in the P/N-region, either positive or negative campaigns can be used for separation at sufficiently high levels. Therefore the high type candidate runs a positive campaign of level k^p_H if $k^p_H \le k^n_H$ and a negative campaign of level k^n_H otherwise. An immediate implication of Proposition 1 is that banning negative campaign never benefits the high type candidate.

[17] Lemma 1 also implies that the interim belief specified in the proof of Proposition 1 is the only one satisfying the Intuitive Criterion of Cho and Kreps (1987). Further, under Lemma 1, the same refinement rules out other separating equilibria in which type (α_H, β_H) runs a higher level of campaign than the least cost separating level.

COROLLARY 1. *Banning negative campaign has no effect on the equilibrium campaign choices in the P-region and in the P/N-region when the least cost separating equilibrium is a positive campaign. Otherwise, the high type candidate runs a positive campaign at a higher cost in the P/N-region and pools with the low type running an uninformative campaign in the N-region.*

In the United States, the marked increase in the amount and intensity of negative advertising in recent elections, especially since the 2004 presidential race, has lent support to the policy proposal of banning or at least limiting negative campaigns. Corollary 1 suggests that doing so can only hurt the high type by either raising his cost of signaling or making signaling altogether impossible. The voter is also worse off if the ban results in pooling through uninformative campaigns, because she loses an opportunity to learn the qualifications of the candidates.

3.2. *Comparative Advantage in Positive or Negative Campaigns.* The high type candidate in the P/N-region is in the best position: He is stronger than the low type and faces a weaker rival than the low type does. This is why the high type can separate from the low type by running either a positive or a negative campaign and chooses the less costly one in equilibrium. Proposition 1, however, is silent on what determines one kind of campaign is less costly than the other for a given high type candidate.

To answer this question, consider the following comparative statics exercise: Fix the low type (α_L, β_L) and compare the equilibrium choice of campaign target by two different high type candidates in the P/N-region. Any high type candidate in this region can deter the low type from imitation by running a positive campaign such that, from rewriting (2),

$$\alpha_H - \beta_H - (\alpha_L - \beta_L) = \alpha_H - \Pi\big(\alpha_L, \alpha_H; k_H^p\big) + C\big(k_H^p\big), \tag{4}$$

or a negative one such that, from rewriting (3),

$$\alpha_H - \beta_H - (\alpha_L - \beta_L) = \Pi\big(\beta_L, \beta_H; k_H^n\big) - \beta_H + C\big(k_H^n\big). \tag{5}$$

The left-hand sides of (4) and (5) are identical, representing the overall strength of the high type over the low type. Since $\alpha_H > \alpha_L$ and $\beta_H < \beta_L$, the right-hand sides of (4) and (5) are increasing in k_H^p and k_H^n, respectively, by Lemma 1. Thus, for any high type candidate with the same overall strength, the term $\alpha_H - \Pi(\alpha_L, \alpha_H; k_H^p)$ represents his advantage in positive campaigns. The greater is this term, the less successful the low type is in imitating the high type, and thus the lower is the level k_H^p that the high type needs to run to deter the low type. Similarly, $\Pi(\beta_L, \beta_H; k_H^n) - \beta_H$ represents the high type's advantage in negative campaigns.

Because the candidate's type is two-dimensional, it is generally difficult to compare the campaign levels k_H^p and k_H^n for two arbitrary high types.[18] To draw unambiguous conclusions from the present comparative statics exercise, we assume that $\alpha_H < \frac{1}{2} < \beta_H$. Under this assumption, we show that holding the overall strength of the high type $\alpha_H - \beta_H$ constant, his advantage in positive campaigns increases in α_H whereas his advantage in negative campaigns decreases in it. Observe that when $\alpha_H = \frac{1}{2}$, the voter is most responsive to the realized campaign signals for any given campaign level. Because she has no prior knowledge of the candidate, her posterior belief of the candidate after a favorable signal is the farthest away from her posterior belief after an unfavorable one. When $\alpha_H < \frac{1}{2}$, as α_H increases, the above difference in her posterior beliefs increases as she becomes more responsive to the realized signals. Intuitively, as α_H increases, the voter interprets both realized signals more favorably, but she increases the upgrade of her posterior belief of the candidate after a favorable signal more than she reduces the downgrade after an unfavorable signal. Because the low type is a weaker candidate than the high type and

[18] Clearly, for a fixed β_H, a high type candidate with a greater α_H needs to run a higher level positive or a higher level negative campaign to separate. The same is true if α_H is fixed. But this does not help us characterize whether either of these two high type candidates should run a positive or negative campaign in the first place.

is therefore less likely to generate a favorable signal in any positive campaign, an increase in α_H makes it harder for him to imitate the high type through positive campaigns. Symmetrically, when $\beta_H > \frac{1}{2}$, a decrease in β_H makes it harder for the low type to imitate the high type through negative campaigns.[19]

PROPOSITION 2. *Suppose that $\alpha_H < \frac{1}{2} < \beta_H$. For the same overall strength of the high type, a simultaneous increase in the candidates' strengths leads to a greater comparative advantage in positive campaigns for the high type and results in a lower least cost separating level if the high type runs a positive campaign and a higher level if he runs a negative campaign.*

Formally, Proposition 2 establishes the existence of a *boundary* that divides the P/N-region into a positive campaign area adjacent to the P-region and a negative one adjacent to the N-region (the blue curve in Figure 1). For any overall strength of the high type candidate, there is a unique pair (α_H, β_H) on the boundary such that he is indifferent between a positive and a negative campaign of the same level. At (α_H, β_H), the high type has the same advantage in positive and negative campaigns (the right-hand side of (4) and (5) are equal). As α_H and β_H increase at the same rate so that the overall strength remains the same, the least cost separating equilibrium involves a positive campaign of a decreasing level. Conversely, as α_H and β_H decrease at the same rate, the least cost separating equilibrium involves a negative campaign of a decreasing level. Thus all the high type candidates above the boundary have a comparative advantage in positive campaigns and all those below have a comparative advantage in negative campaigns.

Along this boundary, as the overall strength of the high type candidate $\alpha_H - \beta_H$ increases, the least cost separating equilibrium level increases. This is clearly true if the boundary is monotonically decreasing in the P/N-region, since the right-hand side of condition (4) increases in α_H and the right-hand side of condition (5) decreases in β_H. But even if the boundary is not monotonically decreasing, the fact that for the same β_H, a higher α_H leads to a higher level of positive campaign and that the high type is indifferent between a positive campaign and a negative campaign of the same level means that the equilibrium level has to increase along the boundary.[20] Simple algebra can also show that the boundary falls between the lines defined by $\alpha_H + \beta_H = 1$ and $\alpha_H + \beta_H = \alpha_L + \beta_L$ in the α_H-β_H diagram. See Figure 1. In the special case where $\alpha_L + \beta_L = 1$, the boundary is simply the line connecting (α_L, β_L) to $(\frac{1}{2}, \frac{1}{2})$.

Proposition 2 helps us think about a candidate's campaign choices when the voter has different amounts of prior knowledge about candidate *a* or *b*'s qualifications. Suppose that candidate *b* is well known such that β_H is sufficiently close to β_L; then the high type candidate *a* is more likely to run a positive campaign due to his comparative advantage in positive campaigns.[21] Intuitively, *a* needs to convince the voter he is stronger than the voter's priors whereas he has

[19] The parameter restriction in Proposition 2 is not necessary for the monotone characterization of the high type's comparative advantage. All we need is that $\alpha_H - \Pi(\alpha_L, \alpha_H; k^p)$ increases in α_H for any fixed k^p and, correspondingly, $\Pi(\beta_L, \beta_H; k^n) - \beta_H$ decreases in β_H. This does not hold, for instance, if α_H is close to 1: A further increase in α_H can result in a decrease in $\alpha_H - \Pi(\alpha_L, \alpha_H; k^p)$ for any fixed k^p. Here the advantage of the high type in positive campaigns decreases. This happens because the voter downgrades her posterior belief after an unfavorable signal more than she upgrades it after a favorable signal and because the voter's posterior belief about the candidate responds little to the realized campaign signal.

