# Gold Rush Fever in Business Cycles

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#### Abstract

Gold rushes are periods of economic boom, generally associated with large increases in expenditures aimed at securing claims near new found veins of gold. An interesting aspect of gold rushes is that, from a social point of view, much of the increased activity is wasteful since it contributes mainly to the expansion of the stock of money. In this paper, we explore whether business cycle fluctuations may sometimes be driven by a phenomenon akin to a gold rush. In particular, we present a model where the opening of new market opportunities causes an economic expansion by favoring competition for market share, which is essentially a dissolution of rents. We call such an episode a market rush. We construct a simple model of a market rush that can be embedded into an otherwise standard Dynamic General Equilibrium model, and show how market rushes can help explain important features of the data. We use a simulatedmoment estimator to quantify the role of market rushes in fluctuations. We find that market rushes may account for half the short run volatility in hours worked and a third of the short run volatility of output.

Key Words: Business Cycle, Investment, Imperfect Competition

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### Introduction

There is a large literature aimed at decomposing business cycles into temporary and permanent components. A common finding in this literature is that there is a significant temporary component in business cycle fluctuations; that is, an important fraction of business cycles appears to be driven by impulses that have no long run impact. While technology shocks has arisen as a leading candidate explanation to the permanent component (whether these shocks be surprise increase in technological capacities, or news about future possibilities), there remains substantial debate regarding the driving forces behind the temporary component of macroeconomic fluctuations. Several potential explanation to the temporary component have been advanced and explored in the literature; the most notable being monetary shocks and government spending shocks. While such disturbances indeed create temporary business cycle movements, quantitative evaluation of their effects have generally found that they account for a very small fraction of macroeconomic fluctuations. Hence, the puzzle regarding the driving force behind temporary fluctuations persists. Since the most obvious –and most easily measured – candidates have not been convincingly shown to adequately explain temporary fluctuations, part of the literature has turned to exploring the potential role of shocks that are conceptually more difficult to measure. A prominent example of this alternative line of research is the literature related to sunspot shocks. While several papers have argued that sunspot shocks offer a good explanation to temporary business cycle fluctuations (see Benhabib and Farmer [1999] for a survey), much of the profession has remained skeptical.<sup>1</sup> In this paper we propose and evaluate a theory of temporary business cycle fluctuations which has some similarities with sunspot shocks, in that expectation changes are the initial driving force. However, our approach is fundamentally different since it does not rely on indeterminacy of equilibrium nor on increasing returns to scale. Instead our model builds on the intuition derived from gold rushes, where expectations play an important role but are nevertheless based on fundamentals.

To help motivate our approach, let us briefly discuss properties of a gold rush. For example, consider the case of Sutter's Mill near Coloma, California. On january 24, 1848, James W. Marshall, a carpenter from New Jersey, found a gold nugget in a sawmill ditch. This was the starting point of one of the most famous Gold Rushes in history, the California Gold Rush of 1848-1858. More than 90,000 people made their way to California in the two years following Marshall's discovery, and more than 300,000 by 1854 – or one of about every 90 people then living in the United States. The population of San Francisco exploded from a mere 1,000 in 1848 to 20,000 full-time residents by 1850. More than a century later, the San Francisco 49ers NFL team is still named for the

<sup>&</sup>lt;sup>1</sup> There are at least two reason for why the profession has remained skeptical about the importance of sunspot shocks in business cycles. First, the empirical evidence has not provided great support for the theoretical features of the economy needed to allow for sunspot shocks. Second, the coordination of beliefs implicit in the underlying mechanism is hard to understand.

prospectors of the California Gold Rush. Another famous episode, which inspired Charlie Chaplin's movie "The Gold Rush" and Jack London's book The "Call of the Wild", is the Klondike Gold Rush of 1896-1904. Gold prospecting took place along the Klondike River near Dawson City in the Yukon Territory, Canada. An estimated 100,000 people participated in the gold rush and about 30,000 made it to Dawson City in 1898. By 1910, when the first census was taken, the population had declined to 9,000.<sup>2</sup> As these examples make clear, gold rushes are periods of economic boom, generally associated with large increases in expenditures aimed at securing claims near new found veins of gold. An interesting aspect of many gold rushes is that, from a social point of view, part of the increased activity is wasteful since historically it mainly contributed to the expansion of the stock of money. We are obviously aware that gold rush episodes do not occur at business cycle frequency, but they will serve here as a useful metaphorical example.

In this paper, we explore whether business cycle fluctuations may sometimes be driven by a phenomenon akin to a gold rush. In particular, we present a dynamic general equilibrium model where the opening of new market opportunities causes an economic expansion by favoring competition for market share. We call such an episode a market rush. The market rush may mainly act to redistribute rents between firms with little external gain, in which case the net social value of such a market rush would be minor. By embedding a simple model of market rushes into an otherwise standard Dynamic General Equilibrium model, we cam evaluate whether such a phenomenon is a significant contributor to business cycle fluctuations. To capture the idea of a market rush, we build on an expanding varieties model where agents compete to secure monopoly positions in new markets, as often done in the growth literature (see for example Romer [1987] and Romer [1990]) and in some business cycle models (see for example Devereux, Head, and Lapham [1993]), although we treat the growth in the potential set of varieties as technologically driven and exogenous. In this setting, when agents perceive an increase in the set of technologically feasible products, they invest to set up a prototype firm (or product) with the hope of securing a monopoly position in the new market. It is therefore the perception of these new market opportunities that causes the unset of a market rush and the associated economic expansion. After the initial rush, there is a shake out period where one of the prototypes secures the dominant position in market. The long term effect of such a market rush depends on whether the expansion in variety has an external effect on productivity. In the case where it does not have an external effect, the induced cycle is socially wasteful as it only contributes to the redistribution of market rents. In contrast, when the expansion of variety does exert positive external effects, the induced cycle can have social value but will generally induce output fluctuations that are excessively large.<sup>3</sup>

<sup>&</sup>lt;sup>2</sup>See http://en.wikipedia.org/wiki/Gold\_rushes for further facts and references.

 $<sup>^{3}</sup>$  A potential example of such a process is the "dot com" frenzy of the late 90s, where large investments were made by firms trying to secure a position in the expanding internet market. At the end of this process, there was a large shake out as many firms went bankrupt and only a small percentage survived and obtained a substantial market position. The long run productivity gains and social value associated with this process are still debated.

The aim of the paper is to explore whether expectations about new profit opportunities, as captured by market rushes, could be a significant contributor to macroeconomic fluctuations.<sup>4</sup> Since expectations of new markets are not easily observable, we need to set a demanding standard to evaluate such a story. This is what we do. Not only do we provide a model that is capable of explaining important qualitative features of the business cycle dynamics of consumption, output and hours, but we also examine the quantitative importance of market rushes when we embed them in a model that includes prominent alternative driving forces discussed in the literature.

We begin the paper by presenting in Section 1 a very simple version of the model that can be solved analytically as to highlight the main elements and mechanisms of a market rush. In this model, we show that current economic activity depends positively on the expectation of next period's activity and on the perceived opening of new markets. Hence, when agents learn that the economy is starting a prolonged period of market expansion, this induces an immediate increase in investment and an associated economic expansion. Given the tractability of the model, we can solve it in the presence of a standard technology shock and our market expansion shock. To motivate our approach to the empirical evaluation of the model, we begin in Section 2 by highlighting the properties of this simple model in relation to the empirical properties of a Consumption–Output VECM as first studied by Cochrane [1994]. In particular, we show that our market rush model, augmented with a standard technology shocks, displays several of the qualitative properties of consumption-output VECM. While our model is not the only candidate explanation to the observed bivariate properties of consumption and output, we show that is offers a very simple and salient interpretation. We then turn to a more complete model to evaluate its quantitative properties. Our main finding is that market expansion shocks (market rushes) appear to be an important driving force underlying business cycle fluctuations. Section 4 examines the robustness of this result with respect to allowing for alternative sources of fluctuations, such as investment specific technology shocks, temporary technology shocks and preference shocks (in the technical appendix to this paper, we also discuss the potential role of monetary shocks). Overall, we find that market rushes can explain a sizable fraction of hours and output fluctuations even when allowing for these alternative explanations. Section 5 offers concluding comments.

### 1 An Analytical Model of Market Rushes

In this section, we present a simple analytical model of market rushes. The main element is that, in each period, agents receive information about potential new varieties of goods which could become

<sup>&</sup>lt;sup>4</sup>This paper is related to several papers, both old and recent, which emphasize the role of expectations in affecting business cycles. The newest embodiment of the literature (Beaudry and Portier [2004a], Beaudry and Portier [2004b], Jaimovich and Rebelo [2006]and Christiano, Motto, and Rostagno [2005], Beaudry and Portier [2006]) emphasizes the role of expectations regarding future productivity growth in creating fluctuations.

profitable to produce. In response to these expectations of profits, agents invest in putting on the market a prototype of the new good. Since many agents may invest in such startups, they engage in a winner takes all competition for securing the market of a newly created variety. The winning firm then becomes a monopolist on the market. This position may then be randomly lost at an exogenous rate. Expansion in variety may or may not have a long run impact on productivity, so that the market rush is not forced *a priori* to satisfy the gold rush analogy.

#### 1.1 Model

**Firms:** There exists a raw final good, denoted  $Q_t$ , produced by a representative firm using labor  $h_t$  and a set of intermediate goods  $X_t(j)$  with mass  $N_t$  according to a constant returns to scale technology represented by the production function

$$Q_t = (\Theta_t h_t)^{\alpha} N_t^{\xi} \left( \int_0^{N_t} X_t(j)^{\chi} \mathrm{d}j \right)^{\frac{1-\alpha}{\chi}}, \qquad (1)$$

where  $\alpha \in (0, 1)$ .  $\Theta_t$  is an index of disembodied exogenous technological progress.  $\chi \leq 1$  determines the elasticity of substitution between intermediate goods and  $\xi$  is a parameter that determines the long run effect of variety expansion. Since this final good will also serve to produce intermediate goods, we will refer to  $Q_t$  as the gross amount of final good. Also note that the raw final good will serve as the numéraire. The representative firm is price taker on the markets.

Each existing intermediate good is produced by a monopolist. Just like in many expanding variety models, the production of one unit of intermediate good requires the use of one unit of the raw final good as input. Since the final good serves as a numéraire, this leads to a situation where the price of each intermediate good is given by  $P_t(j) = 1/\chi$ . Therefore, the quantity of intermediate good j,  $X_t(j)$ , produced in equilibrium is given by

$$X_t(j) = (\chi(1-\alpha))^{\frac{1}{\alpha}} \Theta_t N_t^{\phi-1} h_t.$$

$$\tag{2}$$

where  $\phi = \frac{\xi - 1 + (1 - \alpha)/\chi}{\alpha}$ . The profits,  $\Pi_t(j)$ , generated by intermediate firm j are given by

$$\Pi_t(j) = \pi_0 \Theta_t N_t^{\phi-1} h_t, \tag{3}$$

where  $\pi_0 = \left(\frac{1-\chi}{\chi}\right) (\chi(1-\alpha))^{\frac{1}{\alpha}}$ . Equalization of the real wage with marginal product of labor implies

$$W_t = A\Theta_t N_t^{\phi},\tag{4}$$

where  $A = \alpha(\chi(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}$ .

Value added,  $Y_t$ , is then given by the quantity of raw final good,  $Q_t$ , net of that quantity used to produce the intermediate goods,  $X_t(j)$ . Once we substitute out for  $X_t(j)$ , and take away the amount of  $Q_t$  used in the production of  $X_t(t)$ , we obtain

$$Y_t = Q_t - \int_0^{N_t} X_t(j) dj$$
  
=  $A\Theta_t N_t^{\phi} h_t.$  (5)

The net amount of raw final good can serve for consumption,  $C_t$ , and startup expenditures,  $S_t$ , purposes.

$$Y_t = C_t + S_t. ag{6}$$

Variety Dynamics : We denote by  $\mathcal{N}_t$  the number of potential varieties in period t, while  $N_t$ is the number of active varieties -i.e. those which are effectively produced, with  $N_t \leq \mathcal{N}_t$ . In each period, new potential varieties are created at the stochastic growth rate  $\eta_t$ . The  $\mathcal{N}_t$  existing potential varieties of the period become obsolete at an exogenous rate  $\mu \in (0, 1)$ . Therefore, the dynamics for the number of potential products is given by

$$\mathcal{N}_{t+1} = (1 + \eta_t - \mu)\mathcal{N}_t. \tag{7}$$

Note that  $\eta$  brings information about future potentially profitable varieties but does not immediately affect the production function, acting like a news shock.

