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What do interest rates reveal about the functioning of real business cycle models ?

Paul Beaudry^{*,a}, Alain Guay^b

^a Department of Economics, University of British Columbia, Vancouver, BC V6T 1Z1, Canada ^b Department of Economics, Université du Québec, Montréal, Qué. H3C 3P8, Canada

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Abstract

This paper begins by documenting the extent to which the predictions of standard Real Business Cycle (RBC) models are incompatible with observed movements in real interest rates. The main finding of the paper is that extending the baseline model to include habit persistence in consumption and adjustment costs to capital significantly improves the model's empirical performance. In our evaluation of the model's performance, we take special care of estimating and testing predictions of the model using both moments drawn directly from the data and moments calculated after identifying shocks to the stochastic trend.

Key words: Interest rates; Business cycles; Habit formation; Adjustment costs JEL classification: E3

1. Introduction

Explaining the comovements between quantities and prices is relevant to any theory purporting to describe the functioning of a market economy. In Real Business Cycle (RBC) models there are essentially two prices that drive the allocation process for goods and labor: these are the real wage and the real interest rate. There has been a substantial amount of effort devoted to comparing observed

^{*} Corresponding author (also affiliated with CRDE and NBER).

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wage movements with those predicted by RBC models (for an overview of this literature see Hansen and Wright, 1992). However, this interest in confronting RBC models with price data has not been extended to interest rates.¹ This is especially surprising given the preponderant theoretical role played by interest rates in decentralizing incentives in RBC models. Therefore, in this paper we focus on real interest rates as a mean of evaluating and identifying potential improvements to existing RBC models.

There are several reasons why the RBC literature has paid little attention to the predictions of the theory with respect to interest rate movements. In particular, there is no perfect empirical counterpart to the theoretical risk-free interest rate. Measured real interest rates, whether ex-ante or ex-post, may be inappropriate proxies. Nonetheless, we believe that confronting RBC models with interest rate data is potentially fruitful given that there is no special reason to believe that the measurement issue for interest rates is any more severe than for most other macroeconomic variables.

In the first part of the paper we document the extent to which the standard RBC model is incompatible with observed movements in real interest rates. In effect, most RBC models predict a highly procyclical real interest rate since a persistent improvement in technology induces an increase in demand for both consumption and investment that, in the short run, overrides the increase in supply. This causes interest rates to rise as output increases, thereby generating procyclical movements in interest rates. In contrast, over the post-war period, measured real interest rates are best described as either acyclical or slightly countercyclical. This characterization remains apparent even when we use long run restrictions as in King, Plosser, Stock, and Watson (1991) to identify correlations induced by technology shocks.²

This failure of the standard RBC model with respect to interest rates should not be surprising given the well-known shortcomings of the consumption-based assetpricing model;³ nonetheless, it remains problematic considering the significant implicit role played by interest rates in creating incentives to intertemporally substitute in RBC models. Therefore it seems important to examine whether such a failure identifies a major shortcoming of the RBC paradigm or whether it

¹ There are several recent papers that examine implications of RBC models for asset prices, for example, Rouwenhorst (1995), Donaldson, Johnsen, and Mehra (1990), Detemple and Sundaresan (1992), and Cochrane and Hansen (1992). However, none of these papers concentrate on reconciling the observed comovements between interest rates and quantities with those predicted by RBC models. One exception is Novales (1990,1992), who examines the properties of interest rates in an unconventional RBC model where there is no labor and where technology shocks do not affect the marginal productivity of capital.

 $^{^2}$ A closely related methodology was also used by Blanchard and Quah (1989) to identify supply and demand shocks.

³ See Campbell and Cochrane (1995) for a recent discussion of this issue.

only indicates that the particular specifications for technology and preferences are overly simplistic.

The second part of the paper examines whether introducing adjustment costs in capital accumulation and habit formation in consumption improves the model's empirical performance. These two modifications to the model seem warranted for several reasons. First, many empirical studies of investment support the presence of adjustment costs in capital (see for example Shapiro, 1986) and therefore the standard specification of technology in RBC models is likely to be inappropriate. Second, recent research on asset-pricing has found that habit formation in consumption is empirically plausible (Ferson and Constantinides, 1991; Heaton, 1995) and may help explain certain asset-pricing puzzles (see Constantinides, 1990; Campbell and Cohrane, 1995). Third, these two modifications have the potential of improving the model's performance with respect to the correlation between interest rate and output since they reduce the responsiveness of demand to technology shocks. Therefore, these two modifications seem to be a logical starting point to assess whether a modified RBC model can be reconciled with observations on interest rates without becoming counterfactual on other dimensions.

The main result of the paper is that the introduction of adjustment costs to capital⁴ and habit formation considerably improves the model's performance.⁵ In particular, when we use generalized method of moments (GMM) to estimate the degree of habit persistence and adjustment costs using moments induced from identifying shocks to the stochastic trend, we find that the model's predictions become compatible with observed comovements between interest rates and output. Moreover, we find that the model's overidentifying restrictions are not rejected.

The remaining sections of the paper are structured as follows. In Section 2 we document the predictions of a prototypical RBC model for the comovements between output and interest rates. In Section 3 we examine the empirical counterpart to these predictions. To this end, we examine both the correlations found in raw data and the correlations inferred by using the King, Plosser, Stock, and Watson (1991) methodology to identify technology shocks. These latter results are of interest since they allow an empirical evaluation of RBC models which, in principle, is removed from the debate regarding the relative importance of technology shocks in business cycle fluctuations. In Section 4 we introduce adjustment costs to capital and habit formation in consumption to our baseline model and examine whether such modifications help reconcile theory with observations. We estimate and test the model's predictions on several fronts using both the correlations calculated from the raw data and those implied by identifying shocks to the stochastic trend. In particular, we believe that this latter empirical exercise is of interest

⁴ In the seminal paper by Kydland and Prescott (1982), it was noted that non-time-separabilities improved the fit of technology shock model. However, this claim has been placed in doubt by Rouwenhorst (1991).

⁵ In independent work, Boldrin, Christiano, and Fisher (1995) have arrived at a similar conclusion.

in its own right since it offers an example where conditional moments associated with a particular identification strategy are used to estimate and test a dynamic general equilibrium model. Finally, Section 5 offers concluding comments.