[20] To see why the boundary may not be monotone, fix any (α_H, β_H) on the boundary, with the associated separating level k_H. Consider (α'_H, β_H) just to the right, with $\alpha'_H > \alpha_H$ and the associated positive separating level $k_H^{p\prime}$ given by (4) and (5). We have $k_H^{p\prime}, k_H^{n\prime} > k_H$. The boundary is nonmonotone at (α_H, β_H) if (α'_H, β_H) is below the boundary or, equivalently, if $\partial k_H^p/\partial \alpha_H > \partial k_H^n/\partial \alpha_H$ at $k_H^p = k_H^n = k_H$. From Equations (2) and (3), we have

$$\frac{\partial k_H^p}{\partial \alpha_H} = \frac{\partial \Pi(\alpha_L, \alpha_H; k_H)/\partial \alpha_H}{-\partial \Pi(\alpha_L, \alpha_H; k_H)/\partial k_H + C'(k_H)}; \quad \frac{\partial k_H^n}{\partial \alpha_H} = \frac{1}{\partial \Pi(\beta_L, \beta_H; k_H)/\partial k_H + C'(k_H)}.$$

Although $\partial \Pi(\alpha_L, \alpha_H; k_H)/\partial \alpha_H < 1$, we may have $-\partial \Pi(\alpha_L, \alpha_H; k_H)/\partial k_H < \partial \Pi(\beta_L, \beta_H; k_H)/\partial k_H$.

[21] Observe that in the P/N-region, type (α_H, β_H) likely falls into the positive campaign area if β_H is sufficiently close to β_L and into the negative campaign area if α_H is sufficiently close to α_L.

little to reveal about b. If candidate a himself is well known such that α_H is sufficiently close to α_L, but the voter has a lot of uncertainty about candidate b, then a has a comparative advantage in negative campaigns, which are effective in lowering the voter's posterior belief about b. These choices are consistent with empirical findings: Kahn and Geer (1994) show that positive advertising increased the viewers' rating of an unknown candidate's capability in a study of how TV ads influence voters' impression of a candidate, and, more recently, Lovett and Shachar (2010) find that if a candidate's traits are well known by the voters, the candidate is more likely to go negative.

Propositions 1 and 2 show that, despite having the same overall strength, the high type candidate may nonetheless run different kinds of campaigns depending on his comparative advantage. In particular, even a high type candidate in the best position may run a negative campaign, because it is the cost-effective way to boost the voter's posterior belief about him over his rival. Thus, in our model, the high type candidate neither runs a positive campaign because he wants to "hide" his rival's strong qualifications nor a negative campaign to hide his own low qualifications, in contrast with Polborn and Yi (2006). Consequently, the voter's posterior belief of a candidate depends on more than whether he runs a positive or a negative campaign: Her prior knowledge of the strengths of different types of candidates also matters. Instead, the more informative a campaign is, positive or negative, the stronger a candidate is perceived to be.

3.3. *Contrast Campaigns.* So far candidate a can run only one kind of campaign; we now turn to the case of "contrast campaigns" to see whether the high type candidate can do better by running both a positive and a negative campaign. To avoid biasing our results, we assume that the costs of running two campaigns are additive. The total cost of running a positive campaign of level k^p and a negative campaign of level k^n is simply $C(k^p) + C(k^n)$. The following result suggests that our finding that comparative advantage determines the high type's choice of positive versus negative campaigns is not due to the restriction that only one kind of campaign may be used.

PROPOSITION 3. *Suppose that candidate a can simultaneously run a positive and a negative campaign. In the least cost separating equilibrium, the high type candidate runs a single campaign in the P-region and the N-region, and if the campaign cost function is differentiable and concave, he also runs a single campaign in the P/N-region.*

The above result is straightforward in the P-region or the N-region, where the high type candidate can only signal his type successfully using one kind of campaign. Suppose, for instance, a high type candidate in the P-region now runs both a positive campaign of level k^p and a negative campaign of level k^n. To prevent the low type from imitating, it must be that

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k^p) - \Pi(\beta_L, \beta_H; k^n) - C(k^p) - C(k^n). \tag{6}$$

From Equation (2), the total campaign cost in running both k^p and k^n is smaller than the cost of running just k^p_H if $k^p < k^p_H$ and

$$\Pi(\alpha_L, \alpha_H; k^p) - \Pi(\beta_L, \beta_H; k^n) < \Pi\big(\alpha_L, \alpha_H; k^p_H\big) - \beta_H.$$

The above is impossible by Lemma 1 because $\alpha_H > \alpha_L$ and $\beta_H > \beta_L$ in the P-region. Intuitively, if the high type must also run an informative negative campaign, he has to run a higher level positive campaign than in the single-campaign case to deter the low type, who has a comparative advantage in proving that candidate b is less qualified. This increases his total cost of campaigning.

In the P/N-region, for the high type to separate from the low type with two campaigns, the total campaign cost must be low enough such that condition (6) is satisfied. Previous analysis has shown that candidate a can signal his type using either campaign: Condition (6) is satisfied with equality by either $k^p = \frac{1}{2}$ and $k^n = k^n_H$ given by (3) or by $k^p = k^p_H$ and $k^n = \frac{1}{2}$ given by (2). Further, since $\alpha_H > \alpha_L$ and $\beta_H < \beta_L$, Lemma 1 implies that positive and negative campaigns are substitutes in condition (6). Proposition 3 shows that if the cost function is (weakly) concave, the candidate prefers to completely substitute one kind of campaign for the other.[22]

Intuitively, as the high type increases his positive campaign level and simultaneously decreases his negative campaign level to satisfy condition (6), the positive campaign becomes increasingly effective in deterring the low type relative to the negative campaign. This follows because a higher level of positive campaign leads to a greater response by the voter to the realized campaign signal, and the opposite is true for a lower level of negative campaign. If the cost function is concave, then the marginal cost from a higher positive campaign level declines whereas the marginal saving from a lower negative campaign increases. More substitution of positive campaign for negative campaign thus makes the positive campaign more effective in deterring the low type at a smaller cost, leading to a complete substitution. Therefore the high type candidate must choose the informative campaign that is less costly, exactly as in Proposition 1.

Proposition 3 demonstrates that the result that comparative advantage determines the high type's choice of positive versus negative campaigns is not due to the restriction that only one kind of campaign may be used, in contrast with existing work such as Polborn and Yi (2006). This provides further evidence that our characterization is obtained through the channel of misinformation instead of the choice between which dimension to reveal and which to conceal.

3.4. *Independent Information.* Voters often have access to exogenous sources of information outside the control of candidates such as the media. The effect of such additional information on a candidate's campaign choices, interesting in its own right, is also an important component in the analysis of the competing campaigns model in the next section. To be specific, suppose that the voter receives a public signal after the candidate has made his campaign choices, but before she forms her posterior beliefs about the candidates. Assume that this public signal s' is about a's qualifications; the case when she receives a public signal about b is symmetric. For simplicity, assume that this public signal s' is binary with a symmetric structure: s' is either $\bar{s}'$ or $\underline{s}'$ such that k', the probability of $s' = \bar{s}'$ conditional on candidate a being qualified, equals the probability of $s' = \underline{s}'$ conditional on a being unqualified. Also, s' is conditionally independent of the candidate's campaign signal s.