The law of motion of the number of effectively produced goods is driven by an endogenous adoption decision. Any entrepreneur who desires to produce a potential new variety has to pay a fixed cost of  $\kappa > 0$  units of the final good to setup the startup. She does so if the expected discounted sum of profits of a startup exceeds  $\kappa$ . Let  $N_{S,t}$  denote the number of startups and  $S_t = \kappa N_{S,t}$  denote total expenditures on setup costs. A time t + 1 a startup will become a functioning new firm with a product monopoly with an endogenous probability  $\rho_t$ , and existing monopolies disappear at rate  $\mu$ . Therefore, the dynamics for the number of effectively produced goods is given by

$$N_{t+1} = (1 - \mu)N_t + \rho_t N_{S,t}.$$
(8)

The  $N_{S,t}$  startups of period t compete to secure the  $\eta_t \mathcal{N}_t$  new monopoly positions. The successful startups are uniformly drawn from among the  $N_{S,t}$  existing ones. Therefore, the probability that a startup at time t will become a functioning firm at t + 1 is given by  $\rho_t = \min\left(1, \frac{\eta_t \mathcal{N}_t}{N_{S,t}}\right)$ , and the number of new goods created will be  $\min(N_{S,t}, \eta_t \mathcal{N}_t)$ . If it turns out that startups are not profitable enough, so that  $N_{S,t} < \eta_t \mathcal{N}_t$ , not all existing varieties will be exploited and we will have  $N_t < \mathcal{N}_t$ . In order to obtain a tractable solution, we choose parameters to rule out this case of partial adoption. Allocations will have the properties that it is always optimal for entrepreneurs to exploit the whole range of intermediate goods.<sup>5</sup> This implies that there will be no difference in the

<sup>&</sup>lt;sup>5</sup>Such an assumption would be definitively not appealing in a growth perspective, or to account for cross–country income differences (see for Comin and Hobijn [2004]), but seems to us acceptable in a business cycle perspective.

model between the potential and the actual number of varieties in equilibrium, so that  $N_t = \mathcal{N}_t$  $\forall t$ .

Households : The preferences of the representative household are given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left( \log(C_{t+\tau}) + \psi(\overline{h} - h_{t+\tau}) \right), \tag{9}$$

where  $0 < \beta < 1$  is a constant discount factor,  $C_t$  denotes consumption in period t and  $h_t$  is the quantity of labor the household supplies. The household chooses how much to consume, supply labor, hold equities in existing firms  $(\mathcal{E}_t)$  and in startups  $(\mathcal{E}_t^S)$  by maximizing (9) subject to the following sequence of budget constraints

$$C_t + P_t^{\mathcal{E}} \mathcal{E}_t + P_t^S \mathcal{E}_t^S = W_t h_t + \mathcal{E}_t \Pi_t + (1 - \mu) P_t^{\mathcal{E}} \mathcal{E}_{t-1} + \rho_{t-1} P_t^{\mathcal{E}} \mathcal{E}_{t-1}^S,$$
(10)

where  $P_t^{\mathcal{E}}$  is the beginning of period (prior to dividend payments  $\Pi_t$ ) price of an existing monopoly equity,  $P_t^S$  is the price of startups and  $W_t$  is the wage rate.

The first order conditions imply:

$$\psi C_t = W_t \tag{11}$$

$$\frac{1}{C_t}(P_t^{\mathcal{E}} - \Pi_t) = \beta \mathbb{E}_t \left[ \frac{1}{C_{t+1}} (1-\mu) P_{t+1}^{\mathcal{E}} \right]$$
(12)

$$\frac{P_t^S}{C_t} = \beta \mathbb{E}_t \left[ \frac{1}{C_{t+1}} \rho_t P_{t+1}^{\mathcal{E}} \right].$$
(13)

#### 1.2 Equilibrium Allocations

The three last first order conditions can be combined to give:

$$\frac{P_t^S}{\rho_t C_t} = \beta \mathbb{E}_t \left[ \frac{\Pi_{t+1}}{C_{t+1}} \right] + \beta \mathbb{E}_t \left[ \frac{(1-\mu)}{\rho_{t+1} C_{t+1}} \right],\tag{14}$$

which can be simply restated as

$$P_t^S = \beta \rho_t \mathbb{E}_t \sum_{\tau=0}^{\infty} \left[ \beta^{\tau} \frac{C_t}{C_{t+\tau+1}} (1-\mu)^{\tau} \Pi_{t+\tau+1} \right].$$
(15)

This condition states that the price of a startup is equal to the expected discounted sum of future profits. Free entry of startup drives the expected discounted sum of profits (the right hand side of equation (15)) net of the setup cost to zero. Therefore, one has in equilibrium

$$P_t^S = \kappa$$

Using this last equation, the labor demand condition (4), the profit equation (3) and the startup equity market equilibrium condition  $\mathcal{E}_t^S = N_t^S$ , the asset pricing equation (14) rewrites as

$$\left(\frac{S_t}{C_t}\right) = \beta \frac{\psi \pi_0}{A} \left(\frac{\kappa \eta_t}{1 - \mu + \kappa \eta_t}\right) \mathbb{E}_t h_{t+1} + \beta \left(\frac{1 - \mu}{1 - \mu + \kappa \eta_t}\right) \mathbb{E}_t \left[\left(\frac{\kappa \eta_t}{\kappa \eta_{t+1}}\right) \left(\frac{S_{t+1}}{C_{t+1}}\right)\right].$$
(16)

Using the labor demand condition (4) and the resource constraint (6), we get

$$\left(\frac{S_t}{C_t}\right) = \psi h_t - 1. \tag{17}$$

Equation (16) can therefore we written as:

$$(h_t - \psi^{-1}) = \beta \delta_t \frac{\pi_0}{A} \mathbb{E}_t h_{t+1} + \beta \delta_t \mathbb{E}_t \left[ \left( \frac{1}{\delta_{t+1}} - 1 \right) (h_{t+1} - \psi^{-1}) \right],$$
(18)

where  $\delta_t = \kappa \eta_t / (1 - \mu + \kappa \eta_t)$  is an increasing function of the fraction of newly opened markets  $\eta_t$ .

Equation (18) is a key equation of the model. It shows that current employment  $h_t$  depends on  $h_{t+1}$ ,  $\delta_t$  and  $\delta_{t+1}$ , and therefore indirectly depends on all the future expected  $\delta_s$ . As  $\delta_t$  brings news about the future, employment is purely forward looking. The reason why future employment favors current employment can be easily given an economic intuition: higher future employment reflects higher expected profits, which therefore stimulates new entries today. Note that the model exhibit certain salient neutrality properties,<sup>6</sup> as the determination of employment does not depend on either current or future changes in disembodied technological change  $\Theta_t$ .

By repeated substitution, the above equation can be written as a function of current and future values of  $\delta$  only. Given the nonlinearity of equation (18),<sup>7</sup> it is useful to compute a log–linear approximation around the deterministic steady–state value of employment h. The latter is given by:<sup>8</sup>

$$h = \frac{\psi^{-1}(1 - \beta(1 - \delta))}{(1 - \beta\delta\frac{\pi_0}{A} - \beta(1 - \delta))}$$

A condition for positive hours worked in steady state is then that  $\beta \delta(1-\chi)(1-\alpha)/\alpha + \beta(1-\delta) < 1.^9$ and the log–linear approximation takes the form

$$\widehat{h}_t = \gamma \mathbb{E}_t \widehat{h}_{t+1} + \left(\frac{h - \psi^{-1}}{h}\right) \mathbb{E}_t \left[\widehat{\delta}_t - \beta \widehat{\delta}_{t+1}\right]$$

<sup>&</sup>lt;sup>6</sup>This results is due to the functional forms we use for preferences and technology. It is related to (i) the separability between consumption and hours in the utility function; (ii) logarithmic preferences for consumption and (iii) Cobb–Douglas production function.

 $<sup>^{7}</sup>$ We show in the technical appendix to this paper that it is possible to obtain an exact analytical solution to the model in the case of i.i.d. shocks.

<sup>&</sup>lt;sup>8</sup>Note that we used the fact that  $\mathbb{E}_t(\eta_t) = \mu$ , which implies that  $\delta = \frac{\kappa\mu}{1+(\kappa-1)\mu}$  in steady state.

<sup>&</sup>lt;sup>9</sup>This condition obtains by noting that hours worked are positive as long as  $\beta \delta \pi_0 / A + \beta (1 - \delta) < 1$  and plugging the definition of  $\pi_0$  and A.

where  $\hat{h}_t$  and  $\hat{\delta}_t$  now represents relative deviations from the steady state and  $\gamma \equiv \beta \delta(\pi_0/A) + \beta(1-\delta)$ with  $\gamma \in (0,1)$ .<sup>10</sup> Solving forward, this can be written as

$$\widehat{h}_{t} = \left(\frac{h - \psi^{-1}}{h}\right) \left(\widehat{\delta}_{t} - \beta \delta \left(\frac{A - \pi_{0}}{A}\right) \mathbb{E}_{t} \left[\sum_{i=0}^{\infty} \gamma^{i} \widehat{\delta}_{t+1+i}\right]\right).$$
(19)

Note that, as  $\gamma \in (0, 1)$ , the model possesses a unique determinate equilibrium path. Equation (19) reveals that a positive  $\hat{\delta}_t$ , -i.e. an acceleration of variety expansion, causes an instantaneous increase in hours worked, output and investment in startups S. This boom arises as the result of the prospects of profits derived from securing those new monopoly positions. This occurs irrespective of any current change in the technology or in the number of varieties. Such an expansion is therefore akin to a "demand driven" or "investment driven" boom.

Once the equilibrium path of h is computed, output is directly obtained from equation (5). Finally, combining labor demand (4) and labor supply (11), we obtain an expression for aggregate consumption:

$$C_t = \frac{A}{\psi} \Theta_t N_t^{\phi}.$$
 (20)

### 2 Equilibrium Allocations Properties

#### 2.1 Comparison to the social optimum

Optimality properties of those allocations are worth discussing, and it is useful to compute the socially optimal allocations as a benchmark. The social planner problem is given by

$$\operatorname{Max} \mathbb{E}_{t} \sum_{i=0}^{\infty} \left[ \log C_{t+i} + \psi(\overline{h} - h_{t+i}) \right]$$

s.t. 
$$\begin{cases} C_t \leq A\Theta_t N_t^+ h_t - \kappa \eta_t N_{S,t} \\ \mathcal{N}_{t+1} = (1 + \eta_t - \mu)\mathcal{N}_t \\ N_{t+1} = (1 - \mu)N_t + \rho_t N_{S,t} \\ N_t \leq \mathcal{N}_t, \end{cases}$$

with  $\widehat{A} = \alpha (1-\alpha)^{\frac{\alpha}{(1-\alpha)}}$  and where we have already solved for the optimal use in intermediate goods. We assume again here that parameters are such that it is always socially optimal to invest in a new variety, so that  $N_t = \mathcal{N}_t$ . One necessary condition for full adoption to be socially optimal is that the long run effect of variety expansion is positive,  $-i.e \ \phi > 0 \iff \xi > -(1-\alpha)(1-\chi)/\chi$ . The first order condition of the social planner program is given by

$$\frac{\widehat{A}\Theta_t N_t^{\frac{\xi+(1-\alpha)(1/\chi-1)}{\alpha}}}{\widehat{A}\Theta_t N_t^{\frac{\xi+(1-\alpha)(1/\chi-1)}{\alpha}} h_t - \eta_t N_t \kappa} = \psi.$$
(21)

<sup>&</sup>lt;sup>10</sup>This follows from the restriction imposed on parameters to guarantee positive hours worked.

There are many sources of inefficiency in the decentralized allocations. One obvious source is the presence of imperfect competition: *ceteris paribus*, the social planner will produce more of each intermediate good. Another one is the congestion effect associated with investment in startups, because only a fraction  $\rho_t$  of startups are successful. The social planner internalizes this congestion effect, and does not duplicate the fixed cost of startups, as the number of startups created is equal to the number of available slots for optimal allocations.<sup>11</sup> Because of these imperfections, the decentralized allocation differs from the optimal allocation along a balanced growth path.