2. Interest rate movements in real business cycle models

2.1. The prototype real business cycle model

This section quickly overviews the RBC model developed by King, Plosser, and Rebelo (1988a,b) in order to highlight the prediction of this class of models for the comovements between interest rates and output. The environment involves a single good that is produced by labor and capital and where the production possibilities are stochastic. The preferences of the representative agent are assumed to be time-separable and to depend on consumption, C_t , and leisure, L_t , according to

$$U = \sum_{t=0}^{\infty} \beta^{t} (\log(C_{t}) + v(L_{t})), \qquad v'(L_{t}) > 0, \quad v''(L_{t}) < 0.$$
(1)

The production technology is assumed to be of the Cobb-Douglas form with constant returns to scale as in

$$Y_t = K_t^{1-\alpha} (X_t N_t)^{\alpha}, \tag{2}$$

where K_t is the capital stock at the beginning of period t, N_t is the amount of work supplied at period t, and X_t represents the technology index. The stock of capital evolves according to the accumulation equation

$$K_{t+1} = (1 - \delta)K_t + I_t,$$
(3)

where I_t is gross investment and δ is the depreciation rate. The resource constraints are given in

$$L_t + N_t \le H \quad \text{and} \quad C_t + I_t \le Y_t, \tag{4}$$

where H is the time endowment of the representative agent.

In order to encompass several cases covered in the literature, including situations with either a stochastic or deterministic trend, we allow the process governing technology to be represented as the IMA process given in (5), where B represents the lag operator and $\Phi(\cdot)$ is a polynomial,

$$(1-B)\log X_t = \mu + \Phi(B)\varepsilon_t.$$
⁽⁵⁾

The permanent component of this technology process is

$$(1-B)\log X_t^p = \mu + \Phi(1)\varepsilon_t.$$
(6)

The equilibrium allocations for this economy are found by maximizing the expected utility of the representative agent subject to constraints (2) to (5). Since

exact solutions to this problem are not readily available, some approximation method must be adopted. We follow the linear approximation method⁶ detailed in King, Plosser, and Rebelo (1990).⁷

Given the equilibrium stochastic processes for quantities, the stochastic processes for prices can be obtained through either marginal product conditions or through the consumer's marginal rates of substitution. In particular, using the fact that the real interest rate must be equal to the marginal rate of substitution between present consumption and future consumption, the risk-free rate of interest is given by

$$1 + r_t = \mathcal{E}_t \frac{C_{t+1}}{\beta C_t}.$$
(7)

In Eq. (7), r_t is the per-period return on a risk-free bond and $E_t(\cdot)$ is the conditional expectation operator based on information available at time t.

2.2. Predictions of the RBC model regarding interest rates and output

Table 1 reports cross-correlations between interest rates and output implied by the above model for different processes governing technology. The parameters describing preferences and production possibilities are set equal to those in King, Plosser, and Rebelo (1988a), and are presented at the bottom of Table 1. The results are presented in ascending order with respect to the persistence of the technology shock. The first three rows correspond to situations where technology is stationary around a linear trend and the next two rows correspond to cases with a stochastic trend. Correspondingly, in the first three rows the correlations between output and interest rates represent correlations between detrended output and the interest rate, while in rows 4 and 5 output is measured in growth rates.

The first row of Table 1 presents theoretical correlations for the case where technology shocks are purely temporary, that is, $\Phi(B) = (1 - B)$. As can be seen, this type of process for technology generates a negative contemporaneous correlation between output and interest rates. The economic intuition for the negative correlation is quite clear: a temporary technology shock renders present goods

⁶ Dostey and Mao (1992), among others, have found that the King-Plosser-Rebello linear approximation technique may sometimes be inaccurate. However, our own exploration of the issue has not revealed this technique to be inappropriate for the moments of interest in this study.

⁷ This method consists of undertaking the following steps: (1) divide all quantities except labor by the permanent component in technology X_t^p , (2) linearize the first-order conditions associated with this modified system around the steady state, (3) invoke the certainty equivalence principle to derive decision rules. The outcome of these three steps is a state–space representation for the deviation from steady state of the logarithm of K_t/X_t^p , V_t/X_t^p , V_t/X_t^p , N_t . Hence, to be comparable to actual data, all the resulting quantities must be multiplied by the steady state values and the permanent component X_T^p must be reintroduced where it was removed. The final product from this procedure are the stochastic processes for Y_t , C_t , K_t , I_t , and N_t that (approximately) characterize the optimal allocation.

	k						
$\Phi(B)$	-2	~1	0	1	2		
(A) Linear detrended of	utput: corr(log($(Y_t)^T, r_{t-k})$					
(1 - B) (1 - B)/(1 - 0.5B) (1 - B)/(1 - 0.9B)	-0.34 0.17 0.14	-0.36 0.09 0.28	0.38 0.65 0.43	-0.08 0.30 0.40	-0.07 0.13 0.36		
(B) First-differenced ou	tput: corr(⊿ log	$(Y_t), r_{t-k})$					
$\frac{1}{1/(1-0.22B)}$	0.32 0.42	0.34 0.44	0.35 0.44	0.05 0.48	0.05 0.09		
(C) HP-filtered data: co	$\operatorname{orr}(\log(Y_t)^{HP}, r_t^H)$	$\frac{P}{-k}$)					
(1-B)/(1-0.9B)	0.27 0.35	0.57 0.60	0.95 0.98	0.73 0.71	0.53 0.53		

Table 1								
Theoretical	correlations	between	real	output	and	real	interest	rates

Parameters: $\alpha = .58$, $\delta = 0.025$, $\beta = 0.988$, $\mu = 1.004$, Lv''(L)/v'(L) = -1, and N/H = 0.2, where N is the steady state level of hours worked.

more abundant than future goods and therefore the relative price of today's goods must fall. However, once the technology shock is allowed to be sufficiently persistent, this negative correlation disappears. Again the economic intuition is straightforward: with a persistent technology shock, the rise in the marginal product of capital increases investment demand, and simultaneously increases consumption demand through an increase in permanent income. These two induced shifts in aggregate demand offset the shift in aggregate supply causing current goods to become relatively scarce.⁸ The surprising result in Table 1 is that the correlation between output and interest rates becomes positive at rather low degrees of persistence of the technology shock.⁹ In the second row of Table 1, the technology shock is still trend-stationary and possesses an autoregressive parameter equal to 0.5. Even for this rather low level of persistence in the technology shock, the contemporaneous correlation between interest rate and output is positive. However, with such a low autocorrelation in the technology process the model

⁸ We refer to aggregate demand and aggregate supply as located in the real-interest-rate output space, which is the most relevant space for describing goods market equilibrium in the neo-classical framework.

