An Equilibrium Analysis of Information Aggregation in Investment Markets with Discrete Decisions *

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Abstract

This paper proposes a bridge between the Herding literature and the literature on Rational Expectations under asymmetric information. In particular we examine how the presence of discrete investment decisions affects the properties of a market equilibrium where information is costly to acquire. We choose to focus on the case where individual decisions are discrete since this appears to be the key element behind herding results. Our objective is to examine whether the equilibrium occurrence of herding type phenomena is likely to arise when actions are simultaneous (as opposed to sequential) and when prices can convey information. Our main result is that, as long as acquiring information is not too costly, the unique equilibrium outcome of our model is characterized by fluctuations in investment that resemble herding behavior. Specifically, equilibrium realizations of prices and investment may be high simply because uninformed investors are buying under the impression that the high price is a signal of good investment opportunities. Moreover, we find an interesting tradeoff between the size and the frequency of aggregate allocative errors, whereby as the cost of gathering information declines the size of allocative errors increases, even though there occurrence decreases. We believe these results provide new impetus for the view that herding type behavior may be relevant for understanding market fluctuations and even eventually business cycle phenomena.

Key Words: Investment, Information Aggregation, Herding, Equilibrium Randomness.

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1 Introduction

Understanding fluctuations in aggregate investment is a key element towards a better comprehension of business cycle phenomena. Over the last twenty five years, there has been enormous research effort devoted to this task. As discussed by Caballero (1997), these efforts have involved the examination of non-convex adjustment costs, credit market constraints and imperfect information. In this paper, we pursue this line of research one step further by illustrating how certain aspects of aggregate fluctuations in investment may be better understood when some of these elements are allowed to interact. In particular, we examine the equilibrium determination of aggregate investment in a market where information is costly to acquire and individual investment decisions are discrete (or at least bounded). Our analysis explores how the allocation of investment in this environment is affected by the information revealed in prices. We have chosen to focus on the particular case where individual investment decisions are discrete for two reasons. First, as suggested by the investment literature, many investment decisions are lumpy and credit constraints may limit individuals from infinitely replicating such projects. Hence, if such is the case, it appears appropriate to model individual investment decisions as discrete. Second, the herding literature suggests that information may aggregate improperly when decisions are discrete (see Gale (1996) for a summary of work in this area). However, since the herding literature generally examines situations where investors act sequentially, it is of interest to examine whether the insights derived in this literature extend to a market (non-sequential) situation.

The main finding of this paper is that the interaction between decentralized information gathering and discreteness of investment decisions at the individual level greatly affects how equilibrium prices reveal information and how this translates into aggregate investment fluctuations. In effect, we show how the equilibrium forces that shape the aggregation and acquisition of information in our environment give rise to an outcome which necessarily involves equilibrium randomness; that is, occasional and unpredictable periods in which the behavior of aggregate investment is primarily driven by the mass behavior of uninformed investors as opposed to being driven by the decisions of informed traders. In a sense, our model explains as an equilibrium outcome what may be interpreted by an observer as herding behavior or animal spirits. However, as we will discuss, this equilibrium randomness is not due to multiple equilibria, but instead is more akin to the randomness arising endogenously in games involving mixed strategies. It should be emphasized that we are not the first to note the possibility of endogenous equilibrium randomness in a market with dispersed information. In particular, this has been previously discussed in the rational expectations literature by Dutta and Morris (1997) and DeMarzo and Skiadas (1998); however, to our knowledge this is the first paper in which such randomness arises as the unique equilibrium outcome and in which it clearly reflects a necessary balancing of equilibrium forces.

Although our model is very simple and stylized, it offers interesting insight into the forces determining fluctuations in aggregate investment. For example, our model provides an equilibrium explanation to occasional but large allocative errors in investment even when investors are sophisticated. In effect, we show why the probability of a large allocative error is actually higher when individuals can engage in information gathering activities than when they have no access to information. Hence, the model offers an explanation to a type of boom and bust phenomena – that is, periods of excessively high or low investment when interpreted after the fact – even when economies are informationally advanced. The reason for such booms and busts in investment is closely related to the reason for the Grossman and Stiglitz's (1980) result regarding the impossibility of informationally efficient markets; that is, booms and busts arise endogenously in our model as a means of maintaining incentives to acquire information. Therefore, our model indicates that occasional large errors in aggregate investment may be inherent to the well functioning of a market economy since it is only in these periods that informed individuals can reap the benefits of gathering information. However, in contrast to Grossman and Stiglitz's, the equilibrium outcome of our model does not converge to the informationally efficient outcome as noise trading goes to zero. Instead, even as the importance of noise traders go to zero, we show that the information revealed in prices remains bounded away from full information revelation and the possibility of large allocative errors in aggregate investment remains the norm.