The difference between the market and the socially optimal allocations that we want to highlight regards the response to expected future market shocks. It is remarkable that the socially optimal allocation decision for employment (21) is static, and only depends on  $\eta_t$  (positively). This stands in sharp contrast with the market outcome, as summarized by equation (18), in which all future values of  $\eta$  appear. To understand this difference, let us consider an increase in period t in the expected level of  $\eta_{t+1}$ . In the decentralized economy, larger  $\eta_{t+1}$  means more startup investment in t+1 and more firms in t+2. Those firms will affect other firms profits in period t+2 and onwards. Therefore, a period t startup will face more competitors in t + 2, which reduces its current value, and therefore decreases startup investment and output.<sup>12</sup> Such an expectation is not relevant for the social planner, which does not respond to news about future values of  $\eta$ . Therefore, in that simple analytical model, part of economic fluctuations are driven by investors (rational) forecast about future profitability that are inefficient from a social point of view.<sup>13</sup> A stark result is obtained in the case when the returns to variety are null, so that an expansion in the number for varieties has no long run impact on productivity. This case corresponds to  $\phi = \frac{\xi + (1-\alpha)(1-\chi)/\chi}{\alpha} = 0$ . In this particular case, investment in startups occurs in the decentralized equilibrium in response to market shocks, whereas the social planner would choose not to adopt any new good  $(N_t = N_0 \quad \forall t)$ , as implementing new goods costs  $\kappa$  and has no productive effect. In this very case, optimal allocations are invariant to market shocks  $\eta$ , while equilibrium allocations react suboptimally to those shocks. In particular, as hours are only affected by market shocks in equilibrium, all equilibrium fluctuations in hours are suboptimal.

<sup>&</sup>lt;sup>11</sup>Note that we assume here that parameter values are such that it is optimal to adopt all the new varieties. Another potential source of sub–optimality would be an over or under adoption of new goods by the market. As shown in Benassy [1998] in a somewhat different setup with endogenous growth, the parameter  $\xi$  is then crucial in determining whether the decentralized allocations show too much or too little of new goods adoption.

 $<sup>^{12}</sup>$ This is due to the typical "business stealing" effect found in the endogenous growth literature, for example in Aghion and Howitt [1992], and originally discussed in Spence [1976a] and Spence [1976b].

<sup>&</sup>lt;sup>13</sup>The very result that it is socially optimal not to respond to such news is of course not general, and depends on the utility and production function specification. The general result is not that it is socially optimal not to respond to news about  $\eta$ , but that the decentralized allocations are inefficient in responding to news shocks.

#### 2.2 A VECM Representation of the Model Solution

As mentioned in the introduction, it is useful to represent macroeconomic fluctuations as responses to permanent and transitory shocks to a consumption and output autoregressive vector. Here we derive a consumption-output VECM representation of the model solution, that we will later compare to an estimated VECM. We assume that disembodied technical change,  $\Theta_t$ , follows (in log) a random walk without drift

$$\log \Theta_t = \log \Theta_{t-1} + \sigma_\Theta \varepsilon_t^\Theta,$$

where  $\varepsilon_t^{\Theta}$  are *i.i.d.* with zero mean and unit variance, and that the variety expansion shock  $\eta_t$  follow an AR(1) process of the form

$$\log(\eta_t) = \rho \log(\eta_{t-1}) + (1-\rho) \log(\mu) + \sigma_N \varepsilon_t^N,$$

where  $\varepsilon_t^N$  are *i.i.d.* with zero mean and unit variance. In this case, the solution for hours worked is given by

$$\widehat{h}_t = \omega \widehat{\eta}_t, \tag{22}$$

with  $\omega \equiv \frac{h-\psi^{-1}}{h} \frac{(1-\delta)(1-\beta\rho)}{1-\gamma\rho}$ . The logs of consumption and output are given by:

$$\log(Y_t) = k_y + \log(\Theta_t) + \phi \log(N_t) + \log(h_t)$$
(23)

$$\log(C_t) = k_c + \log(\Theta_t) + \phi \log(N_t), \qquad (24)$$

where  $k_c$  and  $k_y$  are constant terms. Using equation (18) to replace  $h_t$  with its approximate solution, it is straightforward to derive the  $MA(\infty)$  representation of the system, which is as follows:

$$\begin{pmatrix} \Delta \log(C_t) \\ \Delta \log(Y_t) \end{pmatrix} = \begin{pmatrix} \sigma_{\Theta} & \frac{\phi L}{1-\rho L} \sigma_N \\ \sigma_{\Theta} & \frac{(\omega(1-L)+\phi L)}{1-\rho L} \sigma_N \end{pmatrix} \begin{pmatrix} \varepsilon_t^{\Theta} \\ \varepsilon_t^N \\ \varepsilon_t^N \end{pmatrix} = C(L) \begin{pmatrix} \varepsilon_t^{\Theta} \\ \varepsilon_t^N \\ \varepsilon_t^N \end{pmatrix}.$$
(25)

We now estimate a similar system on postwar U.S. data, derive its Wold representation, discuss of its properties and compare them with those implied by Equation 25.

#### 2.3 A Target Set of Observations

The set of observations we present here provides a rich though concise description of fluctuations in output, consumption and hours worked. Some of these observations are well known, and some are not. The set of observations presented is meant to capture important features of fluctuations that any business cycle theory should aim to explain. We will use these observations to evaluate the potential role of market rushes in explaining macroeconomic fluctuations. In particular, we will emphasize both qualitative and quantitative properties of the data, which the idea of showing the the simple version of the model derived above offers an explanation to the qualitative features, while a richer models is shown to help explain the quantitative features.

Our goal is to emphasize key time series properties of output, consumption and hours worked. We could do this by directly examining the tri-variate process. However, as is well known, such an approach can depend heavily on the treatment of hours worked as a stationary or non-stationary process. We therefore choose an approach that is robust to the treatment of hours worked. To this end, we begin by reviewing properties of the bi-variate process for consumption and output. More precisely, we study a Consumption–Output system with one co-integrating relation. The main properties of this system were originally discussed in Cochrane [1994]. As in Cochrane [1994], we use two schemes to orthogonalize the innovations of the process: a long run orthogonalization scheme à la Blanchard and Quah [1989], and a short run or impact scheme à la Sims [1980]. At this point, these two schemes should be viewed as devices for helping present properties of the data. There is no claim that these schemes identify structural shocks, and there are no claims that these data should be explained by a model which only has two shocks. One of the properties we will emphasize is that these two schemes deliver almost identical impulse response functions (IRF). As we will show, this property can be traced back to a particular feature of the Wold representation for consumption and output, and we argue that this feature constitutes a qualitative property that business cycle models should try to replicate.

We begin by documenting the impulse responses associated with using a long run othogonalizing scheme  $\dot{a}$  la Blanchard and Quah [1989], and show that almost all of the short run volatility of consumption is associated with the permanent shock, while this shock only accounts for half of the volatility of output. We then use a short run orthogonalizing scheme  $\dot{a}$  la Sims [1980] to show that the resulting output innovation does not explain the long run properties of the two variables, nor the short run properties of consumption. We then formally test for the identity of the temporary shock recovered using the long run scheme and the output shock recovered using the short run scheme. This is done by showing that such an identity is in fact a zero restriction in the long run impact matrix of the Wold representation. We then turn our focus on relating these features with the behavior of hours and we show that the temporary/output innovation recovered from the consumption–output VECM explains most of the short run volatility of hours.

Our empirical analysis is based on quarterly data for the US economy. The sample spans the period 1947Q1 to 2004Q4. Consumption, C, is defined as real personal consumption expenditures on nondurable goods and services and output, Y, is real gross domestic product. Both series are first deflated by the 15–64 U.S. population and expressed in logarithms.<sup>14</sup> Standard Dickey–

<sup>&</sup>lt;sup>14</sup>Consumption is defined as the sum of services and nondurable goods, while output is real gross domestic product. Each variable is expressed in per capita terms by dividing by the 15 to 64 population. The series are obtained from the following links. Real Personal Consumption Expenditures: Nondurable Goods : http://research.stlouisfed.org/fred2/series/PCNDGC96, Real Personal Consumption Expenditures:

Fuller, likelihood ratio and cointegration tests indicate that C and Y are I(1) processes and do cointegrate. We therefore model their joint behavior with Vector Error Correcting Model (VECM), where the cointegrating relation coefficients are [1;-1] (meaning that the consumption to output ratio is stationary). Likelihood ratio tests suggest that the VECM should include 3 lags. Omitting constants, the joint behavior of (C, Y) admits the following Wold representation

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = A(L) \begin{pmatrix} \mu_{1,t} \\ \mu_{2,t} \end{pmatrix},$$
(26)

where L is the lag operator,  $A(L) = I + \sum_{i=1}^{\infty} A_i L^i$ , and where the covariance matrix of  $\mu$  is given by  $\Omega$ . As the system possesses one common stochastic trend, A(1) is not full rank. Given A(1), it is possible to derive a representation of the data in terms of permanent and transitory components of the form

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = \Gamma(L) \begin{pmatrix} \varepsilon_t^P \\ \varepsilon_t^T \end{pmatrix}, \qquad (27)$$

where the covariance matrix of  $(\varepsilon^P, \varepsilon^T)$  is the identity matrix and  $\Gamma(L) = \sum_{i=0}^{\infty} \Gamma_i L^i$ . The  $\Gamma$  matrices solve

$$\begin{cases} \Gamma_0 \Gamma'_0 = \Omega\\ \Gamma_i = A_i \Gamma_0 \quad \text{for} \quad i > 0 \end{cases}$$
(28)

Note that once  $\Gamma_0$  is known, all  $\Gamma_i$  are pinned down by the second set of relations. But, due to the symmetry of the covariance matrix  $\Omega$ , the first part of the system only pin downs three parameters of  $\Gamma_0$ . One remains to be set. This is achieved by imposing an additional restriction. We impose that the 1,2 element of the long run matrix  $\Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i$  equals zero, that is, we choose an orthogonalization where the disturbance  $\varepsilon^T$  has no long run impact on C and Y (the use of this type of orthogonalization was first proposed by Blanchard and Quah [1989]). Hence,  $\varepsilon^T$  is labeled as a temporary shock, while  $\varepsilon^P$  is a permanent one. Figure 1 graphs the impulse response functions of C and Y to both shocks as well as their associated 95% confidence bands, obtained by bootstrapping the VECM. Table 1 reports the corresponding variance decomposition of the process.

These results provide an interesting decomposition of macroeconomic fluctuations. The lower left panel of Figure 1 clearly shows that consumption virtually does not respond to the transitory shock, since it accounts for less than 4% of consumption volatility at any horizon. Conversely consumption is very responsive to the permanent shock and most of the adjustment dynamics take place in less than one year. In other words, consumption is almost a pure random walk, that responds only to permanent shocks and has very little dynamics. On the contrary, short run fluctuations in output are mainly associated with the temporary shocks, which explain more than 60% of output volatility on impact. These patterns are often interpreted as providing evidnce is favor of the permenat

Services : http://research.stlouisfed.org/fred2/series/PCESVC96, Real Gross Domestic Product, 3 Decimal: http://research.stlouisfed.org/fred2/series/GDPC96, Population: 15 to 64, annual: downloaded from http://www.economy.com/freelunch/default.asp.

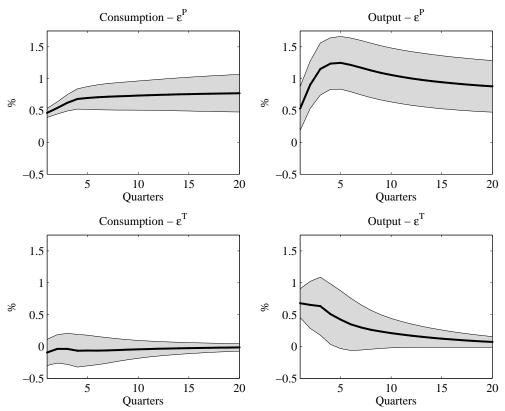


Figure 1: Responses of Output and Consumption to  $\varepsilon^P$  and  $\varepsilon^T$ 

This figure shows the responses of consumption and output to temporary  $\varepsilon^T$  and permanent  $\varepsilon^P$  one percent shocks. These impulse response functions are computed from a VECM (C,Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1-2004Q4. The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VECM.