Since candidate a may run a positive campaign, the voter may observe two signals about candidate a. In this case, her expected posterior belief about a is a weighted average of her beliefs after observing both realized signals. Suppose that the candidate's private belief is α and the voter's interim belief is $\tilde{\alpha}$; then the voter's expected posterior belief, $\Pi(\alpha, \tilde{\alpha}; k, k')$, is given by

$$\begin{aligned}\Pi(\alpha, \tilde{\alpha}; k, k') = {} & \frac{(\alpha k k' + (1-\alpha)(1-k)(1-k'))\tilde{\alpha} k k'}{\tilde{\alpha} k k' + (1-\tilde{\alpha})(1-k)(1-k')} + \frac{(\alpha k(1-k') + (1-\alpha)(1-k)k')\tilde{\alpha} k(1-k')}{\tilde{\alpha} k(1-k') + (1-\tilde{\alpha})(1-k)k'} \\ & + \frac{(\alpha(1-k)k' + (1-\alpha)k(1-k'))\tilde{\alpha}(1-k)k'}{\tilde{\alpha}(1-k)k' + (1-\tilde{\alpha})k(1-k')} \\ & + \frac{(\alpha(1-k)(1-k') + (1-\alpha)kk')\tilde{\alpha}(1-k)(1-k')}{\tilde{\alpha}(1-k)(1-k') + (1-\tilde{\alpha})kk'}.\end{aligned}$$

[22] The cost function may be concave due to the fixed cost of running campaigns. The concavity is sufficient, but not necessary, for the candidate to prefer a single campaign to two informative campaigns. As discussed in the proof, what is required is that the cost function is not too convex such that as the candidate increases his campaign level k^p and decreases his campaign level k^n, the marginal effect on his campaign cost is sufficiently small.

Straightforward algebra show that the partial derivatives of $\Pi(\alpha, \tilde{\alpha}; k, k')$ with respect to k and k' have the same sign as $(\alpha - \tilde{\alpha})$. The following result is the counterpart of Lemma 1.

LEMMA 2. *(i)* $\Pi(\alpha, \tilde{\alpha}; k, k') = \alpha$ *if* $\alpha = \tilde{\alpha}$; *(ii)* $\Pi(\alpha, \tilde{\alpha}; k, k')$ *decreases in* k *and* k' *if* $\alpha < \tilde{\alpha}$; *and (iii)* $\Pi(\alpha, \tilde{\alpha}; k, k')$ *increases in* k *and* k' *if* $\alpha > \tilde{\alpha}$.

Lemma 2 implies that when the additional public signal and the candidate's campaign signal have the same target, candidate *a* himself in this case, it becomes more difficult for the low type candidate to imitate the high type. This can be seen from part (ii) of the above lemma. If $\alpha < \tilde{\alpha}$, then for any given level k of a positive campaign, the candidate's expected payoff is lower than when the voter has no additional information ($k' = \frac{1}{2}$). The additional public signal reduces the low type's possible gain from imitating the high type at any campaign level.

When the public signal has a different target from the candidate's campaign signal, it might seem that the additional information has no impact because it does not affect the voter's evaluation of the campaign signal. This turns out to be false. Suppose that $\alpha_L, \alpha_H < \frac{1}{2}$, which is sufficient to ensure that if the low type is indifferent between running an informative campaign and an uninformative one, the high type strictly prefers the former. Then we have the following result.

PROPOSITION 4. *Suppose the voter receives an additional signal of a sufficiently low level and suppose that* $\alpha_L, \alpha_H < \frac{1}{2}$. *In the least cost separating equilibrium, the high type runs a lower level positive campaign in the P-region and a higher level negative campaign in the N-region than when no additional signal is observed. Moreover, in the P/N-region he runs a lower level of campaign regardless of whether it is positive or negative.*

The additional public signal about candidate *a* enhances the high type's comparative advantage in positive campaigns. Therefore the high type will continue to run positive campaigns if he runs a positive campaign in the absence of such signal, but at a lower level. More interestingly, voters' access to independent evidence outside the control of the candidate may hurt a candidate by raising his signaling cost. When the high type is in the N-region, the new least cost separating equilibrium level is $k_H^{n\prime}$, given by the low type's indifference condition:

$$\alpha_L - \beta_L = \Pi(\alpha_L, \alpha_H; k') - \Pi\big(\beta_L, \beta_H; k_H^{n\prime}\big) - C\big(k_H^{n\prime}\big).$$

Since $\alpha_H < \alpha_L$ and $\beta_H < \beta_L$, a comparison of the above with (3) immediately reveals that $k_H^{n\prime} > k_H^n$ by Lemma 1. Here, the additional information gives the low type candidate a free opportunity to impress the voter with his own strength and thus increases his gain from imitating the high type. This effect forces the high type to run a higher level of negative campaign.[23]

Proposition 4 demonstrates the robustness of the results in Propositions 1 and 2.[24] One difference from the basic model is in the P/N-region: If the high type candidate finds it cheaper to run a negative campaign without the additional signal, his comparative advantage may now change to positive campaigns. This is because the additional signal increases the high type's comparative advantage in positive campaigns by giving the voter an opportunity to observe his own strength.

[23] In the symmetric case of the voter having access to independent information about candidate *b*, the high type *a* can be hurt if he can only use positive campaigns to separate.

[24] The condition on the level of the additional public signal in Proposition 4, given precisely in the proof, is imposed to rule out uninteresting pooling equilibria. If the additional public signal is very informative, the high type does not need to run any costly informative campaign so long as he is stronger than the low type: The equilibrium will be pooling at an uninformative level.

4. COMPETING CAMPAIGNS

One natural question is what happens if both candidate a and b can run an information campaign simultaneously and independently. To identify any new effect arising solely from competition in information provision, we limit attention to an environment identical to the basic one-campaign model except for the possible active campaign from b. In particular, the candidates have the same private information about their qualifications.

We introduce the following underlying information structure for the basic one-campaign model before introducing competing campaigns. Assume that candidate a and the voter share the same prior beliefs about the qualifications of the candidates, $\Pr(q_a, q_b)$. To maintain the two-type structure, candidate a is assumed to receive a private binary signal L or H, with probabilities λ and $1-\lambda$, respectively. The voter knows the value of each $\Pr(H|q_a, q_b)$ but does not observe the signal. Then, given signal H, a's private belief that he is qualified and b is qualified is, respectively,

$$\alpha_H = \frac{\sum_{q_b} \Pr(1, q_b)\Pr(H \mid 1, q_b)}{\sum_{q_a, q_b} \Pr(q_a, q_b)\Pr(H \mid q_a, q_b)}; \quad \beta_H = \frac{\sum_{q_a} \Pr(q_a, 1)\Pr(H \mid q_a, 1)}{\sum_{q_a, q_b} \Pr(q_a, q_b)\Pr(H \mid q_a, q_b)}.$$

Candidate a's private beliefs after signal L are similarly defined.

To ensure that the same amount of information is available as in the basic model, candidate b is assumed to share with a the same prior beliefs and, more importantly, receive the same private signal.[25] In the analysis below, we need to specify the out-of-equilibrium interim beliefs of the voter about the qualifications of the candidates when their campaign choices suggest that their signals disagree according to their equilibrium strategies. To avoid making arbitrary assumptions, consider the hypothetical scenario in which for each (q_a, q_b), candidate b's private signal is perfectly correlated with a's signal with probability ρ and is conditionally independent with probability $1-\rho$, and then let ρ go to 1. As long as $\rho < 1$, the belief about candidate a's strength when the candidates' private signals disagree is well defined, given by

$$\alpha_{HL} = \alpha_{LH} = \frac{\sum_{q_b} \Pr(1, q_b)\Pr(H \mid 1, q_b)(1 - \Pr(H \mid 1, q_b))}{\sum_{q_a, q_b} \Pr(q_a, q_b)\Pr(H \mid q_a, q_b)(1 - \Pr(H \mid q_a, q_b))}.$$

Due to this observation, let α_{HL} be the out-of-equilibrium interim belief of the voter about a. The out-of-equilibrium interim belief for b, $\beta_{HL} = \beta_{LH}$, is similarly defined.