This paper can be viewed as bridging the gap between two segments of literature and highlights the resulting implications for the behavior of aggregate investment. In particular, on the one hand our approach is clearly in the tradition of the rational expectations literature which emphasizes the role of prices in aggregating and transmitting information, as well as the interaction of this process with the incentives to acquire and use private information. On the other hand, by focusing on a situation where individual level decisions are discrete, our approach is also related to the literature on social learning and herding, which focuses on situations where information is dispersed and individual investment decisions are discrete. Work in this area includes Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992), Caplin and Leahy (1993, 1994), Chamley and Gale (1994) and Zeira (1994). However, in contrast to this latter literature which generally adopts a sequential approach and favors a game theoretic analysis, our approach is firmly anchored in Walrasian and Rational Expectation tradition in that market outcomes are analyzed as stable outcome of a situation with simultaneous determination of prices and quantities. In short, our paper examines the implications for aggregate investment of embedding in the rational expectations literature an element found to be important in the social learning literature; that is, discrete decisions. Finally, it should be noted that our paper is very close in spirit to that of Avery and Zemsky (1997) since they have examined whether results found in the herding literature extend to situations in which prices can reveal information. However, the modeling approach we adopt is methodologically very different than that of Avery and Zemsky, whereby they maintain a sequential framework while we adopt a market equilibrium approach.

The remaining sections of the paper are structured as follows. In Section 2, we present the environment under study. [1] In Section 3, we characterize equilibrium behavior. We begin by analyzing a benchmark case in which individuals cannot acquire private information. This benchmark case allows us to introduce the notion of equilibrium we use throughout our analysis. In particular, we follow the work of Dutta and Morris (1997) and DeMarzo and Skiadas (1998) in using an extension of the standard notion of rational expectations equilibria which does not exclude the possibility of endogenous randomness. In Section 3.2 we provide the main results of the paper regarding the determination of prices and aggregate investment when information can be acquired at a cost and individual decisions are discrete. Section 4 concludes. Proofs of all propositions are provided in an appendix.

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[1] The environment we study is similar to that analyzed by Barlevy and Veronesi (1999) in several dimensions. However, the two differ in at least one essential dimension. Our analysis concentrates on the case where noise traders are a small fraction of the market, while Barlevy and Veronesi focus on the opposite case.

2 The Model

It is helpful to introduce our model by referring to a specific example.

Period 1	Period 2
developers: acquire information buy office building	**developers:** sell office building
construction workers: sell office building	**firms:** buy office building

The setting is one where there is a set of developers who must each decide whether to order the construction of an office building from one of many firms in the construction sector. Before doing so, each developer may engage into research about future demand conditions in the office rental market. The objective of a developer is to profit from her investment by meeting future demand for office space. Thus, we think of a developer as an intermediary who profits from transferring goods across time, bringing the supply of office space (say, from the construction sector) and the demand for office space (say, from the services sector) together. Our focus is on the determination of aggregate investment by the developers in a context where the expectation they hold about future demand is fundamental to their behavior. In particular, we will examine this problem in a market economy where there is initially noise trading, but the amount of noise trading is small (in a well defined sense) and will eventually be taken to zero. Hence, our analysis is one where aggregate investment activity is essentially dominated by rational actors. All other aspects of the problem will be kept as simple as possible as to ease exposition.

Formally, we consider a two-period economy. In the first period a continuum of potential investors, with unit measure, must decide whether to purchase or not one unit of capital (a building) at the price p_1. Investment decisions are denoted by $x_i \in \{0, 1\}$, for each agent $i \in [0, 1]$. The price of capital is determined by a supply function $p_1 = s\, Q_1$ $(s > 0)$ where Q_1 is the aggregate quantity in the market. In the second period, each investor has access to a common production technology which transforms one unit of capital into one unit of output. The only decision to be made is how much output y_i to sell at the price p_2 so as to maximize profits. Markets in both periods are competitive and agent i's profits are simply given by

$p_2 y_i - p_1 x_i$, where $y_i \leq x_i$. Before going further, it is worth noting that our main results will carry through under continuous but bounded investment decisions, as we will discuss later, or under risk aversion with discrete decisions.