Horizon		put	Consumption		
	$\varepsilon^T$	$\varepsilon^Y$	$\varepsilon^T$	$\varepsilon^Y$	
1	62%	80%	4%	0%	
4	28~%	46~%	1%	1%	
8	17~%	33%	1%	1 %	
20	10~%	22~%	0%	2%	
$\infty$	0 %	4 %	0%	4%	

Table 1: The Contribution of the Shocks to the Volatility of Output and Consumption

This table shows the k-period ahead share of the forecast error variance of consumption and output that is attributable to the temporary shock  $\varepsilon^T$  in the long run orthogonalization and to the output innovation  $\varepsilon^Y$  in the short run one, for k = 1, 4, 8, 20 quarters and for  $k \longrightarrow \infty$ . Those shares are computed from a VECM (C,Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1-2004Q4.

income hypothesis. However, it must be emphasized that these properties are aggregate properties and not partial equilibrium properties, which implies that a coherent explanation to these patterns requires a general equilibrium model that gives rise to permenent-temporary decomposition with no temporary component in consumption.

Now we consider an alternative orthogonalization that uses short run restrictions

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = \widetilde{\Gamma}(L) \begin{pmatrix} \varepsilon_t^C \\ \varepsilon_t^Y \\ \varepsilon_t^Y \end{pmatrix},$$
(29)

where  $\widetilde{\Gamma}(L) = \sum_{i=0}^{\infty} \widetilde{\Gamma}_i L^i$  and the covariance matrix of  $(\varepsilon^C, \varepsilon^Y)$  is the identity matrix. The  $\widetilde{\Gamma}$  matrices are solution to a system of equations similar to (28). We however depart from (28) as we impose that the 1, 2 element of  $\widetilde{\Gamma}_0$  be equal to zero. Therefore,  $\varepsilon^Y$  can be called an output innovation, and by construction the contemporaneous response of C to  $\varepsilon^Y$  is zero.

Figure 2 graphs the impulse responses of C and Y associated with the second orthogonalization scheme. The associated variance decompositions are displayed in Table 1. The striking result from these estimations is that the consumption shock  $\varepsilon^C$  is almost identical to the permanent shock to consumption ( $\varepsilon^P$  in the long run orthogonalization scheme), so that the responses and variance decompositions are very similar to those obtained using the long run orthogonalization scheme. This observation is further confirmed by Figure 3, which plots  $\varepsilon^P$  against  $\varepsilon^C$  and  $\varepsilon^T$  against  $\varepsilon^Y$ . It is striking to observe that both shocks align along the 45° line, indicating that the consumption innovation is essentially identical to the permanent component.

We now want to link the behavior of hours worked to the above description of output and consumption. In particular, we want to ask how much of the variance of hours worked is associated with the temporary shock (or quasi-equivalently the output shock) versus the permanent shock

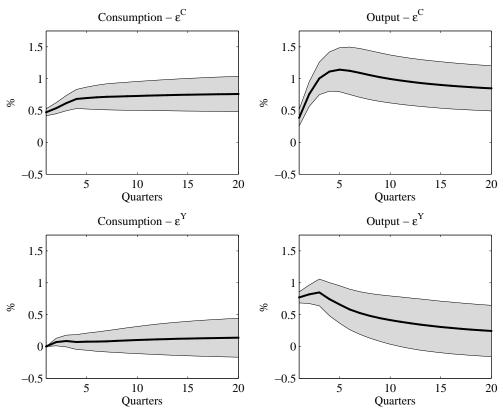
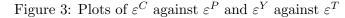
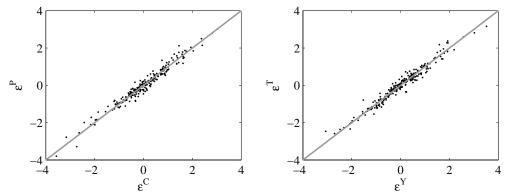


Figure 2: Responses of Output and Consumption to  $\varepsilon^C$  and  $\varepsilon^Y$ 

This figure shows the responses of consumption and output to consumption  $\varepsilon^C$ and output  $\varepsilon^Y$  one percent shocks obtained from a short run orthogonalization scheme. Those impulse response functions are computed from a VECM (C, Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1-2004Q4. The shaded area depicts the 95% confidence intervals obtained from 1000 bootstraps of the VECM.





The left panel plots the estimated permanent innovation  $\varepsilon^P$  (from the long run orthogonalization scheme) against the consumption innovation  $\varepsilon^C$  (from the short run orthogonalization scheme). The right panel plots the estimated temporary innovation  $\varepsilon^T$  (from the long run orthogonalization scheme) against the output innovation  $\varepsilon^Y$  (from the short run orthogonalization scheme). In both panels, the straight line is the 45° line. These shocks are computed from a VECM (C,Y) estimated with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1-2004Q4.

recovered from the consumption-output VECM. It is of interest to evaluate the contribution of these two shocks to the volatility of hours since it allows us to see whether hours can best be described as moving with the temporary component or the permanent component. To do so, we adopt the following approach. Once the innovations  $\varepsilon^P$  and  $\varepsilon^T$  are recovered from the bivariate C-Y VECM, we regress hours worked (in levels or differences) on current and lagged values of these two shocks plus a moving average error term denoted  $\varepsilon^H$ , which we call an hours specific shock.<sup>15</sup> An attractive feature of this approach is that it delivers results which are robust to the specification of hours worked (level or difference).<sup>16</sup> More precisely, we run the regression

$$x_t = c + \sum_{k=0}^{K} \left( \alpha_k \varepsilon_{t-k}^P + \beta_k \varepsilon_{t-k}^T + \gamma_k \varepsilon_{t-k}^H \right), \tag{30}$$

<sup>15</sup>Such a two step strategy amounts to the estimation of the following restricted trivariate moving-average process:

$$\begin{pmatrix} C_t \\ Y_t \\ H_t \end{pmatrix} = \begin{pmatrix} A(L) & 0_{2,1} \\ B(L) & C(L) \end{pmatrix} \begin{pmatrix} \varepsilon_t^P \\ \varepsilon_t^T \\ \varepsilon_t^H \\ \varepsilon_t^H \end{pmatrix},$$

where A(L) is a 2×2 polynomial matrix,  $0_{2,1}$  is a 2×1 vector of zeros, B(L) is a 1×2 polynomial matrix and C(L) is a polynomial in lag operator. A(L),  $\varepsilon^P$  and  $\varepsilon^T$  are recovered from the first step bivariate VECM, while B(L), D(L)and  $\varepsilon^H$  are estimated using a truncated approximation of the third line of the above MA process (which is equation (30)).

 $<sup>^{16}</sup>$  It is well known (see for instance the discussions in Gali [1999], Gali and Rabanal [2004], Chari, Kehoe, and McGrattan [2004], Christiano, Eichenbaum, and Vigfusson [2004]) that specification choice (levels versus first differences) matters a lot for VARs with hours worked. Results show that our procedure is robust to this specification choice.

where  $x_t$  denotes either the (log) hours per capita in levels or in differences. This model is estimated by maximum likelihood, choosing an arbitrarily large number of lags (K = 40). We then compute, for each horizon k the share of the overall volatility of hours worked accounted for by  $\varepsilon^P$ ,  $\varepsilon^T$  and by the hours specific shock  $\varepsilon^H$ . Results are reported in Table 2. The numbers reported in the table

	Level	Specific	cation	n Difference Specification				
Horizon	$\varepsilon^{\mathrm{P}}$	$\varepsilon^{\mathrm{T}}$	$\varepsilon^{\mathrm{H}}$	$\varepsilon^{\mathrm{P}}$	$\varepsilon^{\mathrm{T}}$	$\varepsilon^{\mathrm{H}}$		
1	$19 \ \%$	75 %	6~%	21~%	74~%	5 %		
4	37~%	56~%	7~%	46~%	52~%	2~%		
8	61~%	32~%	7~%	66~%	32~%	2~%		
20	60~%	21~%	19~%	69~%	28~%	3~%		
40	54~%	20~%	26~%	57~%	38~%	5 %		

Table 2: Variance Decomposition of Hours Worked Levels

This table shows the k-period ahead share of the forecast error variance of hours worked to the temporary  $\varepsilon^T$ , the permanent  $\varepsilon^P$  and the hours specific shock  $\varepsilon^H$ . Those shares are computed using a two-step procedure. First  $\varepsilon^T$  and  $\varepsilon^P$  are derived from the estimation of a VECM (C,Y) with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1-2004Q4. Then hours worked (in levels or difference depending on the specification) are projected on current and past values of those innovations plus a moving average term in  $\varepsilon^H$ .

clearly indicate that hours worked are primarily explained by the transitory component at business cycle frequencies.

To summarize, there are four properties of the data that we want to highlight: (i) the permanent shock ( $\varepsilon^P$ ) recovered using a long run restriction in a consumption-output VECM is essentially the same shock as that corresponding to a consumption shock ( $\varepsilon^C$ ) recovered using an impact restriction, (ii) the response of consumption to a temporary shock is extremely close to zero at all horizons, and there are almost no dynamics in the response of consumption to a permanent shock, as it jumps almost instantaneously to its long run level, (iii) the temporary shock (or the output shock in the short run orthogonalization) is responsible for a significant share of output volatility at business cycle frequencies and (iv) hours are largely explained by the transitory shock at business cycle frequencies. These facts emphasize that a substantial fraction of the business cycle action seems to be related to changes in investment and hours worked, without any short or long run implications for consumption. We have investigated the robustness of these findings both against changes in the specification of the VECM — by estimating rather than imposing the cointegration relation, adding additional lags or estimating the VECM in levels — and against the data used to estimate the VECM — we considered total consumption rather than the consumption of nondurables and services, output as measured by consumption plus investment only — and in all these cases we found no major changes in patterns.<sup>17</sup> Since we have emphasized the quasi equivalence between the shocks recovered using a long run restriction, and shocks recovered using an impact restrictions, we also formally test for the equality between  $\varepsilon^Y$  and  $\varepsilon^T$ . We show in the appendix that such a test amount to testing the nullity of the (1,2) element of the long run matrix of the Wold decomposition, denoted  $a_{12}$ . The confidence intervals for the estimate of  $a_{12}$  are obtained from 1000 bootstraps of the long run matrix. The coefficient  $\hat{a}_{12}$  takes an average value of 0.2024 with a 95% confidence interval [-0.2, 0.8]. At a 5% significance level, the hypothesis that the consumption shock is identical to the permanent shock cannot therefore be rejected.

#### 2.4 Orthogonalized Representations of Equilibrium Allocations

Should our simple model be the data generating process, what would give the estimation and orthogonalizations we have just performed? A way to answer would be to simulate data using the model, and then to estimate and orthogonalize a VECM on those simulated data. This is what we will do in a fully fledged version of the model. As our simple model has a tractable analytical solution, it is possible to exactly derive the VECM representation of equilibrium allocations. From the system of equations (25), we obtain he impact and long run effect of the shocks, which are respectively given by C(0) and C(1):

$$C(0) = \begin{pmatrix} \sigma_{\Theta} & 0\\ \sigma_{\Theta} & \omega \sigma_N \end{pmatrix} \text{ and } C(1) = \begin{pmatrix} \sigma_{\Theta} & \frac{\phi}{1-\rho}\sigma_N\\ \sigma_{\Theta} & \frac{\phi}{1-\rho}\sigma_N \end{pmatrix}.$$

The VECM permanent and transitory shocks are then given by

$$\begin{cases} \varepsilon_t^P = \left(\sigma_\Theta^2 + \left(\frac{\phi}{1-\rho}\right)^2 \sigma_N^2\right)^{-1/2} \left(\sigma_\Theta \varepsilon_t^\Theta + \frac{\phi}{1-\rho} \sigma_N \varepsilon_t^N\right) \\ \varepsilon_t^T = \left(\sigma_\Theta^2 + \left(\frac{\phi}{1-\rho}\right)^2 \sigma_N^2\right)^{-1/2} \left(-\frac{\phi}{1-\rho} \sigma_N \varepsilon_t^\Theta + \sigma_\Theta \varepsilon_t^N\right). \end{cases}$$
(31)

Similarly, short run orthogonalization yields

$$\begin{cases} \varepsilon_t^Y &= \varepsilon_t^{\Theta} \\ \varepsilon_t^C &= \varepsilon_t^N. \end{cases}$$
(32)

This simple model shares a lot of dynamic properties with the data when the parameter  $\phi$  is set to zero. This corresponds to the case where  $\xi = -(1 - \alpha)(1 - \chi)/\chi$ , meaning that an expansion in variety exerts no effect on labor productivity.