Perfect correlation between the candidates' signals means that the strengths of the two candidates are either (α_H, β_H) or (α_L, β_L). Since $\alpha_H - \beta_H > \alpha_L - \beta_L$, type (α_H, β_H) candidate b has an incentive to pretend to be type (α_L, β_L) if candidate b ran the only campaign. Thus we refer to both type (α_H, β_H) candidate a and type (α_L, β_L) candidate b as the high type candidates and, correspondingly, type (α_L, β_L) candidate a and type (α_H, β_H) candidate b as the low type. Note that under perfect correlation, regardless of the private signal, one candidate is high type and the other is low type. For ease of comparison with the basic model, we continue to refer to the three regions in the α_H–β_H diagram from a's perspective. Also, Proposition 1 is directly

[25] This assumption rules out the analysis of whether additional information is revealed due to the competition between senders. Kamenica and Gentzkow (2012) consider a general model in which competing senders choose how much information to reveal to the receiver in a symmetric information setting and show that competition cannot decrease the information of the receiver. Li (2012) studies a model in which two senders with possible biases try to influence a receiver and shows that a sender distorts his information less because the other sender's message reduces the marginal value of the information contained in his message.

applicable to b's campaign choices if he were the only one running a campaign. As an example, suppose that type (α_H, β_H) is in the P-region. In the basic one-campaign model, type (α_H, β_H) candidate a can only signal his overall strength over type (α_L, β_L) through a positive campaign. In contrast, if b is the only one making campaigning choices, type (α_L, β_L) candidate b must signal by running a negative campaign, because he is a weaker candidate than type (α_H, β_H) but he faces an even weaker rival than the low type does.

Competing campaigns introduce two new elements into the one-campaign model. First, any attempt to mislead the voter may now be challenged by the information provided by the rival. For instance, type (α_L, β_L) candidate a must now re-evaluate the relative effectiveness of positive versus negative campaigns, because ex post even a favorable campaign signal from a positive campaign might be countered by an unfavorable signal from the negative campaign against him run by candidate b. Second, the way type (α_L, β_L) candidate a influences the voter's interim beliefs through his campaign choices is also affected, because unilateral deviations lead to out-of-equilibrium beliefs $(\alpha_{HL}, \beta_{HL})$. To avoid biasing our results, we make a *neutrality* assumption so that the voter's out-of-equilibrium interim beliefs do not favor either type of candidate: α_{HL} is sufficiently close to the unweighted average of α_H and α_L and symmetrically for β_{HL}.

PROPOSITION 5. *In the competing-campaigns model, under the neutrality assumption, there is a separating equilibrium in which the low type candidate runs an uninformative campaign and the high type runs the same kind of informative campaign as in the one-campaign model. In the P/N-region, there is a separating equilibrium in which the high type candidate runs a lower level campaign.*

Proposition 5 suggests that our results on a candidate's choice of positive versus negative campaigns in the basic model are robust. In the P-region for example, there still exists a separating equilibrium in which the high type candidate a runs a positive campaign and the high type b runs a negative campaign to signal their overall strength. This robustness result owes much to the neutrality assumption, which guarantees that if the low type candidate is just indifferent between his uninformative campaign and imitating the high type in running an informative campaign, then the high type strictly prefers to separate. Moreover, the separating equilibria in Proposition 5 are natural extensions of the least cost separating equilibria in the basic model: If the two high type candidates respectively run the same kind of campaign as they do in the one-campaign case, there does not exist another separating equilibrium with lower campaign cost.[26]

In the P/N-region, Proposition 5 offers a sharper characterization: Competition lowers campaign levels the high types need to run to separate. Regardless of the targets of the high type candidates' equilibrium campaigns, the levels of their campaigns are smaller than k_H^p and k_H^n, their separating levels in the basic model given by (2) and (3). One reason is that in the P/N-region, the campaign levels of the two high type candidates are *strategic substitutes* regardless of their targets: One's campaign level decreases in that of the other.[27] To see why, suppose that the high type a and b run positive campaigns at level k_a^p and k_b^p, respectively. Consider the low type candidate a's incentive to misinform the voter by imitating the high type a. In a separating equilibrium we need

[26] Separating equilibria involving different kinds of campaigns, however, may exist. For instance, in the P/N-region, there exist both a separating equilibrium in which the high type candidate a runs an informative positive campaign and the high type b runs an informative negative campaign and another one in which both high type candidates run positive campaigns. It is not possible to compare these two equilibria in terms of the campaign levels because they are of different kinds, and this model is generally asymmetric with respect to the candidates. However, the proof of Proposition 5 shows that each of the two equilibria is uniquely constructed.

[27] This is true even though the two informative campaigns are never run at the same time: In equilibrium, one and only one informative campaign is run due to the perfect correlation between the candidates' signals.

(7) $$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_{HL}; k_a^p) - \Pi(\beta_L, \beta_{HL}; k_b^p) - C(k_a^p).$$

In the P/N-region, we have $\alpha_L < \alpha_{HL} < \alpha_H$ and $\beta_L > \beta_{HL} > \beta_H$. Thus, an increase in the high type candidate b's positive campaign level k_b^p reduces the low type candidate a's gain from imitating the high type a and vice versa for the high type candidate b. Competing campaigns make it more difficult for the low type to pretend to more qualified when the campaigns have the same targets and more difficult for him to make his rival look less qualified when the campaigns have different targets. In either case, the campaign levels required for separation decrease.

In addition, under competing campaigns, the voter's interim beliefs become more moderate. Specifically, the low type a's campaign choices now have less impact on the voter's interim beliefs because of b's campaign choices. This is true even if b runs no informative campaigns, and thus the voter does not observe more realized campaign signals. The possibility of pretending to be an "intermediate" type $(\alpha_{HL}, \beta_{HL})$ is another difference from our basic model, which can be seen by comparing condition (7) with condition (2). Since $\alpha_L < \alpha_{HL} < \alpha_H$ and $\beta_L > \beta_{HL} > \beta_H$, even if b's campaign is uninformative, the low type candidate a has less incentive to misinform the voter through a positive campaign. In a more general model, both the voter's interim beliefs and the amount of information contained in the candidates' signals will differ from those in the basic model. The two cases we have studied, additional public signal and competing campaigns with perfectly correlated signals, should be viewed as the two polar opposites.

5. WINNER TAKES ALL

In a winner-take-all political system, the candidate wins the election if he convinces the voter that he is more qualified than his rival. The crucial difference from the basic model is that here, only the probability of winning matters, not how much one candidate is better than the other. For simplicity, model candidate a's payoff as

$$\begin{cases} 1 - C(k), & \text{if} \quad \pi_a \geq \pi_b \\ -C(k), & \text{otherwise.} \end{cases}$$

The campaign cost $C(k)$ is assumed to be small for all relevant campaign levels, so that both types can afford any necessary campaigns.[28] Moreover, assume that $\alpha_L, \alpha_H \leq \frac{1}{2}$ and $\beta_L, \beta_H \geq \frac{1}{2}$, or that candidate a is always weaker than b, to allow for a direct comparison with the basic model.

The first difference from the basic model is that a candidate in a winner-take-all system has an incentive to run an informative campaign even if his type is known: The campaign signal has value under complete information. No matter how far candidate a is lagging behind b, he always has a chance of winning if the voter observes a favorable campaign signal that is sufficiently informative to overturn her low initial belief about him. For instance, for type (α_L, β_L) to win under complete information, the minimum level k_L^c of a positive campaign needs to satisfy

$$\frac{\alpha_L k_L^c}{\alpha_L k_L^c + (1-\alpha_L)(1-k_L^c)} = \beta_L.$$

That is, a wins if the realized campaign signal is $\bar{s}$ and his campaign level is at least k_L^c.[29] Similarly, k_H^c is the level type (α_H, β_H) runs under complete information.

The low type remains the one who receives a lower payoff under complete information. Here he needs to run a higher level of campaign to catch up to candidate b: (α_L, β_L) is the low type

[28] Analysis in this section is valid even if campaigns are free. Unlike in the basic model where the campaign cost helps the high type separate from the low type, here it gives the high type more incentives to pool with the low type.

[29] The same level, k_L^c, is required for the low type to win in a negative campaign (when the realized campaign signal about candidate b is unfavorable).

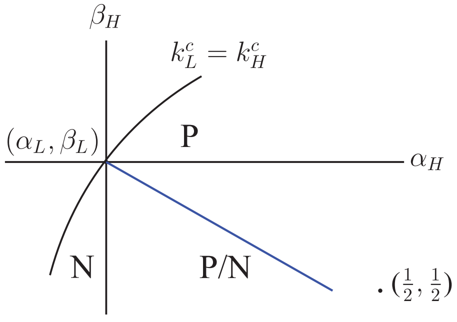

FIGURE 2

WINNER-TAKE-ALL MODEL

if $k_L^c > k_H^c$. Moreover, k_i^c, $i = H, L$, is decreasing in α_i and increasing in β_i. Intuitively, the overall strength of a candidate is inversely related to the campaign level he runs under complete information level. Fix type (α_L, β_L) at the origin; then $k_L^c = k_H^c$ defines a curve such that type (α_H, β_H) is located to the right of (and below) this curve as depicted in Figure 2. Just as before, we classify the parameter space below this curve into three regions: the high type may be located in the P-region, N-region, or P/N-region.