The price of the second period consumption good is given by a function $p_2 = \theta - d\,Q_2$ $(d \geq 0)$, where Q_2 is the aggregate quantity in the market and θ denotes the state of the world and it is assumed to be a random variable

$$\tilde{\theta} = \begin{cases} \theta_h & \text{with probability } \mu \\ \\ \theta_l & \text{with probability } 1-\mu, \end{cases} \tag{2.1}$$

where $\theta_h > \theta_l > 0$ and $\mu \in (1/2, 1)$. Throughout the paper we will adopt the convention that random variables will be denoted with a tilde, while the same variables without a tilde denote particular realizations. In order to guarantee that solutions are interior we assume that $(s+d) > \theta_h$.

Before investment decisions are made, the value of $\tilde{\theta}$ is determined and potential investors can learn the realization of $\tilde{\theta}$ at a cost of $c > 0$. We denote agent i's information acquisition decision by $z_i \in \{0, 1\}$. At the time investment decisions are made, thus, each agent will be either (perfectly) informed, if $z_i = 1$, or uninformed, if $z_i = 0$. [2] In addition to rational investors, there is noise trading. We assume that there is a measure $\tilde{k}$ of noise traders investing in the first period and selling output in the second period, where

$$\tilde{k} = \begin{cases} \bar{k} & \text{with probability } \rho \\ \\ 0 & \text{with probability } 1-\rho, \end{cases} \tag{2.2}$$

with $\rho \in (0, \frac{1}{2})$ [3] and $0 < (1+\rho)\,\bar{k} < \min\{\theta_l/(s+d), 1-\theta_h/(s+d)\}$. This last condition makes

[2] While allowing for noisy signals does not change our main results, it obscures the analysis as it adds exogenous randomness to the economy.

[3] This restriction is not necessary for our main results but allows the statement of Proposition 4 to be much

sure that noise trading is small relative to the market, thus allowing us to concentrate on those instances where market activity is dominated by rational investments. The following tabulation summarizes the exogenous distribution of states of demand and noise:

Table 1: Prob. Dist. of Exogenous States ($\mathbf{Pr}\,(\boldsymbol{\theta}, \boldsymbol{k})$)

	$\tilde{\boldsymbol{k}} = \bar{\boldsymbol{k}}$	$\tilde{\boldsymbol{k}} = \mathbf{0}$
$\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta_h}$	$\mu\,\rho$	$\mu\,(1-\rho)$
$\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta_l}$	$(1-\mu)\,\rho$	$(1-\mu)\,(1-\rho)$

The timing of the model is as follows. In the first period, the investment market opens. When this market is open, there is a simultaneous determination of information acquisition decisions, the first period prices, and the investment decisions. In period 2, demand is realized and investors supply the market, with the second period price adjusting to equate supply and demand. Investors' profits are realized at the end of the second period.

Second Period

Since the interesting aspect of our analysis relates to the outcome in the first period, it is best to immediately solve for the equilibrium outcome of the second period and use the resulting relationship to simplify the analysis throughout the paper. To this end, note that as long as the price of the consumption good p_2 is positive, optimal behavior in the second period implies that every investor supplies all of her output to the market. Market clearing in period 2 then requires that

$$\tilde{p}_2 = \tilde{\theta} - d\,\left(\int_0^1 x_i\; di + \tilde{k}\right), \tag{2.3}$$

and market clearing in period 1 implies

$$\tilde{p}_1 = s\,\left(\int_0^1 x_i\; di + \tilde{k}\right). \tag{2.4}$$

simpler.

Together, the market clearing conditions (2.3) and (2.4) implicitly define the price of the consumption good as a function of the price of capital (p_1) and the state of demand (θ). Let $p_2\left(\tilde{p}_1, \tilde{\theta}\right)$ denote such a function. Each investor's profit is then given by

$$p_2\left(\tilde{p}_1, \tilde{\theta}\right) - \tilde{p}_1 = \tilde{\theta} - \frac{s+d}{s}\,\tilde{p}_1. \tag{2.5}$$

It is worth noting that aggregate investment in period 1 will have an impact on period 2's price which in turn will affect the profitability of period 1's individual investments. This interaction is captured by (2.5) and will be used throughout the paper without further reference.