 $^{^{9}}$ As soon as the degree of persistence is above 0.05, the contemporary cross-correlation between interest rates and output is positive.

underpredicts the autocorrelation in output relative to that found in the data. As noted in King, Plosser, and Rebello (1988a), it takes an autoregressive parameter of approximately 0.9 in the technology process to reproduce the autocorrelation in output observed in the data. The third row in Table 1 reports correlations implied by the model studied in King et al. (1988a), that is, a trend-stationary process with an autoregressive parameter of 0.9. In this case, the correlation between interest rates and output is 0.43, which suggests a very procyclical pattern for the interest rates.¹⁰

The rows 4 and 5 of Table 1 report correlations between output and interest rates for cases where the technology process is difference stationary. Row 4 corresponds to a random walk specification for technology and row 5 corresponds to the case where the growth in technology has an autoregressive parameter of 0.22. This latter case is reported since it corresponds to the parameterization that exactly reproduces the autocorrelation in output growth observed in US data over the period 1954:1–1990:4. In both cases, the model predicts a strong positive correlation between output growth and interest rates.

Rows 6 and 7 report the theoretical correlations found after applying the HP-filter to two of the cases previously discussed.¹¹ Row 6 corresponds to the trend-stationary model with autoregressive parameter 0.9 and row 7 represents the random walk specification of technology. As could be expected, these two processes are rather similar and correspondingly imply similar correlations between output and interest rates after the HP-filter has been applied to the data. It is nevertheless worth noting that these models predict correlations between output and interest rates for HP-filtered data that are close to 1.

In order to further explore the predictions of an RBC model with respect to the pattern of correlations between interest rates and output, we examined the sensitivity of results reported in Table 1 with respect to changes in parameters. In particular, we thoroughly examined whether results changed for reasonable changes in the depreciation rate or the intertemporal elasticity of substitution in consumption.¹² In general, we found the pattern to be very robust. For example, if we doubled the yearly depreciation rate from 10% to 20%, the contemporaneous correlation between output and interest rates actually increased when technology shocks are assumed to be nonstationary, and only falls from 0.43 to 0.31 when technology shocks were assumed to be trend-stationary with an autoregressive parameter equal to 0.9. Similarly, when we change the intertemporal elasticity of

¹⁰ The results reported in Table 1 remain virtually identical if the deterministic trend is set to zero.

¹¹ The HP filter is a stationarity-inducing transformation. It removes nonstationary components that are integrated of order 4 and less. See Hodrick and Prescott(1980) and King and Rebelo (1993) for details regarding this filter.

¹² When we considered variations in the intertemporal elasticity of substitution in consumption, we consider the utility function to be of the class $U(C,L) = C^{1-\sigma}v(L)/(1-\sigma)$. King, Plosser, and Rebelo (1990) show that this class of utility functions is compatible with steady state growth.

substitution in consumption from 1 to 5 in the case technology follows a random walk, the correlation between interest rates and output only falls from 0.35 to 0.21. Overall, we believe that Table 1 reflects a set of robust predictions of the prototypical RBC, that is, the interest rate is strongly procyclical whenever shocks to technology are persistent enough to reproduce substantial autocorrelation in output.¹³

3. The empirical relationship between output and interest rates

3.1. Correlations found in raw data

In Table 2, we report empirical correlations between quarterly data on output and two measures of interest rates. In order to be comparable with the model, the output measure used is per-capita real GNP net of government expenditures.¹⁴ The two interest rates measures are ex-post and ex-ante real returns on threemonth Treasury bills calculated from the prices on the secondary market. The price index for output corresponds to the GNP deflator adjusted for the elimination of government consumption.¹⁵ All empirical results are for the period 1954:1– 1990:4, which has the advantage of excluding the Korean war.¹⁶ In the first column of each table, output is measured relative to a linear trend. In the second column output is in growth rates, and in the last column the HP-filter is applied to output and interest rates.

Panel A of Table 2 presents correlations where real returns are measured expost, and panel B presents sample moments for returns which are measured ex-ante using eight lags of inflation, output, interest rates, and M2 as predictors for inflation.¹⁷ In both cases, the point estimates suggest a correlation between

¹³ This statement may seem to contradict that found in Donaldson, Johnsen, and Mehra (1990) regarding the behavior of the term structure of interest rates over the business cycle. However, the difference is mainly one of terminology. We use the term procyclical to refer to the contemporaneous correlation of a variable with output. In contrast, Donaldson, Johnsen, and Mehra use the term procyclical to refer to the pattern of a variable between the peaks and troughs in output. In general these two concepts do not coincide.

¹⁴ The practice of adjusting the data to exclude the government sector is common in the RBC literature and seems warranted when the government sector is excluded from the model. For examples, see Watson (1991) and King, Plosser, Stock, and Watson (1991).

¹⁵ The data are taken from Citibase. The variables used are (gnp82 - gge82) for output, (fygm3) for Treasury bills, (gnp - gge)/(gnp82 - gge) for the price deflator, (p16) for the population, and M2 is the series constructed by King, Plosser, Stock, and Watson (1991).

 $^{^{16}}$ Guay (1993) shows that these correlations are not very sensitive to the choice of a price index, to the asset price used, or to the particular post-war sample period.

¹⁷ As long as the variance of the conditional covariance between consumption growth and inflation innovations is small, the measured ex-ante real interest rate should be a good proxy for the theoretical risk-free rate.

К	$(\log(Y_t)^T, r_{t-k})^a$	$(\Delta \log(Y_t), r_{t-k})$	$(\log(Y_t)^{HP}, r_{t-k}^{HP})^{\mathrm{b}}$
(A) Outpu	it and ex-post real interest rates	5	
-2	$-0.27 (0.09)^{c}$	-0.04(0.09)	0.09 (0.08)
-1	-0.28(0.08)	-0.04(0.11)	0.07 (0.06)
0	-0.31 (0.08)	-0.13(0.09)	-0.03(0.07)
ì	-0.35 (0.09)	-0.13 (0.13)	-0.13 (0.09)
2	-0.38 (0.10)	-0.13 (0.11)	-0.22 (0.12)
(B) Outpu	it and ex-ante real interest rates	5	
-2	-0.35 (0.11)	-0.04 (0.12)	0.09 (0.09)
-1	-0.36 (0.10)	-0.16(0.13)	0.06 (0.09)
0	-0.35 (0.10)	-0.20 (0.11)	-0.03(0.09)
1	-0.43(0.09)	-0.18 (0.11)	-0.13(0.09)
2	-0.47 (0.09)	-0.12 (0.11)	-0.22 (0.09)

Table 2 Sample correlations between output and ex-post /ex-ante real interest rates, 1954:1-1990:4

^aLinearly detrended logarithm of output.