First of all the system (25) clearly shows that consumption and output do cointegrate (C(1) is not full rank) with cointegrating vector [1;-1]. Second, it shows that consumption is actually a random walk, that is only affected —in the short run as well as in the long run— by technology shocks,  $\varepsilon^{\Theta}$ .

<sup>&</sup>lt;sup>17</sup>All these results are reported in the technical appendix to this paper, available from http://fabcol.free.fr/index.php?page=research.

Output is also affected in the short run by the temporary shock,  $\varepsilon^N$ . Hence, computing, sequentially, our short-run and long-run orthogonalization with this model would imply  $\varepsilon^P = \varepsilon^C = \varepsilon^{\Theta}$  and  $\varepsilon^T = \varepsilon^Y = \varepsilon^N$ , as it can been seen from (31 and (32) in the case  $\phi = 0$ . Finally, it is the temporary shock  $\varepsilon^T$  (which is indeed  $\varepsilon^N$ ) that explains all the variance in hours worked at any horizon, as it can be seen in equation (22). Such a model therefore allows for a structural interpretation of the results we obtained in subsection 2.3. Permanent shocks to C and Y are indeed technology shocks. Consumption does not respond to variety expansion shocks, which however account for a lot of output fluctuations and all the fluctuations in hours worked. Variety expansion shocks create market rushes that are indeed gold rushes, generating inefficient business cycles as the social planner would choose not to respond to them. In effect, these shocks only trigger rent seeking activities, as startups are means of appropriating a part of the economy pure profits.

Although simple, this model illustrates how the market mechanism we have put forward has the potential to account for some extreme properties of the data; in particular, the equivalence of the short and long run identification schemes, and the complete absence of a temporary component in consumption. In the next section, we consider an extended version of the model in which we introduce capital accumulation, possible long run effect of intermediate goods expansions and other real frictions. We use the estimated responses from the long run orthogonalization scheme to estimate the size of the technological and variety expansion shocks. Once those parameters are estimated, we are able to assess the ability of the model to account quantitatively for the facts we documented in subsection 2.3, and therefore decompose economic fluctuations in a meaningful way, using our model as a measurement tool.

### **3** Quantitative Assessment

In this section, we first present the extended model before describing the calibration and estimation procedure. Then we comment on the estimated parameters and derive some implications of the estimated model.

#### 3.1 Model, Calibration and Estimation Procedure

Our emphasis in this work is on the existence of a new type of shock, namely a market rush. In order to gauge the quantitative importance of this shock in the business cycle, we enrich the propagation mechanisms of our baseline model, and estimate it with U.S. data.

**The Extended Model:** We extend the model by including capital accumulation, two types of intermediate goods and habit persistence in consumption. The final good is now produced with

capital,  $K_t$ , labor,  $h_t$  and two types of intermediate goods  $X_t(i)$  and  $Z_t(j)$ , according to

$$Q_t = K_t^{1-\alpha_x-\alpha_z-\alpha_h} (\Theta_t h_t)^{\alpha_h} N_{x,t}^{\overline{\xi}} \left( \int_0^{N_{x,t}} X_t(i)^{\chi} \mathrm{d}i \right)^{\frac{\alpha_x}{\chi}} N_{z,t}^{\widetilde{\xi}} \left( \int_0^{N_{z,t}} Z_t(j)^{\chi} \mathrm{d}j \right)^{\frac{\alpha_z}{\chi}}, \tag{33}$$

with  $\alpha_x, \alpha_z, \alpha_h \in (0, 1), \alpha_x + \alpha_z + \alpha_h < 1$  and  $\chi \leq 1$ . We impose that  $\overline{\xi} = -\alpha_x(1 - \chi)/\chi$  so that variety expansion in intermediate goods X has no long-run impact, and that  $\widetilde{\xi} = (\chi(1-\alpha_x)-\alpha_z)/\chi$ , so that the equilibrium aggregate value added production function is linear in the number of intermediate goods of type Z  $(N_{z,t})$ .  $\Theta_t$  denotes Harrod neutral technical progress, the log of which is assumed to follow a random walk with drift  $\gamma \geq 1$ .

As in the analytical model, the numbers of available varieties evolve exogenously according to

$$\mathcal{N}_{x,t+1} = (1 - \mu + \eta_t^x) \mathcal{N}_{x,t}$$
$$\mathcal{N}_{z,t+1} = (1 - \mu + \eta_t^z) \mathcal{N}_{z,t}.$$

We assume that the stochastic processes for productivity and the market shocks are given by

$$\log(\eta_t^x) = \rho_x \log(\eta_{t-1}^x) + (1 - \rho_x) \log(\mu) + \sigma_x \varepsilon_t^x$$
  

$$\log(\eta_t^z) = \rho_z \log(\eta_{t-1}^z) + (1 - \rho_z) \log(\mu) + \sigma_z \varepsilon_t^z$$
  

$$\log(\Theta_t) = \log(\gamma) + \log(\Theta_{t-1}) + \sigma_\Theta \varepsilon_t^\Theta.$$

We assume that  $\varepsilon^x$ ,  $\varepsilon^z$  and  $\varepsilon^{\Theta}$  are Gaussian white noises with zero means and unit variances.

The setup cost is assumed to grow with technological progress, so that a balanced growth path exists along which the share of setup costs in total resources stays constant:

$$\kappa_t = \Theta_t \kappa$$

As the number of varieties is stationary, a small enough setup cost  $\kappa$  guarantees that full adoption is an equilibrium outcome.

Capital accumulation is governed by the law of motion

$$K_{t+1} = (1-\delta) K_t + \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t,$$

where  $\delta \in (0, 1)$  is the constant depreciation rate. The function  $\mathcal{S}(\cdot)$  accounts for the presence of adjustments costs in capital accumulation. We assume that  $\mathcal{S}(\cdot)$  satisfies  $\mathcal{S}(\gamma) = \mathcal{S}'(\gamma) = 0$  and  $\varphi = \mathcal{S}''(\gamma)\gamma^2 > 0$ . It follows that the steady state of the model does not depend on the parameter  $\varphi$ while its dynamic properties do. Notice that following Christiano, Eichenbaum, and Evans [2005], Christiano and Fisher [2003] and Eichenbaum and Fisher [2005], we adopt the dynamic investment adjustment cost specification. In this environment, it is the growth rate of investment which is penalized when varied in the neighborhood of its steady state value. In contrast, the standard specification penalizes the investment-to-capital ratio. The dynamic specification for adjustment costs is a significant source of internal propagation mechanisms as it generates a hump-shaped response of investment to various shocks.

Finally, we introduce habit persistence in consumption, and the intratemporal utility function is given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left[ \log(C_{t+\tau} - bC_{t+\tau-1}) + \psi(\overline{h} - h_{t+\tau}) \right].$$

Note that introducing adjustment costs to investment and habit persistence, while not affecting the main qualitative properties of the model we have presented in Section 1, will improve the ability of the model to capture the shape of the impulse response function. The model then solves as in the preceding section, except that no analytical solution can be found.

**Calibration:** Our quantitative strategy is to calibrate those parameters for which we have estimates or that we can obtain by matching balanced growth path ratios with observed averages. The time period is a quarter. The discount factor is set such that the household discounts the future at a 3% annual rate. We assume constant markups of 20%, so that  $\chi = 0.833$ . The depreciation rate is equal to 2.5% per quarter, as is common the literature. We assume that the two sets of intermediate goods differ only with regards to the long run impact of a variety expansion, and therefore assume  $\alpha_x = \alpha_z$ .<sup>18</sup> The parameters  $\alpha_h$  and  $\alpha_x$  are set such that the model generates a labor share and a share of intermediate goods in value added of, respectively, 60% (Cooley and Prescott [1995]) and 50% (Jorgenson, Gollop, and Fraumeni [1987]).  $\mu$  and  $\kappa$  are set such that the model generates 15% of total investment. The calibrated parameters are summarized in Table 3.

Preferences							
Discount factor	$\beta$	0.9926					
Technology							
Elasticity of output to intermediate goods	$\alpha_x + \alpha_z$	0.3529					
Elasticity of output to hours worked	$lpha_h$	0.4235					
Depreciation rate	$\delta$	0.0250					
Elasticity of substitution by intermediates	$\chi$	0.8333					
Rate of technology growth	$\gamma$	1.0060					
Monopoly death rate	$\mu$	0.0086					
Setup cost	$\kappa$	1					

 Table 3: Calibrated Parameters

<sup>&</sup>lt;sup>18</sup>This assumption amounts to the choice of the relative variance  $\sigma_x^2/\sigma_z^2$  and can be made without loss of generality.

**Estimation Procedure:** We estimate the following seven parameters: the standard deviation of the technological shock innovation  $\sigma_{\Theta}$ , the persistence parameters  $\rho_x$  and  $\rho_z$ , and the standard deviations  $\sigma_x$  and  $\sigma_z$  of the two market shocks innovations, the habit persistence parameter b and the adjustment cost parameter  $\varphi$ . These parameters are chosen in order to match the output impulse responses of the long run VECM that we have presented in section 2.3, and that are displayed in Figure 1. As the long run orthogonalization scheme cannot recover the model structural shocks (three shocks in the model and only two innovations in the VECM), we cannot directly match the model's theoretical responses with the empirical responses to  $\varepsilon^P$  and  $\varepsilon^T$ . Therefore, we follow a simulated method of moments approach. Let  $\Psi = (\sigma_{\Theta}, \rho_x, \sigma_x, \rho_z, \sigma_z, b, \varphi)$  be the parameters to be estimated, and let M be the column vector of estimated moments to match. We denote by  $\mathcal{M}(\Psi)$  a column vector of the same moments obtained from simulating the model with parameters  $\Psi$ . The set of estimated parameters  $\hat{\Psi}$  is then chosen so as to minimize the distance  $\mathcal{D}$ 

$$\mathcal{D} = \left(\mathcal{M}(\Psi) - M\right)' W \left(\mathcal{M}(\Psi) - M\right),$$

where W is a weighting matrix that is given by the inverse of the covariance matrix of the estimators of M. The simulated moments  $\mathcal{M}(\Psi)$  are obtained by simulating the model over 20 times 232 periods,<sup>19</sup> which is the length of our data sample.

The last issue concerns the choice of moments to match. We aim at matching the impulse responses of output obtained from model generated data to both the permanent and the transitory responses presented in Section 2.3 based on real data. We use the first twenty quarters of the impulse responses in these exercises. We leave as tests of the model its capacity to reproduce the responses of output to the short run orthogonalization scheme and consumption in both scheme. Using the long run scheme, the response of output to a permanent shock displays a hump,<sup>20</sup> and for that reason, we have supplemented the model with habit persistence and adjustment costs in investment. There are therefore forty moments to match. We have two ways to test our model. The first is by making use of the over-identifying restrictions associated with the estimation procedure (seven parameters for forty moments), by means of a J-test, following Hansen [1982]. The second one is to check whether or not the estimated model possesses the properties of the data that we have highlighted in Section 2.3, namely the identity between the short and long run orthogonalization schemes and the importance of the temporary shock in explaining the short run patterns in output and hours worked, but not of consumption. To do so, we will compute two distance statistics,  $\mathcal{D}(C)$  and  $\mathcal{D}(C,Y)$ .  $\mathcal{D}(C)$  is the a measure of the distance between the first twenty coefficients of

<sup>&</sup>lt;sup>19</sup>Michaelides and Ng [1997] have shown that efficiency gains are negligible for a number of simulations larger than 10.

<sup>&</sup>lt;sup>20</sup>Cogley and Nason [1995] have also proposed the estimation of a (C, Y) VECM, and show that the response of output to the temporary shock is hump-shaped. We find in this study that it is the response to the permanent shock that is hump-shaped. This difference comes from the sample period and the choice of the output variable, that is Net Domestic Product in Cooley and Nason and Gross Domestic Product(GDP) in our work. We prefer the use of GDP as it is the most commonly used measure of output in the literature.

consumption IRF as estimated in the data and as estimated from the simulated data. The statistic is computed as:

$$\mathcal{D}(C) = (\mathcal{M}(C,\widehat{\Psi}) - M(C))' W_c(\mathcal{M}(C,\widehat{\Psi}) - M(C)),$$

where M(C) is a vector collecting the impulse responses of consumption to both the permanent and the transitory shocks obtained from the VECM, while  $\mathcal{M}(C, \widehat{\Psi})$  collects the same impulse responses obtained by simulating the theoretical model for the estimated values of the parameters.  $W_c$  is the inverse of the covariance matrix of these moments obtained from the VECM. Similarly,  $\mathcal{D}(C, Y)$ represents the distance between the model and the data for both consumption and output IRFs.