3 Equilibrium Analysis

As is standard in the rational expectations literature, our goal is to determine the equilibrium properties of the set of endogenous random variables – the price $\tilde{p}_1$, the investment levels $\tilde{x}_i$ and the information acquisition decisions $\tilde{z}_i$— as a function of the exogenous random variables ($\tilde{\theta}$ and $\tilde{k}$). In particular, we want to characterize the joint distribution of these variables under the requirements that, for all realizations of the random variables, the first-period investment market clears and individual investors are satisfied with their allocations (x_i, z_i). [4] Moreover, we want to impose the additional weak requirements that allocations are anonymous (the distributions of $\tilde{x}_i$ and $\tilde{z}_i$ are independent of i) and that the price cannot reveal what no one knows, that is, if $z_i = 0$ for all i and for all p_1, then $\tilde{p}_1$ must be independent of $\tilde{\theta}$. In other words, we want to characterize the joint distribution of $\tilde{p}_1, \tilde{x}_i, \tilde{z}_i$ (conditional on all realizations of $\tilde{\theta}$ and $\tilde{k}$) which satisfies these conditions. However, as noted by Dutta and Morris (1997) and DeMarzo and Skiadas (1998), these requirements alone do not necessarily imply that the mapping from states to prices is deterministic. For example, it is possible that the conditional distribution of the price, which we will denote by $\delta(\tilde{p}_1 \mid \theta, k)$, be non-degenerate. Hence, one must take a stand on whether or not it is desirable to force such a distribution to be degenerate. Our choice is to follow Dutta and Morris (1997) and DeMarzo and Skiadas (1998), among others, and not to a

[4] In the interest of clarity of exposition, our discussion no longer refers to the individual rationality and market clearing conditions for period 2 since these are implicit in our definition of $p_2\left(\tilde{p}_1, \tilde{\theta}\right)$, as it has been explained above.

priori force such a deterministic mapping since such a restriction does not appear warranted. This opens the possibility of endogenous randomness to arise as an equilibrium phenomenon, where by endogenous randomness we mean randomness that is not a deterministic function of fundamentals. We think that allowing such a possibility is important in our context since herding type behavior, if it is to arise in a simultaneous-market setting, it is likely to take the form of endogenous randomness.

In order to clearly expose the notion of equilibrium we use in the paper, we will proceed in two steps. In Section 3.1, we consider the case where agents in the economy do not have access to private information. This eliminates momentarily the choice of whether to acquire information or not and hence the presence of informed investors in the market. This simplification will allow us to illustrate the equilibrium concept we use. Our case of interest, where information acquisition is endogenous, is the focus of Section 3.2.

3.1 A Benchmark: The Economy Without Private Information

In this section we examine the case where no one in the economy has access to information. Thus, the economy consists only of rational uninformed agents and noise traders. The object is to characterize the joint distribution of $\tilde{p}_1$ and $\tilde{x}_i$ conditional on θ and k. Given the discreteness of the allocations, any equilibrium that treats individuals equally will necessarily require that individuals receive the good with a probability strictly between zero and one. Hence, it is necessary to discuss the appropriate notion of individual rationality to be used in this case.

The standard notion of individual rationality would simply impose that for all realizations of $\tilde{x}_i$, x_i should optimize the agents objective given prices as indicated below.

$$x_i \in \arg \max_{\hat{x} \in \{0,1\}} \left\{ E\left[\left(p_2\left(\tilde{p}_1, \tilde{\theta}\right) - \tilde{p}_1 \right) \hat{x} \mid p_1 \right] \right\}.$$

However, in the case where there can be information revelation by equilibrium outcomes and where individual allocations have a random component, it appears most reasonable to impose that agents remain satisfied with there allocation even after they observe the realization of $\tilde{x}_i$. In other words, in a market setting where agents are ex ante indifferent between receiving or not a good, agents should not want to change (or re-optimize) their allocation after it is delivered to them. The most obvious way to guarantee that such "regret" does not occur is to also require the following ex post individual rationality condition

$$x_i \in \arg \max_{\hat{x} \in \{0,1\}} \left\{ E\left[\left(p_2\left(\tilde{p}_1, \tilde{\theta}\right) - \tilde{p}_1 \right) \hat{x} \mid p_1, x_i \right] \right\}.$$

It is worth noting that ex ante individual rationality immediately implies ex post individual rationality whenever the optimal allocations are not random. The extra requirement of ex post individual rationality only has bite whenever the distribution of $\tilde{x}_i$ conditional on the price is non-degenerate.