^bWe apply the Hodrick-Prescott filter to both series.

^cStandard errors in parentheses are calculated using the Newey and West (1987) procedure.

output and interest rates that is negative. In relation to the theoretical moments presented in Table 1, the sample moments do not lie within a 95% confidence interval of any of the parameterizations that imply substantial persistence in output (rows 3 to 7).¹⁸ In particular, the theoretical and sample moments are statistically furthest apart for the HP-filtered data even though the corresponding sample moments are close to zero.¹⁹ In order to further document the extent to which the data and the model are at odds, we performed a joint test on the five predicted correlations, $corr(\Delta Y_t, r_{t-i})$, i = -2, -1, 0, 1, 2, under the assumption that technology follows a random walk. The resulting statistic, which has an asymptotic $\chi^2(5)$ distribution under the null that the model is right, produced a value of 55.7 which is clearly rejected at standard confidence levels.²⁰

The inference that we draw from Tables 1 and 2 is that there is a discrepancy between theory and measurement. However, before trying to address this

¹⁸ The standard errors for this comparison are calculated from the data. We also found this result when we used standard errors inferred by simulating the model. In fact, in this latter case standard errors are substantially smaller.

¹⁹ Prescott, Guenther, Kehoe, and Manuelli (1983) also find the empircal correlation between HP-filtered output and the real interest rate to be slightly negative.

²⁰ This statistic is the value of $(m - \hat{m})^{\ell} \hat{V}^{-1}(m - \hat{m})$, where *m* corresponds to the five predicted moments, \hat{m} corresponds to the empirical analogs, and \hat{V} is an estimated variance-covariance matrix for \hat{m} derived using the procedure described in Newey and West (1987).

difficulty theoretically, it is important to explore whether it may simply be caused by a measurement issue. In particular, evaluating an RBC model based on sample moments drawn directly from the data is only valid if technology shocks are the sole source of variation in the data. However, it is generally accepted that technology shocks account for at most 70% of variation in output at business cycle frequencies (see the discussion by Kydland and Prescott, 1991, on this issue) and possibly much less (see Eichenbaum, 1991). Therefore, the sample moments presented in Table 2 are likely to be contaminated by other shocks (including measurement error) and may thus be inappropriate for direct comparison with the theoretical moments drawn from an RBC model. In order to explore this possibility, we exploit in the following subsection the methodology used by King, Plosser, Stock, and Watson (1991) (hereafter KPSW) to identify variations in the data that are likely to reflect technology shocks. Our approach complements their work by calculating estimates of cross-correlations induced by technology shocks instead of concentrating exclusively on impulse responses.

3.2. Estimating correlations induced solely by technology shocks

There are two results from the work of King, Plosser, Stock, and Watson (1991) that we draw upon in order to refine our measure of empirical correlations. First, KPSW note that if a system of variables possesses one common stochastic trend and that innovations to this trend are orthogonal to all other innovations affecting the system, then the dynamics induced by the stochastic trend can be identified. Second, KPSW note that a RBC model generates a system of variables with a unique stochastic trend under the assumption that the technology process possesses a unit root and that all other shocks to the system induce stationary responses. Therefore, if actual data is thought to be generated by a RBC model in which innovations to technology shocks are uncorrelated with other shocks to the system and where the technology shocks are the only source of long-term growth, then the correlations induced solely by technology shocks can be inferred from observations on raw data.

Before applying the KPSW methodology to the identification of technology shocks, it is important to examine whether our data are compatible with the restrictions implied by the approach. In particular, we focus on the vector ($\log Y_t$, $\log C_t$, $\log I_t$, r_t). As discussed in KPSW, if this vector is generated by a RBC model where long-term growth is driven by a stochastic trend to technology, then the three first variables should be nonstationary and the interest rate variable should be stationary. Moreover, the first three variables should be pairwise cointegrated with cointegrating vectors equal to (-1, 1).²¹ The data we use to examine these properties, and thereby test the appropriateness of the KPSW

²¹ These implications for the stochastic growth model are clearly presented in KPR (1988b) and therefore not derived again here.

methodology, are quarterly US observations over the period 1954:1–1990:4. The variables used correspond to the logarithms of per-capita real GNP net of government spending, per-capita real private consumption, per-capita real private investment, and ex-post²² real return of three-month Treasury bills.²³

As is well known, augmented Dickey-Fuller tests generally indicate that the processes for output, consumption, and investment can be characterized as I(1)processes with drift, which is consistent with the null hypothesis behind the KPSW methodology. When we test this characterization with our data, the respective statistics are -3.31, -2.18, and -2.53 when we follow the procedure suggested by Campbell and Perron (1991) to choose the number of lags to include in the test. For the real interest rate, the null hypothesis is that it is a stationary variable. Using the test developed by Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) (1992), we find that the stationarity of the ex-post real interest rate cannot be rejected at the 5% level. In fact, the test statistic takes a value of 0.399 (the critical value is 0.461). Note that this test takes the null hypothesis to be stationarity, which is implied by the model, as opposed to the Dickey-Fuller test that takes the null hypothesis to be nonstationarity. Concerning the hypothesis that $\log Y_t$, $\log C_t$, and $\log I_t$ are pairwise cointegrated, the statistics associated with the hypothesis that $\log Y_t - \log C_t$ and $\log Y_t - \log I_t$ are stationary using the test developed by KPSS are respectively 0.352 and 0.058, which again cannot be rejected at the 5% level.

The aforementioned results all provide evidence in support of the long-run restrictions implied by the stochastic growth model. In light of these results, we pursue the method suggested by KPSW to recover the structural moving-average representation of the data from the reduced-form representation. The reduced-form moving-average representation for the variables $\Delta \log Y_t$, $\Delta \log I_t$, r_t was obtained from a vector-error-correction model where eight lags of each variable were included in addition to the cointegration vectors. The structural moving-average representation for these variables can be written as

$$\Delta X_t = \mu + \Gamma(B)\eta_t,$$

where $\Delta X_t = (\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, \Delta r_t)'$, $\eta_t = (\eta_{1t}, \eta_{2t}, \eta_{3t}, \eta_{4t})'$ is the vector of structural disturbances, $\Gamma(\cdot)$ is a polynomial, and $\Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i$ is the matrix of long-run multipliers.