#### 3.2 Estimation Results

Table (4) reports the estimated values of the seven parameters of interest, together with the values of the J-statistics for over-identification. First of all note that the model is not rejected by the data as the associated J-state is low. A key result is that  $\sigma_z$  is not significantly different from zero. Those market shocks that contribute to business cycle volatility are therefore the ones with no long run effects, those that have been shown to create inefficient fluctuations. The model therefore essentially reduces to a model with only one set of intermediate goods and with  $\phi = 0$ . This result does not mean that variety expansion does not contribute to growth, but that it does so in a way which, for our purpose, is not distinguishable from Harrod neutral technical progress.

The impulse responses of the VECM estimated on the artificial data generated from the model are presented on Figure 4, together with the ones estimated with the data. The confidence bands are the one computed from the data. Note that the IRFs obtained from artificial data as generated by the model lie within the confidence bands, which confirms that the model does a good job not only on impact and in the long run (as shown by the low level of the J-stat), but also for much of the dynamics. This is confirmed by the  $\mathcal{D}(C)$  statistic. This statistic is distributed according to a chi-square with 33 degrees of freedom under the null hypothesis of equality between the IRFs obtained from the VECM on both historical and simulated data for consumption. The test statistic value is 42.51 with an associated p-value of 12%. The implications of the model for consumption dynamics are not rejected by the data. Similarly, we perform the same test for the joint behavior of consumption and output. Again, the model is not strongly at odds with the data ( $\mathcal{D}(C, Y)=92.78$ with p-value 6%).

The model already displays two of the three properties of the data that we put forward previously: (i) there is virtually no dynamics in the response of consumption to the permanent shock, as it affects permanently and almost instantaneously the level of consumption and (ii) the temporary shock is responsible for a significant share of output volatility at business cycle frequencies.

It is now of interest to test whether the model also possesses the first property: (i) the permanent

Persistence of the X market shocks	$ ho_x$	0.9166
		(0.0336)
Standard dev. of X market shocks	$\sigma_x$	0.2865
		(0.0317)
Persistence of the Z market shocks	$ ho_x$	0.9164
		(0.6459)
Standard dev. of Z market shocks	$\sigma_z$	0.0245
		(0.1534)
Standard dev. of the technology shocks	$\sigma_{\Theta}$	0.0131
		(0.0015)
Habit persistence parameter	b	0.5900
		(0.1208)
Adjustment cost parameter	$\varphi$	0.4376
		(0.3267)
J-Stat		17.40
		[0.99]
$\mathcal{D}(C)$		42.51
		[0.12]
$\mathcal{D}(C,Y)$		92.78
		[0.06]

 Table 4: Estimated Parameters

This table first presents the estimated parameters, as obtained from a Simulated Method od Moments estimation of the model. Standard errors are in parenthesis. The last three lines display J-statistics and Distance statistics. These statistics are distributed chi-square, and the p-value for testing their nullity is given in brackets.

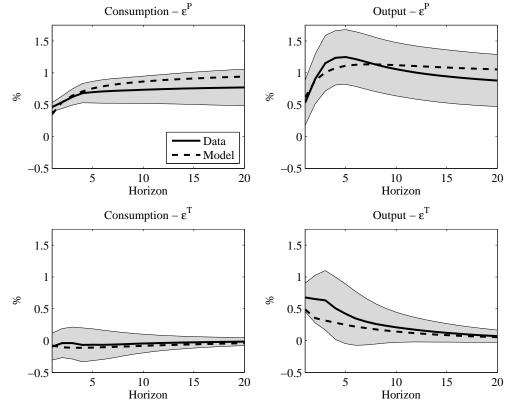


Figure 4: Impulse Response Functions: Data versus Model (Long Run Orthogonalization Scheme)

This figure compares the responses of consumption and output to permanent and transitory shocks (long run orthogonalization scheme), as estimated from the data (continuous line) and from model simulated data (dashed line). More precisely, the dashed line is the average over 1000 replications of the model simulation, VECM estimation and orthogonalization. The shaded area represents the 95% confidence intervals obtained from 1000 bootstraps of the VECM estimated with actual data.

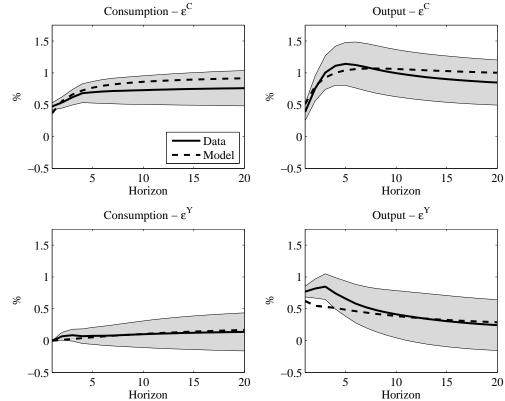


Figure 5: Impulse Response Functions: Data versus Model (Short Run Orthogonalization Scheme)

This figure compares the responses of consumption and output to consumption and output shocks (short run orthogonalization scheme), as estimated from the data (continuous line) and from model simulated data (dashed line). More precisely, the dashed line is the average over 1000 replications of the model simulation, VECM estimation and orthogonalization. The shaded area represents the 95% confidence intervals obtained from 1000 bootstraps of the VECM estimated with actual data.

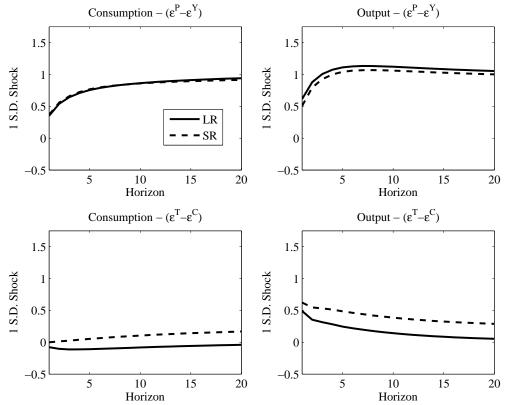


Figure 6: Theoretical Impulse Response Functions (Long Run versus Short Run)

This figure compares the responses of consumption and output to a permanent (continuous line, labeled LR for Long Run) or consumption shock (dashed line, labeled SR for Short Run) and to a transitory (continuous line) or output shock (dashed line), as estimated from model simulated data. More precisely, each line is the average over 1000 replications of the model simulation, VECM estimation and short or long run orthogonalization.

shock to consumption  $\varepsilon^P$  is approximately identical to the  $\varepsilon^C$  shock recovered from a consumption– output VECM. We therefore perform our test for the equality between  $\varepsilon^Y$  and  $\varepsilon^T$  in the data generated by the model. We then generate 1000 replications of the model simulations. In 87% of the cases we have the property that the (1, 2) element of the long run effect matrix of the Wold decomposition of a (C, Y) VECM is not significantly different from zero. Figure 5 reports the estimated impulse response functions as obtained from the VECM in the data and using the simulated data of the model assuming a short–run orthogonalization scheme. Again, the fit is extremely good. In Figure 6 we report the IRFs of output and consumption as obtained from the short–run and long–run orthogonalization scheme of the VECM estimated with simulated data. The figure clearly shows that the profiles of the IRFs are very similar. The model is therefore able to reproduce those three salient features of the data that we have put forward in section 2.3.

Table 5 reports the variance decomposition of hours worked both in the data and in the model, as computed from the same regression as equation (30). As can be seen from the table, hours volatility is primarily accounted for by the transitory shock over the short run horizon. In fact the model predicts that 65% of the overall volatility of hours can be accounted for by transitory shocks at the one period horizon, to be compared to the 75% found in the data. Again the model seems to perform remarkably well along this dimension.

 Table 5: Variance Decomposition of Hours Worked

		Data			Model	
Horizon	$\varepsilon^{\mathrm{P}}$	$\varepsilon^{\mathrm{T}}$	$\varepsilon^{\mathrm{H}}$	$\varepsilon^{\mathrm{P}}$	$\varepsilon^{\mathrm{T}}$	$\varepsilon^{\mathrm{H}}$
1	$19 \ \%$	75~%	6~%	35~%	65~%	0%
4	37~%	56~%	7~%	19~%	81~%	0%
8	61~%	32~%	7~%	24~%	76~%	0 %
20	60~%	21~%	19~%	28~%	72~%	0 %
40	54~%	20~%	26~%	28~%	72~%	0~%

This table shows the k-period ahead share of the forecast error variance of hours worked to the temporary  $\varepsilon^T$ , the permanent  $\varepsilon^P$  and the hours specific shock  $\varepsilon^H$ , as estimated from the data and from the simulated data. Those shares are computed using a two-step procedure. First  $\varepsilon^T$  and  $\varepsilon^P$  are derived from the estimation of a VECM (C,Y) with one cointegrating relation [1;-1], 3 lags, using quarterly per capita U.S. data over the period 1947Q1-2004Q4 for the actual data and 232 periods for simulated data. Then hours worked in levels are projected on current and past values of those innovations plus a moving average term in  $\varepsilon^H$ . In the case of the model, those numbers are averages over the 20 replications used during the estimation process.

#### 3.3 Business Cycle Accounting

Once estimated, the model can be used to evaluate the importance of the market rush phenomena in the U.S. business cycle. In effect, the model allows for a meaningful (structural) variance decomposition of fluctuations as reported in Table 6.

As expected from the estimation results, the market shock that exerts a permanent effect on output does not contribute to the dynamics of the model. Indeed, the market shock that has no impact on productivity,  $\varepsilon^x$ , accounts for more than one third of output volatility<sup>21</sup> and about 85% of hours worked on impact. On the contrary, and as expected, consumption is almost solely explained by the permanent technology shock.

Horizon	Output			Con	Consumption			Hours		
	$\varepsilon^{\Theta}$	$\varepsilon^x$	$\varepsilon^{z}$	$\varepsilon^{\Theta}$	$\varepsilon^x$	$\varepsilon^{z}$	$\varepsilon^{\Theta}$	$\varepsilon^x$	$\varepsilon^{z}$	
1	64 %	36~%	0 %	94~%	6%	0 %	$15 \ \%$	85~%	0%	
4	86~%	14~%	$0 \ \%$	95~%	5~%	0 %	19~%	81~%	0%	
8	92~%	8~%	$0 \ \%$	96~%	4%	0 %	32~%	68~%	0%	
20	96~%	3~%	1~%	98~%	1~%	1~%	40~%	59~%	1%	
$\infty$	96~%	0 %	4%	96~%	0~%	4%	41~%	57~%	2%	

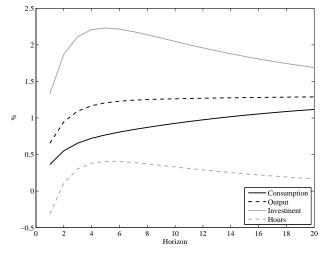
Table 6: Contribution of Shocks to the Business Cycle in the Estimated Model

This table reports the forecast error variance decomposition of consumption, output and hours worked when the estimated model is used as the forecasting model.

Figures 7, 8 and 9 show the theoretical responses of the main variables of the model to the structural shocks. Note that responses are qualitatively different from one shock to another, which allows for a proper identification of the contribution of each of them. The responses to the permanent shock (Figure 7) display comovements of investment, output and consumption, as well as a negative initial response of hours, explained by the role of habit persistence in consumption. The response to the unproductive market shock (Figure 8) displays a boom in output, hours worked and investment in the short-run, while consumption hardly responds. It is clear from this figure that such a shock is likely to contribute to the identified transitory shock of the (C, Y) VECM. The market shock drives the discounted sum of expected future profits of a startup upwards, making it worthwhile to invest in startups. This creates a boom in total investment, and also leads firms to raise their demand for labor. Therefore output increases — creating an expansion without any changes in productivity. Furthermore, as this shock essentially creates a competition for rents, it is unproductive and consumption almost does not respond. The response to the productive market

 $<sup>^{21}</sup>$ It is worth noting that the response of output to the transitory shock hinges on the response of all components of investment except residential investment (see the technical appendix). We take this observation as an additional fact in favor of our story in which entrepreneurs investment play a major role.