With this extended notion of individual rationality in mind, we can now present a concise definition of equilibrium.

Definition 1 *An equilibrium for the benchmark economy is a joint distribution for $\tilde{p}_1$ and $\tilde{x}_i$ conditional on θ and k, such that for all realizations of both endogenous and exogenous variables*

(a) *Individual rationality is satisfied.*

(b) *The investment market clears.*

As is standard in the rational expectations literature, an equilibrium consists of a conditional distribution for the endogenous variables which are the price and the allocations. Condition (a) requires that individual allocations are optimal given the information revealed in the price and, if relevant, the information revealed in the realization of $\tilde{x}_i$. Note that, in addition, we are imposing an anonymity condition which requires all individuals to be treated ex ante identically irrespective of their identity i.

The equilibrium analysis of this simple benchmark economy is designed to illustrate the following phenomena. First, that randomness in individual allocations arise naturally, and often necessarily, as an equilibrium outcome in the presence of discrete decisions. Secondly, that our generalization of the conventional notion of a *rational expectations equilibrium* to allow for endogenous randomness in prices **does not** necessarily give rise to such non-fundamental randomness. In effect, in this case, it will not give rise to an endogenous randomness in prices.

In what follows, it will be helpful to have defined the prices that would prevail in period 1 if all agents had perfect information about future demand. Therefore, let $p^f(\theta) \equiv \theta s/(s+d)$, for $\theta \in \{\theta_l, \theta_h\}$, and note that $p^f(\theta)$ is the unique price p_1 that solves $p_2(p_1, \theta) - p_1 = 0$. We will refer to $p^f(\theta)$ as the full-information price in state θ.

Proposition 1 *The unique equilibrium of the benchmark economy is such that* [5] [6]

$$
\begin{aligned}
supp[\delta] &= p^n = \mu\, p^f(\theta_h) + (1-\mu)\, p^f(\theta_l), \\
Pr(p_1 = p^n \mid k, \theta) = \delta(p^n \mid k, \theta) &= 1, \quad \forall\, k\, \theta, \\
\Pr(x_i = 1 \mid \theta, k, p^n) &= \begin{cases} s^{-1}p^n - \bar{k} & \text{if} \quad k = \bar{k} \\ s^{-1}p^n & \text{if} \quad k = 0. \end{cases}
\end{aligned}
$$

Thus, the equilibrium in this case is rather simple and intuitive, with the price obviously not conveying any information about the state of demand since no one has such information. What is interesting to note about the benchmark economy is how the market equilibrium is supported. In particular, we emphasize the absence of randomness in aggregate investment despite the presence of noise traders in the economy. The reason is that sophisticated investors will go in and out of the market to accommodate the (unobserved) movements in noise trading. This implies that allocation rules must be necessarily random, at least if one insists that market allocations must treat individuals symmetrically. Note that aggregate investment in period 1 is given by $s^{-1}p^n$ and is deterministic even though the aggregate investment made by the noise traders is random.

To highlight the nature of the equilibrium, Figure 1 plots investment demand for the out-

[5] As is common in the literature, we are disregarding the possibility of equilibria which reveal information that no one has.

[6] We denote the support of prices by $\text{supp}[\delta] = \left\{p_1 \in \Re_{++} \mid \delta(p_1 \mid k) > 0 \text{ for some } k \in \left\{0, \bar{k}\right\}\right\}$.

comes where $\tilde{k} = 0$ and $\tilde{k} = \bar{k}$. The important feature is that the market price is insensitive to the behavior of the uninformed. Two elements are driving this result. First, it must be that $\bar{k}$ is sufficiently small. If noise traders can take over the entire market, then aggregate investment will trivially fluctuate above $s^{-1}p^n$ when $\tilde{k} = \bar{k}$. Second, note that demand by the uninformed is perfectly elastic at the market price p^n. Otherwise, if the uninformed agents' market demand were downward sloping, the market price would react to changes in the behavior of the uninformed. For example, this would happen if individuals were risk averse and investment decisions continuous. Conversely, the price will not fluctuate if investment decisions are discrete (as assumed here) or if agents are risk neutral with continuous but bounded decisions (this latter case will be discussed below). This elastic property of the market demand, which is a direct implication of discrete individual decisions, will be key to understanding the equilibrium derived in the following section. We now turn to discussing the notion of allocative errors for this economy.