 $^{^{22}}$ We also examined the system where inflation and nominal interest rates replaced the ex-post interest rates. This modification caused the system to possess at least one more stochastic trend and therefore an additional identifying restriction was needed to determine technology shocks. When we used a Choleski decomposition as the additional identification restriction, as suggested by KPSW, we found very similar results for the inferred ex-ante interest rate as those reported here. See Guay (1993) for details.

²³ The variables used are the same as those identified in Footnote 12. In addition, from Citibase, consumption corresponds to (gce82/p16) and investment to (gif82/p16).

Under the maintained assumption that the only nonstationary driving force is technology, the matrix of long-run multipliers implied by our model is given by 24

$$\Gamma(1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

when η_{1t} is defined as the innovation in technology. By exploiting the structure of $\Gamma(1)$ and the assumption that η_{1t} is orthogonal to the transitory shocks $(\eta_{2t}, \eta_{3t}, \eta_{4t})$, the reduced-form moving-average representation can be inverted as to recover the impulse responses associated with shocks η_{1t} to the stochastic technological trend. The resulting impulse responses (or structural moving-average representation) can then be used to calculate correlations induced by technology shocks.

In panel A of Table 3 we report the estimated cross-correlation matrix between $\Delta \log Y_t, \Delta \log C_t, \Delta \log I_t, r_t$ inferred using this identification strategy. In order to facilitate comparisons, panel B of Table 3 reports the same statistics calculated from the raw data. The most striking feature emerging from the two panels are the similarities: for example, the ranking of variances is identical and the pattern of correlations is remarkably close. This observation is comforting given that raw data are usually used to evaluate RBC models. Nevertheless, the two sets of moments do differ in several respects. As should be expected, the contemporaneous correlations between output and consumption is much higher for moments induced by shocks to the stochastic trend. For our purpose, the most interesting finding in the table is that the contemporaneous correlation between output and interest rates is even more negative in panel A than in panel B. In fact, the point estimates given in panel A suggest that shocks to technology induce a countercyclical movement in interest rates. When we perform a joint test on the five correlations, corr($\Delta \log Y_t, r_{t-i}$), i = -2, -1, 0, 1, 2, we now obtain a test statistic of 10.0 which is still clearly rejected at the 10% level (*p*-value = 0.075).²⁵

In summary, the examination of interest rate movements indicate a deviation between theory and observation; the prototypical RBC model predicts a strong positive relationship between interest rates and output while the data does not support this claim. Furthermore, we have shown that this discrepancy does not disappear when we use the cointegration restrictions implied by the stochastic growth model to identify technology shocks.

²⁴ The first column of $\Gamma(1)$ is normalized to 1.

²⁵ For the calculation of this statistic, we use 1000 Monte Carlo simulations to estimate the variancecovariance matrix of the five moments.

(A) Moment	s inferred from shocks	to the stochastic trend					
	Relative standard deviations						
$x = \sigma_x / \sigma_y$	$\Delta \log(y)$	$\Delta \log(c)$	$A \log(I)$	r 0.12			
	1.00	0.48	1.45				
	Cross-correlatio	Cross-correlations between $(\Delta \log(y_t), x_{t-k})$					
-2	0.22	0.14	0.37	-0.09			
- 1	0.24	0.15	0.73	-0.12			
0	1.00	0.97	0.79	-0.42			
1	0.24	0.24	0.30	-0.09			
2	0.22	0.19	0.10	-0.07			
(B) Sample	moments drawn directly	y from data					
(B) Sample	moments drawn directly Relative standar	y from data d deviations					
(B) Sample $x =$	moments drawn directly Relative standar 	y from data d deviations $A \log(c)$	∆ log(1)	ŗ			
(B) Sample $x = \sigma_x / \sigma_y$.	moments drawn directly Relative standar $\Delta \log(y)$ 1.00	y from data rd deviations $A \log(c)$ 0.57	∆ log(1) 2.06	r 0.53			
(B) Sample $x = \sigma_x / \sigma_y$	moments drawn directly Relative standar $\Delta \log(y)$ 1.00 Cross-correlatio	y from data d deviations $\Delta \log(c)$ 0.57 ns between $(\Delta \log(y_t),$	$\frac{\Delta \log(I)}{2.06}$	r 0.53			
(B) Sample $x = \sigma_x / \sigma_y$ -2	moments drawn directly Relative standar $\Delta \log(y)$ 1.00 Cross-correlatio 0.19	y from data y from data $\Delta \log(c)$ 0.57 ns between $(\Delta \log(y_t))$, 0.18	$\frac{\Delta \log(I)}{2.06}$ x_{t-k} 0.26	r 0.53 0.04			
(B) Sample $x = \frac{\sigma_x}{\sigma_y}$ $-\frac{2}{-1}$	moments drawn directly Relative standar $\Delta \log(y)$ 1.00 Cross-correlatio 0.19 0.32	y from data d deviations $\Delta \log(c)$ 0.57 ns between $(\Delta \log(y_t))$, 0.18 0.24	$\frac{\Delta \log(I)}{2.06}$ $\frac{x_{t-k}}{0.26}$ 0.26 0.48	r 0.53 0.04 0.04			
(B) Sample $x = \frac{\sigma_x / \sigma_y}{-2}$	moments drawn directly Relative standar $\Delta \log(y)$ 1.00 Cross-correlatio 0.19 0.32 1.00	y from data d deviations $A \log(c)$ 0.57 ns between $(A \log(y_t))$, 0.18 0.24 0.62		r 0.53 -0.04 -0.04 -0.13			
(B) Sample $x = \frac{\sigma_x / \sigma_y}{-2}$ -2 -1 0 1	moments drawn directly Relative standar $\Delta \log(y)$ 1.00 Cross-correlatio 0.19 0.32 1.00 0.32	y from data d deviations $A \log(c)$ 0.57 ns between $(A \log(y_t))$, 0.18 0.24 0.62 0.35		r 0.53 -0.04 -0.04 -0.13 -0.13			

Table 3 Cross-correlation matrix 1954:1-1990:4

4. Reconciling theory with observation

In this section we examine whether simple modifications to preferences and technology can help reconcile the RBC model with observations on interest rates. As emphasized in Section 2, the prototypical RBC model predicts a procyclical interest rate due to a strong and immediate response of both consumption and investment to persistent technology shocks. However, the type of response of demand to technology shocks depends on the particular specification of preferences and production possibilities adopted. For example, if firms find it costly to adjust capital or if the marginal utility of present consumption depends positively on past consumption, then the demand response following a technology shock would be diminished relative to the base case. Since there exists empirical evidence for both adjustment cost to capital and habit persistence in consumption, it seems natural to examine whether allowing for such nonseparabilities significantly reduces the predicted correlation between output and interest rates and thereby realigns theory with evidence. However, it is important that we do not focus exclusively on interest rate movements when evaluating these modifications since introducing such changes may adversely affect the predictions of the model on other fronts. Consequently, the approach we adopt is (1) extend the prototype model to include adjustment cost to capital and habit persistence in consumption, (2) estimate by GMM the parameters associated with habit persistence and adjustment costs, (3) test whether such changes help or hinder the model's performance on different dimensions including its capacity to explain the observed comovements between interest rates and output. In accordance with our previous discussion, we estimate and test the model using both moments observed in the raw data and moments induced by identifying shocks to the stochastic trend.