The second difference from the basic model is that, under complete information, each type prefers the kind of campaign that has a higher chance of winning. For instance, since $\alpha_L < \frac{1}{2} < \beta_L$, the low type will run a negative campaign under complete information if and only if

$$\alpha_L k_L^c + (1 - \alpha_L)(1 - k_L^c) < (1 - \beta_L) k_L^c + \beta_L (1 - k_L^c),$$

or $\alpha_L + \beta_L < 1$. If so, then the low type has a *preference for negative campaigns*, and he has a preference for positive campaigns if $\alpha_L + \beta_L > 1$. For any given campaign level, *a* prefers the campaign in which the voter's prior belief about the target is closer to $\frac{1}{2}$ and is thus more responsive to the realized campaign signal. Also, the low type's winning chance falls in his campaign level if it is above k_L^c, because a more informative campaign is more likely to generate an unfavorable signal about *a* or a favorable signal about *b*. Similar analysis applies to the high type.

Intuitively, in the winner-take-all model, both types of candidate *a* only want to run the least informative campaign that can overturn the voter's initial low opinion. Consequently, there does not exist a separating equilibrium in which both types of candidate run the same kind of campaign. For the same kind of campaign, say positive ones, if the voter's interim belief is such that a campaign level is sufficient for one type of candidate to win when the realized signal is favorable, then it must also be sufficient for the other type to win. But since a more informative campaign merely reduces *a*'s winning chances regardless of type, the type running a higher level of campaign in the putative equilibrium strictly prefers to deviate to the lower one: It increases his winning chance at a lower cost. Hence separation must involve opposite kinds of campaigns in this model.

To ease exposition, we only discuss the case when the low type has a preference for negative campaigns ($\alpha_L + \beta_L < 1$); the other case is similar. In any separating equilibrium (if it exists), the low type candidate must run his preferred campaign: a negative one at level k_L^c. Moreover, there exists a unique $k_L^p \in (\frac{1}{2}, k_L^c)$ such that the low type is indifferent between his preferred negative campaign of level k_L^c and a positive campaign of a lower level k_L^p, given by

$$(8) \qquad (1 - \beta_L) k_L^c + \beta_L (1 - k_L^c) - C(k_L^c) = \alpha_L k_L^p + (1 - \alpha_L)(1 - k_L^p) - C(k_L^p).$$

The right-hand side of (8), the low type's payoff if he pretends to be a high type by running a positive campaign, does not depend on the voter's interim beliefs so long as a favorable signal leads to a win.[30] Consequently, this (possible) separation level k_L^p does not depend on the high type's characteristics. The equilibrium condition for separation, then, is for the high type to prefer not to pool with the low type candidate by running a negative campaign at the level k_L^c:

$$\begin{aligned}(9)\qquad &\alpha_H \max\{k_H^c, k_L^p\} + (1-\alpha_H)\big(1-\max\{k_H^c, k_L^p\}\big) - C\big(\max\{k_H^c, k_L^p\}\big)\\ &\geq (1-\beta_H)k_L^c + \beta_H\big(1-k_L^c\big) - C\big(k_L^c\big).\end{aligned}$$

If condition (9) is satisfied, the high type has a comparative advantage in positive campaigns.[31]

PROPOSITION 6. *Suppose that the low type candidate prefers negative campaigns under complete information. There is a unique least cost separating equilibrium in which the low type candidate runs a negative campaign and the high type runs a positive campaign if the latter has a comparative advantage in positive campaigns; otherwise, there is a pooling equilibrium in which both types run a negative campaign of the same level.*

The high type candidate should run the kind of campaign in which he has a comparative advantage as before. The least cost separating equilibrium, however, looks different due to the payoff discontinuity. It is easiest to understand the least cost separating equilibrium when the overall strength of the high type candidate is so high that he has no need to run a very informative campaign under complete information ($k_H^c \leq k_L^p$). In this case, condition (9) implies a linear *positive boundary* in the P/N-region (the blue line in Figure 2). Above the positive boundary, the high type candidate separates with a positive campaign of level k_L^p from the low type. Moreover, this separating level is completely determined by the low type's characteristics. The weaker is the low type's preference for negative campaigns (as α_L increases and/or β_L decreases), the more tempted he is to imitate the high type, who then needs to run a higher level of positive campaign to separate.[32] If $k_H^c > k_L^p$, however, the high type has to be willing to run a higher level of positive campaign than k_L^p to separate from the low type: k_L^p is not high enough to convince the voter.[33]

Qualitatively similar to the basic model, above the positive boundary, the high type has a comparative advantage in positive campaigns and a comparative advantage in negative campaigns below the boundary. To see why, consider the example of the N-region, which lies below the boundary. Because $\alpha_H < \alpha_L$ and $\beta_H < \beta_L$ in the N-region, for any given level of campaign, the high type has a higher probability of getting an unfavorable signal for his rival b and a lower probability of getting a favorable signal for himself than the low type. As a result, he is more likely to win the election in a negative campaign, but less likely to win in a positive campaign. The fact that the low type prefers a campaign of k_L^c to a positive campaign of either level k_L^p or k_H^c means that the high type strictly prefers the negative campaign of level k_L^c. Intuitively, since

[30] This contrasts with the basic model in which the low type's expected payoff varies continuously with the voter's interim belief. In the winner-take-all model, the voter's interim belief only matters discontinuously, making it more difficult for a high type candidate to signal his type through choices of campaign levels.

[31] The maximum operator on the left-hand side of condition (9) arises because if $k_L^p < k_H^c$, the campaign level k_L^p is high enough to deter the low type, but not high enough for type (α_H, β_H) to convince the voter that he should win even after a favorable signal. See the Online Appendix for details.

[32] In the case where $\alpha_L + \beta_L = 1$ and thus the low type has no preference between the two kinds of campaigns, the positive boundary is simply $\alpha_H + \beta_H = 1$, a line connecting to (α_L, β_L) to $(\frac{1}{2}, \frac{1}{2})$ in the α_H-β_H diagram.

[33] There is a critical type (α_H, β_H) on the positive boundary such that condition (9) holds as an equality with the corresponding complete information level $k_H^c = k_L^p$. For all high types closer to (α_L, β_L) than this critical type, the boundary between a separating equilibrium and a pooling equilibrium is instead given by (9) with $\max\{k_H^c, k_L^p\} = k_H^c$. That is, when $k_H^c > k_L^p$, the high type may not run a positive campaign in a separating equilibrium above the boundary, unlike in the basic model. Since this does not affect our result qualitatively, we relegate the complete characterization to the proof of Proposition 6.

$\alpha_H + \beta_H < \alpha_L + \beta_L < 1$ here, the high type's preference for negative campaigns makes him unwilling to run a positive campaign of at least level k_L^p to separate. Similarly, above the boundary, the high type candidate prefers a positive campaign because he has a higher probability than the low type of getting a favorable signal for himself and thus winning the election.[34]

When separation is impossible—if type (α_H, β_H) is located below the positive boundary—both types run the same negative campaign. In any pooling equilibrium, the high type candidate is more likely to win the election. Unlike the basic model, however, the "wrong" candidate may be elected ex post in a winner-take-all system. To see this, observe that in any such equilibrium, the pooling campaign level is below the low type's complete information level k_L^c, otherwise the low type candidate would deviate to his complete information campaign. But by definition, the low type always loses if his campaign level is below k_L^c under complete information. Therefore the low type candidate wins with a positive probability when he should not have won.