Definition 2 *For any $p_1 \in supp[\delta]$ and $\theta \in \{\theta_l, \theta_h\}$, an allocative error is defined as $s^{-1}p_1 - s^{-1}p^f(\theta)$.*

We find it informative to define allocative errors relative to the first-best allocation $s^{-1}p^f\left(\tilde{\theta}\right)$ which would be attained under full information. It follows from Proposition 1 that, in equilibrium, allocation errors are given by

$$s^{-1}p^n - s^{-1}p^f(\theta) = \begin{cases} -(1-\mu)\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right) & \text{if} \quad \tilde{\theta} = \theta_h \\ \\ \mu\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right) & \text{if} \quad \tilde{\theta} = \theta_l. \end{cases} \tag{3.1}$$

Allocative errors are a fraction of the maximum feasible error $s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)$, weighted by the extent to which the realization of the state of demand was unexpected. Of course, the presence of ex post allocative errors in an economy with exogenous noise is expected. However, anticipating our results below, it turns out that even larger allocative errors than that given by (3.1) occur with positive probability when individuals have the possibility of purchasing information before making investment decisions. This is the case we examine in what follows.

3.2 Endogenous Information Acquisition

In this section we want to reintroduce the possibility of information gathering at a cost $c > 0$. This extension requires us to simply extend our previous definition of equilibrium as to express the restrictions to be satisfied by the joint distribution of all three endogenous random variables $\tilde{x}_i, \tilde{z}_i$ and $\tilde{p}_i$ conditional on θ and k. Once again, the only important step is to establish the appropriate individual rationality requirements for $\tilde{x}_i, \tilde{z}_i$. This is set out in the following.

Definition 3 *The joint distribution of $\tilde{x}_i, \tilde{z}_i$ and $\tilde{p}_i$ satisfies individual rationality if for all realizations*

(a) *x_i is optimal given the information set $\{p_1, z_i\theta\}$, that is*

$$x_i \in \arg \max_{\hat{x}\in\{0,1\}} \left\{ E\left[\left(p_2\left(\tilde{p}_1, \tilde{\theta}\right) - \tilde{p}_1 \right) \hat{x} \mid p_1, z_i\theta \right] \right\}$$

and

(a.1) *x_i remains optimal with respect to the information set $\{p_1, z_i\theta, x_i\}$, that is, when the information set is expanded to include x_i.*

(b) *z_i is optimal given the information set $\{p_1\}$, that is*

$$\begin{aligned} z_i \quad \in \quad & \arg \max_{\hat{z}\in\{0,1\}} \left\{ (1-\hat{z}) \max_{\hat{x}\in\{0,1\}} \left\{ E\left[\left(p_2\left(\tilde{p}_1, \tilde{\theta}\right) - \tilde{p}_1 \right) \hat{x} \mid p_1 \right] \right\} \right. \\ + \quad & \left. \hat{z}\, E\left[\max_{\hat{x}\in\{0,1\}} \left\{ E\left[\left(p_2\left(\tilde{p}_1, \tilde{\theta}\right) - \tilde{p}_1 \right) \hat{x} - c \mid p_1, \theta \right] \right\} \mid p_1 \right] \right\}. \end{aligned}$$

and

(b.1) *z_i remains optimal with respect to the information set $\{p_1, z_i\}$, that is, when the information set is expanded to include z_i.*

The notion of individual rationality we use here is very close to that presented earlier. In particular, Conditions (a) and (b) incorporate the standard optimality conditions, where in (a) we include θ in i's information set if $z_i = 1$. Note that Condition (b) implies that we are in effect allowing information acquisition decisions to be determined simultaneously with prices since the optimality of z_i is set conditional on the price. Note that the added condition (a.1) and (b.1) only has bite if allocations or information acquisition decisions are random conditional on the price, in which case, it insures that if agents are indifferent about either the outcome x_i or z_i conditional on the price, they remain satisfied with their allocations after learning the outcome. With this notion of individual rationality, we can now easily proceed to define an equilibrium for the case with endogenous information acquisition.