4.1. Extending the baseline model

We consider two modifications to the baseline model presented in Section 2. First, the consumer's preferences are assumed to exhibit habit persistence as represented by Eq. (8). Attention is restricted to this very simple form of habit persistence since such parsimony has the advantage of limiting the number of parameters to estimate and therefore allows for more powerful tests of the model.

$$U = \sum_{t=0}^{\infty} \beta^{t} (\log(C_{t} - vC_{t-1}) + v(L_{t})), \quad v > 0.$$
(8)

Second, the installation of new capital is assumed to be resource consuming so that the economy's total resource constraint needs to be modified according to

$$C_t + I_t \le Y_t - \frac{q(K_{t+1} - \gamma K_t)^2}{2K_t}, \qquad q > 0,$$
 (9)

where $\gamma = \exp(\mu + \Phi(1)\varepsilon_t)$. In Eq. (9), the term $q(K_{t+1} - \gamma K_t)^2/2K_t$ represents the costs associated with installing new capital. The precise formulation of adjustment costs has been chosen so that a steady state (around trend growth μ) exists and is invariant to the size of the adjustment costs. The steady state of this economy and the exact derivation of the state-space representation are discussed in the Appendix.²⁶

The two new parameters of the model, v and q, are referred to respectively as the habit persistence parameter and the adjustment cost parameter. The relationship between these parameters and the sensitivity of demand to technology shocks

 $^{^{26}}$ The interest rates implied by the model are still calculated from the marginal rate of substitution in consumption, but this no longer corresponds exactly to Eq. (7) because the presence of habit formation modifies the marginal utility of consumption.

is quite clear. On the one hand, a positive value for the habit persistence parameter has the effect of attenuating the contemporaneous response of consumption to an increase in permanent income since consumers prefer smooth transitions over jumps. On the other hand, a positive value for q reduces the impact on investment of an increase in the marginal product of capital since adjustment costs to capital also favor smoothing. These two modifications of the model should intuitively contribute to a reduction of the contemporaneous correlation between interest rates and output.

In order to examine the relevance of this extention, we begin by estimating v and q by GMM and testing whether these parameters are significantly different from zero. In performing this estimation, we take all other parameters of the model to be equal to those used in Section 2 and we assume that the process governing technology is a random walk with drift. This limited focus allows us to concentrate exclusively on the effects of adding habit formation and adjustment cost to the baseline model. More precisely, our estimation procedure consists of minimizing, by choice of $\theta = (v, q)$, the following quadratic form:

$$\theta_T = \underset{\theta \in \Theta}{\arg\min(m(\theta) - \hat{m}_T)' \hat{\Omega}_T^{-1}(m(\theta) - \hat{m}_T)}.$$
(10)

In the above minimization, the vector $m(\theta)$ represents a set of population moments implied by the model for given values of v and q, the vector \hat{m}_T represents corresponding sample moments, and $\hat{\Omega}_T$ represents an estimated variance– covariance matrix for these moments. In the cases where we compare predicted moment with those found in raw data, the variance–covariance matrix is estimated directly from the data using the Andrews–Monahan (1992) prewhitened kernel estimator with automatic bandwidth procedure. However, in the cases where we compare predicted moments with those computed after identifying shocks to the stochastic trend, the variance–covariance matrix is computed by Monte Carlo simulations.²⁷ Since the model does not give us an explicit expression for the relationship between structural parameters and implied moments, we solve the minimization problem using a standard numerical optimizing routine based on numerical derivatives.

The minimized value of the quadratic form (10) (called Q-statistic), multiplied by the number of observations, has, under the null, an asymptotic χ^2 distribution (with the number of degrees of freedom being equal to the number of elements in m minus 2). Therefore, we test the extended model's 'fit' using this statistic. The GMM framework also allows us to test whether the model's predictions regarding the correlations between output growth and real interest rates is consistent with its empirical counterpart. This last test is particularly relevant given our discussion of the previous section.

²⁷ The variance-covariance matrices for these moments are calculated using 1000 simulations of the impulse response function as discussed by Thomas A. Doan (1988, pp. 10-4,10-5).

Table 4 reports results of this estimation when applied to different sets of moments. The results are divided between panels A and B, where in panel A the empirical moments corresponding to \hat{m} have been calculated directly from raw data, while in panel B the empirical moments are induced from shocks to the stochastic trend. Accordingly, in panel A the maintained null hypothesis is that technology shocks are the only driving force behind economic fluctuations, while in panel B the maintained hypothesis is that shocks to technology are the sole driving force behind the stochastic trend in the data.

In both panels of Table 4 we use three different sets of moments to estimate v and q. Our reason for using different sets of moments is to provide information regarding the robustness of the results. We have chosen the moments to reflect those most often considered in the RBC literature as well as those especially related to the focus of the current paper. Results reported in the first row of Table 4 (case 1) correspond to the case where eight moments are used for the estimation of v and q. They are the relative volatilities (with respect to output) of consumption, investment, employment, and the real interest rate as well as the contemporaneous correlation of these variables with output.²⁸ The second row corresponds to the case where twelve moments are used (case 2); these include the same eight moments as those used in the first row plus four cross-correlations between output and interest rates, that is, $corr(\Delta y_i, r_{i+i})$ for i = -2, -1, 1, 2. Finally, the set of twelve moments used in row 3 correspond to the same eight moments used in row 1 plus first-order autocorrelations of output, consumption, interest rates, and employment.