Two consequences of Proposition 6 are immediate. First, banning one campaign can never increase the voter's welfare because doing so (weakly) increases pooling and hence the probability the wrong candidate is elected. Second, since separation is impossible within the same kind of campaign and since there are only two kinds of campaigns, only the lowest type can possibly be separated from the rest if there are more than two types. In that case, we should expect to see two groups of candidates each running one kind of campaign at the same level.

6. CONCLUDING REMARKS

This article provides a natural framework to examine misinformation, the idea that the sender can manipulate the way that the receiver interprets the realization of an information structure by changing the precision of the information structure. Although we focus on information campaigns in the political economy context, the insights that arise from the analysis should be broadly applicable. The two-type model presented here is clearly too limited to explore the full potential of this new signaling game. We now discuss several interesting ways to extend this model.

With only two types of candidates, the equilibrium characterization in Proposition 1 needs no restriction on the candidate's type because the least cost separating equilibrium is determined by the incentives of the low type to imitate the high type. To further understand the nature of the least cost separating equilibrium, or to study the candidate's behavior when there are multiple types, it is necessary to rank a candidate's incentives to misinform the voter according to his type. The Online Appendix presents a single crossing condition, which can be used to rule out pooling equilibria in this model and to generalize the model to multiple types. In addition, we introduce a counterpart of this condition for the case of continuously distributed campaign signals.

Our separation result in the basic model relies on the assumption that campaign levels, possibly with some noise, are observable to the voter. If the campaign level is unobservable, that is, if the signal is jammed as in Fudenberg and Tirole (1986), the realized campaign signal alone may fail to provide the voter with any information because the candidate may have no incentive to run an informative campaign. It is easy to see that when the campaign level is not observable, there exists a pooling equilibrium in which both types of candidates run an uninformative campaign if their probability of obtaining the favorable campaign signal is decreasing in their campaign level.

Finally, in the basic model, the candidate's payoff is continuous in the voter's posterior beliefs, and in the winner-take-all model, his payoff is discontinuous. In the former case, the payoff continuity implies that if the voter knows the candidate's type, which is the case when her interim belief is correct, the information contained in the campaign signal has no value to the candidate.

[34] Type (α_H, β_H)'s comparative advantage is not only driven by his preference under complete information. Since $k_L^p < k_L^c$, the slope of the linear part of the positive boundary is greater than -1, implying that some high type *a* who prefer negative campaigns under complete information also have a comparative advantage in positive campaigns.

Instead, in the latter case, the payoff discontinuity implies that the campaign signal has positive value to the candidate. In situations when both winning the election and the margin of winning are important, the candidate's payoff functions exhibit features from both models above. In such a hybrid model, the value of information to the candidate in an informative campaign is generally non-zero as in the winner-take-all case, which has interesting implications on the candidates' campaign choices.

APPENDIX

A. *Proofs*

PROOF OF PROPOSITION 1. In any separating equilibrium, type (α_L, β_L) candidate a must run no informative campaign and receive a payoff of $\alpha_L - \beta_L$. Moreover, if in a separating equilibrium, type (α_H, β_H) candidate a runs a positive campaign of level $k^p > \frac{1}{2}$ or a negative campaign of $k^n > \frac{1}{2}$ to separate from type (α_L, β_L), the following incentive constraints must be satisfied:

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k^p) - \beta_H - C(k^p), \tag{A.1}$$

$$\alpha_H - \beta_H - C(k^p) \geq \alpha_L - \beta_L, \tag{A.2}$$

$$\alpha_L - \beta_L \geq \alpha_H - \Pi(\beta_L, \beta_H; k^n) - C(k^n), \tag{A.3}$$

$$\alpha_H - \beta_H - C(k^n) \geq \alpha_L - \beta_L. \tag{A.4}$$

First, consider the case of $\alpha_H > \alpha_L$, $\beta_H > \beta_L$. Observe that at $k^p = \frac{1}{2}$, the left-hand side of (A.1) is smaller than the right-hand side, whereas at $k^p = 1$, the left-hand side is greater than the right-hand side. Also, the right-hand side of (A.1) decreases in k^p by Lemma 1, and thus the campaign level $k_H^p \in (\frac{1}{2}, 1)$ defined in (2) is the unique level such that (A.1) holds with equality. Moreover, substituting (A.1) at k_H^p into (A.2), we require

$$\alpha_H - \Pi\big(\alpha_L, \alpha_H; k_H^p\big) \geq 0,$$

which is always true when $\alpha_L < \alpha_H$. Now, we show that separation in negative campaigns is impossible in the P-region. Adding up (A.3) and (A.4), we require

$$\Pi(\beta_L, \beta_H; k^n) \geq \beta_H,$$

which contradicts the assumption that $\beta_L < \beta_H$ in the P-region. The interim belief supporting the equilibrium is (α_L, β_L) if $k^p < k_H^p$ and (α_H, β_H) if $k^p \geq k_H^p$ for any positive campaign of some level k^p and (α_L, β_L) for any negative campaign.

By a symmetric argument, one can show that in the case of $\alpha_H < \alpha_L$, $\beta_H < \beta_L$, the unique least cost separating equilibrium level is k_H^n given by (3). Finally, if $\alpha_H \geq \alpha_L$, $\beta_H \leq \beta_L$, with at least one strict inequality, type (α_H, β_H) can separate from type (α_L, β_L) by either running a positive campaign of level k_H^p or by running a negative campaign of level k_H^n. The least cost separating level is the positive campaign of k_H^p if $k_H^p \leq k_H^n$ and a negative campaign of level k_H^n otherwise. The interim belief that supports this equilibrium is (α_L, β_L) if $k^p < k_H^p$ and (α_H, β_H) if $k^p \geq k_H^p$ for any positive campaign of some level k^p and (α_L, β_L) for any negative campaign and (α_L, β_L) if $k^n < k_H^n$ and (α_H, β_H) if $k^n \geq k_H^n$ for any negative campaign of some level k^n. ■

PROOF OF PROPOSITION 2. Fix type (α_L, β_L) and suppose that $\alpha_L < \alpha_H < \frac{1}{2}$ and $\beta_L > \beta_H > \frac{1}{2}$. We claim that for each $\mu \in (\alpha_L - \beta_L, 0)$, there is a unique set of solutions (α_H, β_H) and k_H to

$$\alpha_H - \beta_H = \mu, \tag{A.5}$$

$$\mu - (\alpha_L - \beta_L) = \alpha_H - \Pi(\alpha_L, \alpha_H; k_H) + C(k_H), \tag{A.6}$$

$$\mu - (\alpha_L - \beta_L) = \Pi(\beta_L, \beta_H; k_H) - \beta_H + C(k_H). \tag{A.7}$$

Define

$$\Delta(\alpha; k) = \frac{\alpha k}{\alpha k + (1-\alpha)(1-k)} - \frac{\alpha(1-k)}{\alpha(1-k) + (1-\alpha)k}.$$

Then,

$$\alpha_H - \Pi(\alpha_L, \alpha_H; k_H) = (2k_H - 1)(\alpha_H - \alpha_L)\Delta(\alpha_H; k_H),$$
$$\Pi(\beta_L, \beta_H; k_H) - \beta_H = (2k_H - 1)(\beta_L - \beta_H)\Delta(\beta_H; k_H).$$

It is straightforward to verify that

$$\Delta(\alpha; k) = \frac{\alpha(1-\alpha)(2k-1)}{(\alpha k + (1-\alpha)(1-k))(\alpha(1-k) + (1-\alpha)k)} > 0$$

for all $\alpha \in (0, 1)$ and $k > \frac{1}{2}$, and that

$$\frac{\partial \Delta(\alpha; k)}{\partial \alpha} = \frac{(1-2\alpha)(2k-1)k(1-k)}{(\alpha k + (1-\alpha)(1-k))^2(\alpha(1-k) + (1-\alpha)k)^2},$$

which has the same sign as $1 - 2\alpha$ for all $k \in (\frac{1}{2}, 1)$. Thus, at $\alpha_H = \beta_L + \mu$ and $\beta_H = \beta_L$, the right-hand side of (A.6) is strictly larger than the right-hand side of (A.7), and the opposite is true at $\alpha_H = \alpha_L$ and $\beta_H = \alpha_L - \mu$. Further, as α_H decreases from $\beta_L + \mu$ to α_L and simultaneously β_H decreases from β_L to $\alpha_L - \mu$ so that Equation (A.5) remains satisfied, the right-hand side of (A.6) decreases for any fixed k_H whereas the right-hand side of (A.7) increases. The proposition then follows immediately from Lemma 1, as the right-hand side of both (A.6) and (A.7) increases in k_H. ■