Figure 7: Model Response to a Permanent Technology Shock  $\varepsilon^{\Theta}$ 



This figure displays the responses of consumption, total investment, hours worked and output to a technology innovation of one standard-deviation, as computed from the estimated model.

shock (Figure 9) is quite different, which again allows for a proper identification. First the shock exerts a permanent effect on consumption, investment and output. Second, output does not move much in the short run, which makes this shock unlikely to contribute to the temporary shock of the VECM. Third, the wealth effect of this permanent shock dominates the short run dynamics of hours, investment and output. They all go down in the short run, as a consequence of the presence of habit persistence and adjustment cost to investment.

Table 7 displays some statistics of the Hodrick-Prescot (HP) filtered simulated series. As can be seen from the table, the model performs well in matching the ranking of volatilities and the comovements of the US business cycle, although investment and hours appear to be not volatile enough.

### 4 Alternative Models

In this section we explore the robustness of the result of the previous section which indicated that non-productive market shocks (market rushes) may be an significant source of macroeconomic fluctuations. Our approach is to add alternative shocks to our model, one at a time, re-estimate it and then see whether the resulting structural variance decompositions substantially change our estimates of the relative contribution of market rushes for output, consumption and hours fluc-

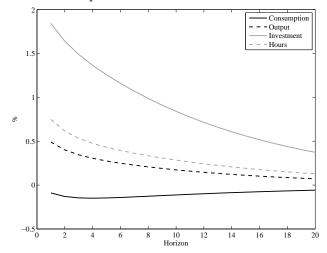


Figure 8: Model Response to a Non-Productive Market Shock  $\varepsilon^x$ 

This figure displays the response of consumption, total investment, hours worked and output to a non-productive market shock of one standard-deviation, as computed from the estimated model.

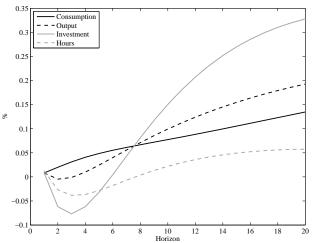


Figure 9: Model Response to a Productive Market Shock  $\varepsilon^z$ 

This figure displays the response of consumption, total investment, hours worked and output to a productive varieties innovation of one standard-deviation, as computed from the estimated model.

		Data		Model			
	$\sigma_x$	$ ho(\cdot,y)$	$ ho(\cdot,h)$	$\sigma_x$	$ ho(\cdot,y)$	$ ho(\cdot,h)$	
$\overline{y}$	1.69	_	—	1.44	_	_	
c	0.78	0.78	_	0.78	0.79	_	
I Total	6.81	0.86	_	3.56	0.95	_	
h	1.90	0.88	_	1.12	0.66	_	
y/h	0.87	0.50	0.09	1.09	0.64	-0.14	

Table 7: Some Moments of the Simulated Data (HP-filtered)

This table reports moments calculated both model generated data and actual data. Series are taken in logs and detrended applying the HP-filter with smoothing parameter  $\lambda = 1600$ .

tuations. We consider four types of shocks.<sup>22</sup> First we investigate the implications of adding an investment specific shock in our model. We examine this case in considerable detail since it has received substantial attention in the literature and we recognize that the omission of investment specific shocks could create a substantial bias in favor of finding that market rushes are important even if they are not. The two other real shocks we consider are a temporary total factor productivity shock and a preference shock. A fourth possibility involves examining the role of a monetary shock in a New–Keynesian type of model. As this fourth possibility requires writing a substantially different model, we leave the exposition of the model and results to the technical appendix. As can be seen in the appendix, we do not find that allowing a monetary shock to compete with our market rush shock changes our findings regarding the relevance of market rushes for fluctuations.

One simplification we make in analyzing these cases is that we remove the Z shock (the market shock with productivity effects), and thereby focus on models with three shocks: a permanent TFP shock, a non-productive market shock and a third shock which can be either an investment specific shock, a temporary TFP shock or a preference shock. The omission of the Z shock appears reasonable since we saw in the last section that it is not playing any substantial role in fluctuations. The production function therefore reduces to<sup>23</sup>

$$Q_t = K_t^{1-\alpha_x-\alpha_h} (\Theta_t h_t)^{\alpha_h} N_{x,t}^{\overline{\xi}} \left( \int_0^{N_{x,t}} X_t(i)^{\chi} \mathrm{d}i \right)^{\frac{\alpha_x}{\chi}}$$

 $<sup>^{22}</sup>$ Obviously we do not claim to have exhausted the list of possible explanations, but want to point out what we think are the four more widespread explanations in the profession, as we can identify them from introspection and comments during seminars.

<sup>&</sup>lt;sup>23</sup>An implication of this simplification is that the elasticity of output with respect to the intermediate good,  $\alpha_x$ , is now greater as it is still calibrated to match on the share of intermediate goods in output. This implies a mechanical reduction in the volatility of the market shock  $\eta_t^x$  (see footnote 18).

#### 4.1 Investment Specific Shock

The first shock we consider is an investment specific shock  $\dot{a} \, la$  Greenwood, Hercowitz, and Krussell [2000] or Fisher [2002]. We modify the resource constraint, which is now given by

$$Y_t = C_t + S_t + e^{-\zeta_t} I_t$$

where  $\zeta$  is the investment specific shock. In his VAR study, Fisher [2002] argues that investment specific shocks ought to exert a permanent effect and should be modeled as a random walk. We will consider both the case where the investment specific shock is modeled as a random walk and the case where it is modeled as a stationary autoregressive process.

We first estimate a model with a permanent investment specific shock, our market shock and a permanent TFP shock. As a first pass, the model is estimated by matching the response of output to both the permanent and the transitory component, as was done in the previous section. This experiment is labeled PIS-1 in Table 8 which reports the estimation results and in Table 9 which reports the associated variance decomposition. In this version the model fits the data very well. The model does relatively well in accounting for the joint dynamics of consumption and output (as indicated by the value of the  $\mathcal{D}(C, Y)$ ). What is interesting is that the volatility of the permanent investment specific shock is estimated to be very close to zero. Its contribution to output, consumption and hours worked volatility is essentially zero. In contrast, our market shock remains a very important component in explaining hours and output.

We replicate the estimation of this model by now matching both output and consumption responses, instead of only matching the output response. This experiment is labeled PIS–2 in the tables. The model is not rejected by the data (p–value=0.86), and the volatility of the investment specific shock is now close to that usually obtained by fitting the relative price of investment. Nevertheless, the variance decomposition reported in Table 9 shows that the investment specific shock is not an important source of business cycle fluctuations, while the market shock remains important. These results suggest that our market shock is not simply picking up some component of output and consumption fluctuations that is induced by permanent investment specific shocks.

These pessimistic results regarding the contribution of the investment specific shock to fluctuations may come from its specification as a random walk. We therefore also consider a stationary representation of the investment specific shock. The TIS–1 experiment then corresponds to a situation where the model parameters are estimated so as to match the impulse responses of output. One difficulty with this model is that it estimates a volatility for the innovation to the investment specific shock that is about three times as high as in the data. The interesting result for our conjecture, as shown in Table 9, is that the investment specific shock does not undermine the contribution of the market shock to accounting for the business cycle. The market shock still accounts for 42% of output volatility in the short–run while the investment specific shock only accounts for less than 5% of the output volatility at the same horizon. Furthermore, the same invariance result obtains for hours worked and for consumption.

One may however be worried that the investment specific shock may not be well identified by this procedure since it considers only output responses at the estimation stage. We therefore add the impulse responses of consumption to our list of moments to match so as to add information to the system. This experiment is labeled TIS-2 in the tables. This experiment still gives some strong support to the model (p-value=0.87). Now the estimated process of the investment specific shock is very much in line with what would result from an estimation from the relative price of investment series. The key results in terms of variance decomposition are left unaffected. The market shock still accounts for 40% of output volatility in the short–run while the investment specific shock only accounts for less than 3%. Likewise hours worked are mainly explained by the market shock (94% on impact), while consumption is only explained by the technological shock. In other words, the investment specific shock appears to explain little of economic fluctuations when one allows for our market shock, while the market shock continues to explain a substantial fraction of fluctuations when the investment specific shock is included.<sup>24</sup>

#### 4.2 Transitory Technological Shock

We now consider a version of the model in which we allow for a temporary shock to total factor productivity in addition to a permanent TFP shock and our market shock. Technology is now given by

$$Q_t = e^{\zeta_t} K_t^{1-\alpha_x-\alpha_h} (\Theta_t h_t)^{\alpha_h} N_{x,t}^{\overline{\xi}} \left( \int_0^{N_{x,t}} X_t(i)^{\chi} \mathrm{d}i \right)^{\frac{\alpha_x}{\chi}},$$

where  $\zeta$  is a stationary AR(1) autoregressive process. The parameters are estimated so as to match the impulse responses of consumption and output. The experiment is labeled T.T. in the tables. As can be seen from Table 9, the introduction of this shock does not undermine the contribution of the market shock to output volatility in the business cycle. The market shock still account for about 38% of output volatility in the short–run and more than 95% of that of hours worked. In fact, the transitory technology shock acts as a substitute for the permanent technology shock in explaining fluctuations. This can be seen from the variance decomposition of consumption that clearly shows that consumption is mainly accounted for by technology shocks — about 93% in the

 $<sup>^{24}</sup>$ At first pass, these observations may appear at odds with the results in Fisher [2002] which suggest, using an identified VAR approach, that investment specific shocks explain a substantial fraction of fluctuations. One way to reconcile these two sets of observations is to recognize that the investment specific shocks identified in Fisher [2002] may be capturing part of the effects we associate with market rushes. In particular, if changes in the quality of equipment proceed and signal the opening of new markets, then the shocks identified in Fisher [2002] could be generating macroeconomic fluctuations mainly through their effect on market rushes as opposed to working through standard capital accumulation incentives.

	PIS-1	PIS-2	TIS-1	TIS-2	T.T.	T.P.
b	0.6108	0.3125	0.6457	0.3062	0.3420	0.3877
	(0.1229)	(0.1921)	(0.1180)	(0.2184)	(0.1869)	(0.1472)
$\varphi$	0.4195	0.2534	0.6099	0.2775	0.3125	0.3699
	(0.3227)	(0.3201)	(0.6675)	(0.4235)	(0.2645)	(0.3228)
$\sigma_{\Theta}$	0.0131	0.0088	0.0126	0.0089	0.0062	0.0075
	(0.0017)	(0.1592)	(0.0017)	(0.0016)	(0.0044)	(0.0037)
$ ho_x$	0.9117	0.8919	0.9143	0.8967	0.9195	0.9075
	(0.0323)	(0.0395)	(0.0374)	(0.0420)	(0.0234)	(0.0259)
$\sigma_x$	0.1575	0.1859	0.1594	0.1775	0.1768	0.1825
	(0.0217)	(0.0349)	(0.0197)	(0.0266)	(0.0278)	(0.0297)
$ ho_T$	—	—	0.5328	0.8478	0.9143	0.8799
			(0.2742)	(0.4974)	(0.1148)	(0.1959)
$\sigma_T$	0.0003	0.0038	0.0118	0.0032	0.0046	0.0068
	(0.0243)	(0.0082)	(0.0137)	(0.0048)	(0.0021)	(0.0030)
J-stat	17.31	60.96	14.89	59.48	54.65	50.56
	[0.99]	[0.86]	[1.00])	[0.87]	[0.95]	[0.98]
$\mathcal{D}(C,Y)$	99.42		92.34			
	[0.03]		[0.06])			

 Table 8: Estimation Results

<u>Note:</u>  $\rho_T$  and  $\sigma_T$  denote respectively the persistence parameter and the volatility of the third shock. Standard errors are in parenthesis, p-value in brackets.