There is a clear pattern to the results in Table 4. First, regardless of the sets of moments used, our estimates of v and q are highly significant both individually and jointly. Moreover, the point estimates range mainly between 8 and 17 for q and between 0.3 and 0.5 for v. In order to evaluate whether these estimates are of reasonable magnitude, it is helpful to refer to the studies of Shapiro (1986) and Ferson and Constantinides (1991). For example, the estimate of adjustment costs found in Table II column (d) in Shapiro, which represents the specification closest to our formulation, corresponds to a value of q of 8.²⁹ It is worth noting that this value for q implies a loss in output of only 0.4% when the growth of capital is 1.0% above its long-run value. As for the estimate of habit persistence, using quarterly data, Ferson and Constantinides estimate v to be between 0.2 and 0.7.³⁰ Both these comparisons suggest that our parameter estimates are consistent with previous studies.

²⁸ Note the moments are calculated for the growth rate of all the nonstationary variables.

²⁹ The estimates of g_{kk} in Shapiro (1986) must be multiplied by the average manufacturing output (1967\$) over the period 1955–80 to be comparable with our parameter q.

³⁰ Ferson and Constantinides also find estimates of v around 0.95. However, we believe that these estimates should be disregarded since they imply that the marginal utility of consumption would have been negative at several occasions over their sample period.

	q (Std. error)	v (Std. error)	Joint test (p-value)	Q-stat. ^e (p-value)	$\operatorname{corr}(AY_t, r_t)$
(A) Results	s using moments di	rawn directly from	data		
Case 1 ^a	10.87 (0.041)	0.440 (0.002)	1998 (0.00)	31.26 (0.00)	-0.49
Case 2 ^b	9.64 (0.378)	0.347 (0.021)	8540 (0.00)	40.36 (0.00)	-0.30
Case 3 ^c	10.79 (0.410)	0.374 (0.018)	3148 (0.00)	36.74 (0.00)	-0.39
	$\chi^2(1)$	(p-value)	χ ² (5)	(p-value)	
Test ^d	4.245	(0.05)	9.726	(0.08)	

Table 4

Results from estimation and testing of extended model

(B) Results using moments inferred by shocks to stochastic trend

Case 1 ^a	11.05	0.468	6.09	3.11	-0.48
	(0.051)	(0.025)	(0.04)	(0.29)	
Case 2 ^b	7.90	0.529	19.46	4.654	-0.31
	(2.847)	(0.166)	(0.00)	(0.91)	
Case 3 ^c	17.305	0.335	22.674	9.42	-0.59
	(8.68)	(0.015)	(0.00)	(0.47)	
	$\chi^2(1)$	(<i>p</i> -value)	$\chi^{2}(5)$	(p-value)	
Test ^d	0.2846	(0.65)	1.830	(0.83)	

^aThere are eight moments used in estimation: four relative volatilities and four contemporaneous correlations with output growth.

^bThere are twelve moments used in estimation: the same eight moments as under (a) plus four cross-correlations between interest rates and output growth.

^cThere are twelve moments used in estimation: the same eight moments as under (a) plus four first-order autocorrelations.

^dReports tests of whether the model's predictions with respect to interest rate-output correlations are consistent with observation. The $\chi^2(1)$ corresponds to a test of only the contemporaneous correlation. The $\chi^2(5)$ corresponds to a test of the five correlations corr($\Delta Y_t, r_{t-i}$), i = -2, -1, 0, 1, 2.

eReports test of model's overidentifying restrictions.

The second element to note in Table 4 is that the model's overidentifying restrictions, as expressed by the *Q*-statistic, are not rejected by the data under the hypothesis that permanent technology shocks are the driving force behind the stochastic trend in the data. However, these same overidentifying restrictions are strongly rejected when moments taken directly from the data are used to test the model. We interpret this evidence as suggesting that the model's predictions are more consistent with the view that technology shocks are not the only driving force behind economic fluctuations.

Finally, the most important element to remark from Table 4 is that the addition of habit persistence and adjustment costs now leads the model to predict a negative contemporaneous correlation between output growth and interest rates that ranges between -0.30 and -0.59. A formal test of the model's predictions along this front corresponds to a test for a subset of moments. To this end, we apply the LR-type test proposed by Eichenbaum, Hansen, and Singleton (1988), which corresponds to a test of whether a r-dimensional subset of moment conditions is equal to zero. The test consists of 1) evaluating the quadratic form (10) for the whole set of moment conditions, and 2) evaluating (10) without the r-dimensional subset of moment conditions and substract the value of the quadratic form obtained from 2) to that obtained from 1), all multiplied by the number of observations. Under the null hypothesis, the resulting statistic has an asymptotic $\gamma^2(r)$ distribution. We have followed this procedure to test both the model's prediction regarding the contemporaneous correlation between output growth and interest rates as well as the model's prediction for the five moments, $corr(\Delta Y_t, r_{t-i})$, i = -2, -1, 0, 1, 2. First, we perform this test using our basic set of eight moments. Second, we perform the test using twelve moments, that is, our basic set of eight moments plus the four additional cross-correlations between output growth and interest rates. The bottom row of panels A and B of Table 4 report the resulting statistics and the associated *p*-values.

When we use moments induced by shocks to the stochastic trend to perform this test, we see from panel B that the model's predictions with respect to the correlation between interest rates and output are not rejected by the data at standard confidence levels. Moreover, this holds true for both the predictions regarding only the contemporaneous correlation as well as the predictions regarding the set of five cross-correlations. In contrast, when we use moments drawn directly from the data to estimate and test the model, we find that the model's predictions are rejected by the data at the 10% level but not at the 5% level, and again this holds for both the contemporaneous correlation and the set of five correlations. Overall, we interpret the evidence as suggesting that the addition of habit persistence and adjustment costs considerably improves the model's capacity to explain the co-movement between output and interest rates. However, the model only fits adequately when we compare its predictions with moments induced by shocks to the stochastic trend.

5. Conclusion

The first message of this paper is that standard real business cycle models induce demand responses to technology shocks that are too strong to be consistent with observations. In particular, this difficulty with RBC models becomes obvious when predicted correlations between output and interest rates are compared with their empirical counterparts. In effect, the prototypical RBC model driven by persistent technology shocks predicts a highly procyclical movement in interest rates, while the data does not support this claim. Moreover, this discrepancy between theory and observation becomes especially evident when we use longrun restrictions implied by stochastic growth to identify correlations induced by technology shocks.