PROOF OF PROPOSITION 3. It is shown in the text that running both a positive and a negative campaign cannot reduce the total cost of separation for type (α_H, β_H) candidate in the P-region or the N-region. Now, suppose that type (α_H, β_H) is in the P/N-region. In the least cost separating equilibrium, for type (α_H, β_H) to run two campaigns to separate from type (α_L, β_L), the campaign levels k^p and k^n must satisfy condition (6) with equality. Suppose that the high type increases k^p while simultaneously decreasing k^n, starting from $k^p = \frac{1}{2}$ and $k^n = k_H^n$ given by (3), such that (6) continues to hold with equality. Then, the infinitesimal changes $dk^p > 0$ and $dk^n < 0$ must satisfy

$$dk^p \left(\frac{\partial \Pi(\alpha_L, \alpha_H; k^p)}{\partial k^p} - C'(k^p) \right) = dk^n \left(\frac{\partial \Pi(\beta_L, \beta_H; k^n)}{\partial k^n} + C'(k^n) \right).$$

The total cost $C(k^p) + C(k^n)$ changes by $C'(k^p)dk^p + C'(k^n)dk^n$, which has the same sign as

$$\frac{C'(k^p)}{C'(k^n)} + \frac{\partial\Pi(\alpha_L, \alpha_H; k^p)/\partial k^p}{\partial\Pi(\beta_L, \beta_H; k^n)/\partial k^n}. \tag{A.8}$$

The first ratio in expression (A.8) is always positive, and it is weakly decreasing as k^p increases and k^n decreases if C is concave. By Lemma 1, $\partial\Pi(\alpha_L, \alpha_H; k^p)/\partial k^p$ is negative and decreasing as k^p increases when $\alpha_L < \alpha_H$ and $\partial\Pi(\beta_L, \beta_H; k^n)/\partial k^n$ is positive and decreasing as k^n decreases when $\beta_L > \beta_H$. Therefore the second ratio in expression (A.8) is negative and decreasing as k^p increases and k^n decreases. Moreover, at $k^p = \frac{1}{2}$ and $k^n = k^n_H$ given by (3), expression (A.8) is positive because $\partial\Pi(\alpha_L, \alpha_H; k^p)/\partial k^p = 0$. Together, we have that when C is concave, expression (A.8) can change sign at most once from positive to negative.[35] Thus the total campaign cost is minimized at either $k^p = \frac{1}{2}$ and $k^n = k^n_H$ given by (3), or $k^p = k^p_H$ given by (2) and $k^n = \frac{1}{2}$. ■

PROOF OF PROPOSITION 4. Fix type (α_L, β_L). We start by considering the case when type (α_H, β_H) is in the P-region. If there exists a separating equilibrium in positive campaigns, the following two conditions must hold for some campaign level k^p:

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k^p, k') - \beta_H - C(k^p); \tag{A.9}$$

$$\alpha_H - \beta_H - C(k^p) \geq \Pi(\alpha_H, \alpha_L; k') - \beta_L. \tag{A.10}$$

By Lemma 2, $\Pi(\alpha_L, \alpha_H; k^p, k')$ decreases in both k^p and k' since $\alpha_L < \alpha_H$ in the P-region. Let $k'' \in (\frac{1}{2}, 1)$ be uniquely defined by $\alpha_L - \beta_L = \Pi(\alpha_L, \alpha_H; k'') - \beta_H$. This is the upper bound on k' in the statement of the proposition. Then, for all $k' < k''$, there exists a unique value of $k^p \in (\frac{1}{2}, 1)$, say $k^{p\prime}_H$, such that (A.9) holds with equality. Condition (A.10) holds for $k^p = k^{p\prime}_H$ if

$$\alpha_H - \Pi\big(\alpha_L, \alpha_H; k^{p\prime}_H, k'\big) \geq \Pi(\alpha_H, \alpha_L; k') - \alpha_L.$$

Note that the left-hand side is greater than $\alpha_H - \Pi(\alpha_L, \alpha_H; k')$ by Lemma 2. Further,

$$\begin{aligned}\alpha_H - \Pi(\alpha_L, \alpha_H; k') &= (2k' - 1)(\alpha_H - \alpha_L)\Delta(\alpha_H; k') \\ &> (2k' - 1)(\alpha_H - \alpha_L)\Delta(\alpha_L; k') = \Pi(\alpha_H, \alpha_L; k') - \alpha_L,\end{aligned}$$

where Δ is defined in the proof of Proposition 2, and the inequality follows because $\alpha_L < \alpha_H < \frac{1}{2}$.

In the P-region, if there exists a separating equilibrium in negative campaigns, then

$$\alpha_L - \beta_L \geq \Pi(\alpha_L, \alpha_H; k') - \Pi(\beta_L, \beta_H; k^n) - C(k^n); \tag{A.11}$$

$$\alpha_H - \beta_H - C(k^n) \geq \Pi(\alpha_H, \alpha_L; k') - \beta_L. \tag{A.12}$$

However, because $\Pi(\alpha_L, \alpha_H; k') > \Pi(\alpha_L, \alpha_H; k^{p\prime}_H, k')$ and $\Pi(\beta_L, \beta_H; k^n) < \beta_H$, the right-hand side of condition (A.11) is strictly larger than that of condition (A.9) for any k^n. It follows that if there exists a level k^n that satisfies condition (A.11) with equality, it must be that $k^n > k^{p\prime}_H$. Thus in the least cost separating equilibrium the high type runs a positive campaign of level

[35] Further, as one increases k^p and decreases k^n from $k^p = \frac{1}{2}, k^n = k^n_H$, the second ratio of expression (A.8) is negative and decreasing at an increasing rate because $\pi(\alpha, \tilde{\alpha}; k)$ is convex in k. Therefore if the first ratio, which is positive and increasing when $C(k)$ is convex, increases sufficiently slowly, expression (A.8) can still change sign at most once, and thus the above argument applies.

$k_H^{p\,\prime}$. Depending on whether there is a separating equilibrium in negative campaigns, we can construct the interim belief accordingly, similar to the proof of Proposition 1. Also, because $\Pi(\alpha_L, \alpha_H; k_H^{p\,\prime}, k') < \Pi(\alpha_L, \alpha_H; k')$, the equilibrium campaign level $k_H^{p\,\prime}$ is strictly lower than k_H^p given by (2) in the basic model.

Second, suppose that type (α_H, β_H) is in the N-region. If there exists a separating equilibrium in which type (α_H, β_H) runs a negative campaign of level k^n, then (A.11) and (A.12) hold. Because $\beta_L > \beta_H$, the right-hand side of (A.11) strictly decreases in k^n. Next, because $\alpha_L > \alpha_H$, $\Pi(\alpha_L, \alpha_H; k')$ increases in k' and is larger than α_H. Therefore for any k' there exists a unique level $k_H^{n\,\prime} \in \left(\frac{1}{2}, 1\right)$ such that (A.11) holds with equality. At this level, condition (A.12) holds strictly. This is because in the N-region, $\beta_L > \beta_H$ implies $\Pi(\beta_L, \beta_H; k^n) > \beta_H$, and $\alpha_H < \alpha_L < \frac{1}{2}$ implies

$$\begin{aligned}\alpha_L - \Pi(\alpha_H, \alpha_L; k') &= (2k' - 1)(\alpha_L - \alpha_H)\Delta(\alpha_L; k') \\ &> (2k' - 1)(\alpha_L - \alpha_H)\Delta(\alpha_H; k') = \Pi(\alpha_L, \alpha_H; k') - \alpha_H.\end{aligned}$$

Arguments similar to that of the P-region can show that either there is no separating equilibrium in positive campaigns or it involves a separating level higher than $k_H^{n\,\prime}$. Thus there exists a unique least cost separating equilibrium with $k_H^{n\,\prime}$. Finally, because $\Pi(\alpha_L, \alpha_H; k') > \alpha_H$ in the N-region, $k_H^{n\,\prime}$ is strictly greater than k_H^n given by (3) in the basic model.