Horizon		Output		Cor	nsumpti	on	Hours		
	$\varepsilon^{\Theta}$	$\nu^x$	ζ	$\varepsilon^{\Theta}$	$\nu^x$	ζ	$\varepsilon^{\Theta}$	$ u^x$	ζ
PIS-1: $\zeta$ =	=Permai	nent Inv	estmen	t Specific	Shock				
1	64 %	36~%	0 %	95~%	5 %	0 %	$15 \ \%$	85~%	0 %
4	87~%	13~%	0 %	96~%	4%	$0 \ \%$	20~%	80~%	0 %
8	93~%	7~%	0 %	97~%	3~%	0 %	34~%	66~%	0 %
20	97~%	3~%	0 %	99~%	1~%	0 %	42~%	58~%	0 %
$\infty$	100~%	0 %	0 %	100~%	0 %	0 %	44~%	56~%	0 %
PIS-2: $\zeta$ =	=Permar		restmen	t Specific					
1	55~%	45 %	0 %	84 %	16~%	0 %	0 %	99~%	1 %
4	76~%	22~%	2~%	87~%	13~%	0 %	12~%	82~%	6%
8	84~%	13~%	3~%	90~%	10~%	0 %	19~%	72~%	9~%
20	90~%	6~%	4%	95~%	4%	1~%	24~%	64~%	11~%
$\infty$	96~%	0 %	4%	96~%	0 %	4%	26~%	63~%	11~%
TIS-1: $\zeta$	=Tempo	rary Inv	vestmen	t Specifie	c Shock				
1	53~%	42 %	$5 \ \%$	93~%	6 %	1 %	19~%	73~%	8 %
4	76~%	15~%	9~%	95~%	5~%	$0 \ \%$	12~%	60~%	28~%
8	87~%	9~%	4~%	96~%	4~%	0 %	23~%	57~%	20~%
20	95~%	4~%	1~%	98~%	2~%	0 %	32~%	52~%	16~%
$\infty$	100~%	0 %	0 %	100~%	0 %	0 %	34~%	50~%	16~%
TIS-2: $\zeta$	=Tempo	rary Inv	vestmen	t Specifie	c Shock				
1	56~%	42 %	$2 \ \%$	84 %	$15 \ \%$	1 %	0 %	94~%	6 %
4	75~%	20~%	5~%	87~%	12~%	1~%	11~%	73~%	16~%
8	83~%	13~%	4~%	91~%	8~%	1~%	19~%	67~%	14~%
20	92~%	6~%	2~%	96~%	3~%	1~%	25~%	63~%	12~%
$\infty$	100~%	0 %	0 %	100~%	0 %	0 %	26~%	62~%	12~%
T.T.: $\zeta =$	Tempora	ary Tech	nology	Shock					
1	$21 \ \%$	38~%	41 %	44 %	17~%	39~%	0 %	98~%	2~%
4	29~%	20~%	50~%	51~%	17~%	32~%	4 %	72~%	23~%
8	37~%	15~%	48~%	57~%	13~%	30~%	7~%	66~%	27~%
20	54~%	10~%	36~%	69~%	6~%	25~%	9~%	68~%	23~%
$\infty$	99~%	0 %	0 %	100~%	0 %	0 %	10~%	66~%	24~%
T.P.: $\zeta = \zeta$	Tempora	ry Pref	erence S	Shock					
1	27~%	39~%	34 %	55~%	$15 \ \%$	30~%	1 %	53~%	46 %
4	40~%	20~%	40~%	64~%	13~%	23~%	3~%	34~%	63~%
8	50~%	14~%	36~%	71~%	10~%	19~%	5~%	31~%	64~%
20	70~%	8~%	22~%	82~%	5~%	13~%	7~%	33~%	60~%
$\infty$	100~%	0 %	0 %	$100 \ \%$	0 %	0 %	8 %	33~%	59 %

 Table 9: Variance Decomposition

short-run — and that the split between the two shocks is about half-half. This can be explained from the estimated AR(1) process of the technology shock that clearly shows that the persistence of this shock is high ( $\rho_T = 0.91$ ). Hence, it exerts a strong wealth effect on the consumption decision and it is not surprising that this shock competes with the permanent TFP one.

#### 4.3 Transitory Preference Shock

We now introduce a transitory preference shock that shifts the labor supply. Preferences are now given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \left[ \log(C_{t+\tau} - bC_{t+\tau-1}) + \psi e^{\zeta_{t+\tau}} (\overline{h} - h_{t+\tau}) \right],$$

where  $\zeta$  is the temporary preference shock, that is assumed to follow a AR(1) process. The model is estimated so as to match the impulse responses of consumption and output. Again we find that the market shock accounts for about 40% of output volatility in the short–run and does not contribute much to the volatility of consumption (less than 15%). As far as output and consumption are concerned, the introduction of the preference shock mainly undermines the role of the technology shock. For instance, out of the 61% of output volatility not explained by the market shock about 35% is accounted for by the preference shock. The only dimension along which the preference shock competes with the market shock is in the determination of hours worked. About half of hours worked volatility can be accounted for by the preference shock. We however do not view it as seriously calling into question the potential role of the market shock in the business cycle, as the preference shock mainly reduces the explanatory power of the permanent shock rather than undermining the role of the market shock.

#### 4.4 Discussion

One important question relates to the interpretation of "a new market" and the associated empirical observations with regards to its cyclical properties.<sup>25</sup> Our metaphor of new markets describes all new ways of introducing new products given existing technology or using new technologies, although our estimations seem to favor the former interpretation rather than the later, as we do not estimate any significant long run effect of new goods creation. Broadly speaking, a new market ranges from producing a newly invented product (say cellular phones) to producing old goods with newly developed uses (fiber–optic cable networks once the use of the internet has exploded) or new ways of designing old products (say producing shirts of a fashionable new color). Given this broad interpretation, it is difficult to obtain a comprehensive measure of our new market margin. In a very narrow sense, one could associate new markets with new firms, and therefore look at Net Business

 $<sup>^{25}\</sup>mathrm{We}$  have here benefited from comments and discussion with Nir Jaimovich.

Formation. Net Business Formation is without ambiguity procyclical in the U.S., which is also one of our model predictions if we literally associate N with the number of firms. The problem is that the evidence suggests that smaller firms typically make up the majority of entrants and exits, which is insufficient to account for a large share of hours worked and output variance at short horizons. A less restrictive interpretation is to look at variations in the number of establishments and franchises as an additional channel affecting the number of "operating units". The Business Employment Dynamics database documents job gains and job losses at the establishments level at the quarterly frequency for the period between the third quarter of 1992 and the second quarter of 2005. Using these observations, Jaimovich [2004] finds that more than 20% of the cyclical fluctuations in job creation is accounted for by opening establishments, which is already a sizable number. Another dimension which could be associated to the new market margin is variation in the number of franchises. As Lafontaine and Blair [2005] show, numerous firms in a variety of industries have adopted franchising as a method of operation. Sales of goods and services through the franchising format amounted to more than 13% of real Gross Domestic Product in the 1980s and 34% of retail sales in 1986. Jaimovich [2004] documents that the variations in the number of franchises are procyclical at the business cycle frequency, which is again in line with the predictions of our model. We take this empirical evidence, together with anecdotal evidence and the evidence obtained by estimating our model, as supporting the idea that agents expectations about the possibility of new markets is likely an important driving force of the business cycle.

### 5 Conclusion

This paper presented theory and evidence in support of the idea that expectations of new market openings may be a key component in business cycles fluctuations. In particular, we proposed a model where the opening of new market opportunities causes an economic expansion by favoring competition for market share. We called such an episode a market rush in analogy to a gold rush. We first studied a simple analytical model of market rushes and showed that it displays certain important qualitative that are consistent with the data, namely that the business cycle is, to a large extent, associated with a non-permanent effect on output and hours and that does not move consumption at any frequency. We then provided a quantitative evaluation of a more complete model, which embedded market rushes as one component, and estimated it using simulated method of moments. The estimation results suggest that market rush phenomenon is a significant contributor to business cycle fluctuations and this finding is robust to the inclusion of many alternative shocks discussed in the literature.

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## Appendix

This appendix proposes a test for the equality between  $\varepsilon^{Y}$  and  $\varepsilon^{T}$  (or equivalently between  $\varepsilon^{C}$  and  $\varepsilon^{P}$  as the shocks are pairwise orthogonal). We show that, for the VECM under consideration, this equality corresponds to a particular zero in the long run impact matrix of the Wold decomposition. Consider the Wold representation (26), and consider the following representation of the process:

$$\begin{pmatrix} \Delta C_t \\ \Delta Y_t \end{pmatrix} = B(L) \begin{pmatrix} \nu_{1,t} \\ \nu_{2,t} \end{pmatrix},$$
(34)

where  $B(L) = \sum_{i=0}^{\infty} B_i L^i$  and the variance covariance matrix of  $(\nu_1, \nu_2)$  is the identity matrix. We want here to perform an overidentified orthogonalization and impose at the same time that *(i)* the shock  $\nu_2$  has no impact effect on *C* and *(ii)* no long run effect on *C* and *Y*. More precisely, we look for a matrix *S* such that  $\mu = S\nu$  and  $B_i = A_i S$ . Imposing a zero impact effect of  $\nu_2$  on *C* implies

$$s_{12} = 0.$$
 (35)

The matrix giving the long run effect of  $\nu$  on both variables is given by  $\hat{B} = \hat{A}S$ , where  $\hat{A} = \sum_{i=0}^{\infty} A_i$ . Imposing the long run restriction  $\hat{b}_{12} = 0$  implies

$$\hat{a}_{11}s_{12} + \hat{a}_{12}s_{22} = 0. \tag{36}$$

When the two series cointegrate, the matrix  $\widehat{A}$  rewrites

$$\widehat{A} = \left(\begin{array}{cc} \widehat{a}_{11} & k\widehat{a}_{11} \\ \widehat{a}_{21} & k\widehat{a}_{21} \end{array}\right),$$

where k is a real number.

When  $\hat{a}_{12} \neq 0$  — which occurs when both k and  $\hat{a}_{11}$  are non zero — equations (35) and (36) imply that the second column of S is composed of zeros, meaning that S is not a full rank matrix. In other words, the two restrictions cannot hold at the same time. On the contrary, if  $\hat{a}_{12} = 0$  when either k or  $\hat{a}_{11}$  are zero — the long run and short run constraints are simultaneously satisfied. This suggests that a convenient way of testing whether both the short and long run constraints are satisfied is to test for the nullity of a particular coefficient of the long run matrix  $\hat{A}$  of the Wold representation of the process,  $\hat{a}_{12}$ . The following proposition states this result in a more general case where the two series need not cointegrate.

**PROPOSITION 1** Consider a bivariate process whose Wold decomposition is given by

$$\left(\begin{array}{c} \Delta X_t^1\\ \Delta X_t^2 \end{array}\right) = A(L) \left(\begin{array}{c} \mu_{1,t}\\ \mu_{2,t} \end{array}\right),$$

with  $A(L) = \sum_{i=0}^{\infty} A_i L^i$ , with  $A_0 = I$ , where the covariance matrix of  $\mu$  is given by  $\Omega$ , and where the matrix of long run effect  $\widehat{A} = A(1) = \sum_{i=0}^{\infty} A_i$  is non singular. Consider a structural representation

$$\left(\begin{array}{c} \Delta X_t^1\\ \Delta X_t^2 \end{array}\right) = B(L) \left(\begin{array}{c} \nu_{1,t}\\ \nu_{2,t} \end{array}\right),$$

where  $B(L) = \sum_{i=0}^{\infty} B_i L^i$ , the covariance matrix of  $\nu$  is the identity matrix and  $\mu = S\nu$ . Then, the two following statements are equivalent

- (a) If the second structural shock  $\nu_2$  has no short run impact on  $X_1$ , i.e.  $s_{12} = 0$ , then it has no long run impact on  $X_1$ , i.e.  $\hat{b}_{12} = 0$ , and conversely.
- (b) The (1,2) element of the long run effect matrix of the Wold decomposition is zero, i.e.  $\hat{a}_{12} = 0$

**Proof :** We first prove that (a) implies (b). Assume that

$$S = \begin{pmatrix} s_{11} & 0\\ s_{21} & s_{22} \end{pmatrix} \quad \text{and} \quad \widehat{B} = \begin{pmatrix} \widehat{b}_{11} & 0\\ \widehat{b}_{21} & \widehat{b}_{22} \end{pmatrix}$$

Inverting S, we obtain

$$S^{-1} = \begin{pmatrix} 1/s_{11} & 0\\ -s_{21}/(s_{11}s_{22}) & 1/s_{22} \end{pmatrix}.$$

The long run effect matrix of the Wold decomposition is  $\widehat{A} = \widehat{B} \times S^{-1}$  and one can easily check that  $\widehat{a}_{12} = 0$ .

We then prove that (b) implies (a). We have the relation  $\widehat{AS} = \widehat{B}$ . We assume that  $\widehat{a}_{12} = 0$ . Then,  $\widehat{b}_{12} = \widehat{a}_{11}s_{12}$ . As  $\widehat{A}$  is assumed to be nonsingular,  $\widehat{a}_{11} \neq 0$ , so that  $\widehat{b}_{12} = 0 \iff s_{12} = 0$ . Q.E.D.