The second message of the paper is that simple and plausible modifications of the model, such as the introduction of adjustment cost to capital accumulation and habit persistence in consumption, improve the model's capacity to explain the comovement between output and interest rates. However, we do not find that allowing for adjustment costs and habit persistence generally renders the model's predictions consistent with correlations drawn directly from raw data. It is only when we use moments calculated after identifying shocks to the stochastic trend that we find the model's predictions are not rejected. Therefore we believe that adjustment costs to capital and habit persistence contribute to our understanding of the mechanism by which technology shocks are propagated through the economy, but we nevertheless do not find that these modifications are sufficient to justify the view that permanent technology shocks are the sole force driving economic fluctuations.

Appendix

The Lagrangian for the problem where all quantities except labor are divided by the permanent component (X_t^p) is

$$L = \sum_{t=0}^{\infty} \beta^{t} [\log(c_{t} - vc_{t-1}/\gamma) + v(L_{t}) + \log X_{t}^{p}] + \sum_{t=0}^{\infty} \lambda_{t} \beta^{t} \left[A_{t} k_{t}^{(1-\alpha)} N_{t}^{\alpha} - c_{t} - \gamma k_{t+1} + (1-\delta)k_{t} - \frac{q\gamma^{2}(k_{t+1}-k_{t})^{2}}{2k_{t}} \right],$$

where $(X_t^s)^{\alpha} = A_t$ and $\gamma_t = \exp(\mu + \phi(1)\varepsilon_t)$.

The first-order conditions are

$$\frac{1}{c_t - vc_{t-1}/\gamma} - \frac{(\beta v)}{\gamma} \left(\frac{1}{c_{t+1} - vc_t/\gamma}\right) = \lambda_t, \tag{A.1}$$

$$\frac{\partial v(1-N_t)}{\partial N_t} = \lambda_t \alpha A_t k_t^{(1-\alpha)} N_t^{(\alpha-1)}, \tag{A.2}$$

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$$\lambda_{t+1}\beta \left[(1-\alpha)A_{t+1}k_{t+1}^{-\alpha}N_{t+1}^{\alpha} + (1-\delta) + \frac{q}{2}\gamma^{2}\left(\frac{k_{t+2}-k_{t+1}}{k_{t+1}}\right)^{2} + q\gamma^{2}\left(\frac{k_{t+2}-k_{t+1}}{k_{t+1}}\right) \right] = \lambda_{t}\gamma + \lambda_{t}\gamma^{2}q\left(\frac{k_{t+1}-k_{t}}{k_{t}}\right),$$
(A.3)

$$A_{t}k_{t}^{(1-\alpha)}N_{t}^{\alpha} - c_{t} - \gamma k_{t+1} + (1-\delta)k_{t} - \frac{q\gamma^{2}(k_{t+1}-k_{t})^{2}}{2k_{t}} = 0.$$
(A.4)

We approximate the first-order conditions near the stationary point where each variable is expressed in terms of the percentage deviation from its stationary value. Accordingly,

$$\frac{\beta \nu/\gamma}{(1-\nu/\gamma)}\hat{c}_{t+1} - \left(\frac{1+\beta(\nu^2/\gamma^2)}{1-\nu/\gamma}\right)\hat{c}_t + \frac{\nu/\gamma}{(1-\nu/\gamma)}\hat{c}_{t-1} = \hat{\lambda}_t \left(1-\frac{\beta\nu}{\gamma}\right), \quad (A.5)$$

$$\xi_{II} \frac{N}{1-N} \hat{N}_t = \hat{\lambda}_t + \hat{A}_t + (1-\alpha)\hat{k}_t - (1-\alpha)\hat{N}_t, \qquad (A.6)$$

$$\hat{\lambda}_{t+1} + \eta_A \hat{A}_{t+1} - \alpha \eta_A \hat{k}_{t+1} + \alpha \eta_A \hat{N}_{t+1} + \beta q \gamma \hat{k}_{t+2} - \beta q \gamma \hat{k}_{t+1} = \hat{\lambda}_t + q \gamma \hat{k}_{t+1} - q \gamma \hat{k}_t,$$
(A.7)

$$s_c \hat{c}_t + s_i \varphi \hat{k}_{t+1} - s_i (\varphi - 1) \hat{k}_t = \hat{A}_t + \alpha \hat{N}_t + (1 - \alpha) \hat{k}_t, \qquad (A.8)$$

where

$$\eta_A = [\gamma - \beta(1 - \delta)]/\delta, \qquad \varphi = \frac{\gamma}{\gamma - (1 - \delta)}, \qquad \xi_{II} = \frac{L\delta^2 v(L)/\delta L\delta L}{\delta v(L)/\delta L},$$

and s_c and s_i represent the share of consumption and investment in output at the steady state. The steady state of the economy depends on structural parameters by the following relations:

$$\frac{k}{y} = \frac{\beta(1-\alpha)}{\gamma - \beta(1-\delta)},\tag{A.9}$$

$$s_i = \frac{i}{y} = (\gamma - (1 - \delta))\frac{k}{y}, \tag{A.10}$$

$$N = \left(\frac{\partial v(1-N)}{\partial N}\right)^{-1} \frac{(\gamma - \beta v)}{(\gamma - v)} (1 - si), \tag{A.11}$$

$$1 + r = \frac{\gamma}{\beta}.\tag{A.12}$$

The steady state is thus affected by the introduction of habit formation but unaffected by adjustment costs.

Let h_t be the consumption habit at time t. For the specification of our model, the habit persistence at time t depends only on consumption at time t - 1, $h_t = c_{t-1}$. We then substitute \hat{c} by \hat{h} in Eqs. (A.6)–(A.7). With the expressions (A.6) and (A.8), we can obtain optimal decisions rules for the shadow price $\hat{\lambda}_t$ and the variable \hat{N}_t as a function of the state variables \hat{k} , \hat{h} at time t + 1 and t, and temporary component of the technological process A_t .

Using these decision rules and Eqs. (A.5) and (A.7), we can derive a secondorder linear system in capital stock (\hat{k}_{t+2}) and habit persistence (\hat{h}_{t+2}) as a function of the exogenous technical process. We can then rewrite this system as a first-order system in \hat{k}_{t+2} , \hat{k}_{t+1} , \hat{h}_{t+2} , and \hat{h}_{t+1} . Given the law of motion of the technological process, we can solve this system, subject to the transversality conditions, to produce a unique solution sequence for the capital stock and the habit persistence. With this solution and the relations for $\hat{\lambda}$ and \hat{N} , we obtain approximate solutions for output, investment, labor productivity, and real interest rates, where we need only to reintroduce the permanent component before computing population moments.

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