In the P/N-region, similar arguments can show that there always exists a separating equilibrium in positive campaigns of level $k_H^{p\,\prime}$, and $k_H^{p\,\prime} < k_H^p$. Also, if there exists a separating equilibrium in negative campaigns, then the separating level is $k_H^{n\,\prime}$, which is strictly lower than k_H^n because $\alpha_H > \alpha_L$ implies $\Pi(\alpha_L, \alpha_H; k') < \alpha_H$ in the P/N-region. The least cost separating equilibrium is the less costly of the two campaigns. ■

PROOF OF PROPOSITION 5. We construct separating equilibria where, for each realized type, the low type candidate runs an uninformative campaign. For the high types, there are two cases: Type (α_H, β_H) candidate a and type (α_L, β_L) candidate b run campaigns with the same target or they run campaigns with different targets.

In the first case, suppose that in equilibrium, type (α_H, β_H) candidate a runs a positive campaign of some level k_a^p and (α_L, β_L) candidate b runs a negative campaign of level k_b^n; the case of the high type a running a negative campaign and the high type b running a positive campaign is symmetric. We argue that there is always such an equilibrium when (α_H, β_H) is either in the P-region or in the P/N-region. The necessary equilibrium conditions are

$$\alpha_L - \beta_L \geq \Pi\big(\alpha_L, \alpha_{HL}; k_a^p, k_b^n\big) - \beta_{HL} - C\big(k_a^p\big), \tag{A.13}$$

$$\alpha_H - \beta_H - C\big(k_a^p\big) \geq \alpha_{HL} - \beta_{HL}, \tag{A.14}$$

$$\beta_H - \alpha_H \geq \beta_{HL} - \Pi\big(\alpha_H, \alpha_{HL}; k_a^p, k_b^n\big) - C\big(k_b^n\big), \tag{A.15}$$

$$\beta_L - \alpha_L - C\big(k_b^n\big) \geq \beta_{HL} - \alpha_{HL}. \tag{A.16}$$

Suppose that conditions (A.13) and (A.15), respectively the incentive constraints for type (α_L, β_L) candidate a and type (α_H, β_H) candidate b, hold with equality. Substituting condition (A.13) into (A.14) and condition (A.15) into (A.16), we need the following conditions for (A.14) and (A.16) to hold:

$$\alpha_{HL} - 2\beta_{HL} + \Pi\big(\alpha_L, \alpha_{HL}; k_a^p, k_b^n\big) \leq \alpha_H - \beta_H + \alpha_L - \beta_L \leq \alpha_{HL} - 2\beta_{HL} + \Pi\big(\alpha_H, \alpha_{HL}; k_a^p, k_b^n\big).$$

By Lemma 2, $\Pi(\alpha_L, \alpha_{HL}; k_a^p, k_b^n) < \alpha_{HL} < \Pi(\alpha_H, \alpha_{HL}; k_a^p, k_b^n)$ in either P-region or P/N-region. Thus under the neutrality assumption, the above conditions are satisfied so that no high type candidate wants to pretend to be type $(\alpha_{HL}, \beta_{HL})$ when the low type candidate is indifferent. It is then simple to specify a set of interim beliefs to support the separating equilibrium.

Next, we show that there exists a unique pair of campaign levels $k_a^p, k_b^n \in (\frac{1}{2}, 1)$ such that both condition (A.13) and (A.15) hold with equality, which are then the equilibrium levels for the high type candidates. Let $\overline{k}_a^p$ be such that condition (A.13) holds with equality at $k_b^n = \frac{1}{2}$. This is well defined because $\alpha_L - \beta_L < \alpha_{HL} - \beta_{HL}$ in both the P-region and the P/N-region. Similarly, let $\overline{k}_b^n$ be such that condition (A.15) holds with equality at $k_a^p = \frac{1}{2}$. Let $r_a(k_b^n)$ be the value of k_a^p such that condition (A.13) holds with equality for each $k_b^n > \frac{1}{2}$. Since $\alpha_L < \alpha_{HL}$ in the P-region and in the P/N-region, by Lemma 1 and 2, $r_a(k_b^n)$ decreases as k_b^n increases. We claim that the function r_a is well defined for all $k_b^n \leq \overline{k}_b^n$, that is, $r_a(\overline{k}_b^n) > \frac{1}{2}$. From condition (A.13), this claim is equivalent to

$$\alpha_{HL} - \Pi\big(\alpha_L, \alpha_{HL}; \overline{k}_b^n\big) \leq \alpha_{HL} - \alpha_L - \beta_{HL} + \beta_L.$$

By the definition of $\overline{k}_b^n$, we have

$$\Pi\big(\alpha_H, \alpha_{HL}; \overline{k}_b^n\big) - \alpha_{HL} = \alpha_H - \alpha_{HL} - \beta_H + \beta_{HL} - C\big(\overline{k}_b^n\big).$$

Using the function Δ defined in the proof of Proposition 2, we have

$$\begin{aligned}\alpha_{HL} - \Pi\big(\alpha_L, \alpha_{HL}; \overline{k}_b^n\big) &= \big(2\overline{k}_b^n - 1\big)(\alpha_{HL} - \alpha_L)\Delta\big(\alpha_{HL}; \overline{k}_b^n\big);\\ \Pi\big(\alpha_H, \alpha_{HL}; \overline{k}_b^n\big) - \alpha_{HL} &= \big(2\overline{k}_b^n - 1\big)(\alpha_H - \alpha_{HL})\Delta\big(\alpha_{HL}; \overline{k}_b^n\big).\end{aligned}$$

Thus, the claim is true if $\alpha_{HL} = \frac{1}{2}(\alpha_H + \alpha_L)$ and $\beta_{HL} = \frac{1}{2}(\beta_L + \beta_H)$, and hence by continuity also holds under the neutrality assumption. A symmetric argument establishes that the function $r_b(k_a^p)$ given by the value of k_b^n such that condition (A.15) holds with equality is well defined for all $k_a^p \in [\frac{1}{2}, \overline{k}_a^p]$. Now, by taking derivatives we can verify that $r'_a(k_b^n) < r'_b(k_a^p)$ whenever they intersect under the assumption of $\alpha_{HL} = \frac{1}{2}(\alpha_H + \alpha_L)$. It follows immediately that r_a and r_b have a unique intersection at some k_a^p, k_b^n, with $k_a^p \in (\frac{1}{2}, \overline{k}_a^p)$ and $k_b^n \in (\frac{1}{2}, \overline{k}_b^n)$.

Finally, in the P/N-region, we have $\beta_H < \beta_{HL} < \beta_L$. Comparing (A.13) to (2), and (A.15) to (3), we immediately obtain that the equilibrium levels for the high type candidates are strictly lower than their respective levels k_H^p and k_H^n.

In the second case, suppose that in equilibrium, type (α_H, β_H) candidate a runs a positive campaign of some level k_a^p and (α_L, β_L) candidate b runs a positive campaign of level k_b^p; the case of the two types running negative campaigns is symmetric. The equilibrium condition for the low type candidate a is (7); the condition for the low type b is

$$\beta_H - \alpha_H \geq \Pi\big(\beta_H, \beta_{HL}; k_a^p\big) - \Pi\big(\alpha_H, \alpha_{HL}; k_b^p\big) - C\big(k_b^p\big). \tag{A.17}$$

The rest of the argument is analogous to the first case. ■

Proof of Proposition 6. See the Online Appendix.

B. *Robustness of Least Cost Separation.* In models with more than two types or with richer information structure, it is necessary to rank each type of candidate's incentives to imitate all types of candidate above him to study whether equilibrium separation is possible. In the Online Appendix, we present a single crossing condition and show that the separation result in the basic model is robust if this condition is satisfied.

Supporting Information

Additional Supporting Information may be found in the online version of this article:

Online Appendix

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