Definition 4 *An equilibrium is a joint distribution for $\tilde{p}_1$, $\tilde{x}_i$ and $\tilde{z}_i$ conditional on the different realizations of θ and k, such that for all realizations of the random variables (both endogenous and exogenous)*

(a) *Allocations satisfy individual rationality.*

(b) *The market for the investment good clears.*

(c) *If $z_i = 0$ for all i and for all $p_1 \in supp[\delta]$, then $\Pr(\theta_h \mid p_1) = \mu$ for all $p_1 \in supp[\delta]$.*

As before, an equilibrium consists simply of a joint distribution for the endogenous variables which satisfy individual rationality and market clearing. Once again, we restrict attention to anonymous allocation rules, whereby all individuals are treated according to their information and we make explicit that prices cannot convey information that no one has obtained (Condition (c)).

The equilibrium object under study, which is a joint distribution for $\tilde{z}_i$, $\tilde{x}_i$ and $\tilde{p}_1$, is rather complex. In order to describe its properties, we therefore proceed in two steps. We begin by focusing on the properties of the price and the aggregate level of investment. This has the advantage of immediately pointing out the most relevant properties of the equilibrium. In particular, these are the features that are most interesting from an aggregate point of view and these properties happen to be uniquely defined by the model. Once these properties are described and explained, we then illustrate how the joint distribution of all three endogenous variables can be constructed. That is, we will show how to construct micro-allocations (x_i, z_i) that support the noted price distribution and the aggregate allocations as an equilibrium phenomenon. It should be once again emphasized that the approach we adopt is clearly in the Walrasian and Rational Expectations tradition in that we will not explain how the equilibrium price distribution comes about; instead we will simply describe the price distribution that satisfies the equilibrium requirements. [7]

The first aspect of the equilibrium we want to emphasize relates to the support for the price $\tilde{p}_1$ and its relationship to whether or not there is information acquisition. This is done in the following proposition.

Proposition 2 *If $c \geq \bar{c} \equiv \mu(1-\mu)(\theta_h - \theta_l)$, there is no information acquisition and the equilibrium support of $\tilde{p}_1$ is the singleton given by*

$$p^n = \mu \, p^f(\theta_h) + (1-\mu)\, p^f(\theta_l) \, .$$

If $c < \bar{c}$, then there is always some information acquisition and the equilibrium support of $\tilde{p}_1$ is the couple $\left\{p^l, p^h\right\} \in R^2_{++}$ given by

$$p^h = \left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}} \right) p^f(\theta_h) + \left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}} \right) p^f(\theta_l) \, ,$$

$$p^l = \left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}} \right) p^f(\theta_h) + \left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}} \right) p^f(\theta_l) \, .$$

The first aspect to note from Proposition 2 is that, if the cost of information acquisition is too high ($c \geq \bar{c} \equiv \mu(1-\mu)(\theta_h - \theta_l)$), there will be no information acquisition and the

[7] Recently, there has been substantial effort at providing a non-cooperative game-theoretic foundation to Rational Expectations Equilibria. See, for example, Pesendorfer and Swinkels (2000) and Jackson (1999). In principle, it would be of interest to also provide an explicit non-cooperative foundation to the equilibrium presented here. However, we view this as beyond the scope of the current paper.

equilibrium will replicate the one discussed in the previous section. Since this case is now well understood, we will focus on the case where $c < \bar{c}$. In this latter case, the proposition indicates that the equilibrium will be characterized by a two-point price distribution. As can be easily seen, this price distribution is characterized by a high price p^h which is above the pooling price $p^n = \mu\, p^f(\theta_h) + (1-\mu)\, p^f(\theta_l)$ but below the full information price $p^f(\theta_h)$. Accordingly, the low price p^l will be below the pooling price but above the full information price $p^f(\theta_l)$. Moreover, as c converges either to 0 or $\bar{c}$, these prices converge respectively to the full information prices or the single pooling price.

An immediate implication of Proposition 2 is that, as long as the cost of acquiring information is sufficiently low ($c < \bar{c}$), the aggregate level of investment will also be characterized by a two-point distribution. In effect, the market clearing condition implies that aggregate investment must fluctuate between the two quantities on the supply curve given by $s^{-1}p^h$ and $s^{-1}p^l$. Hence, from an aggregate point of view Proposition 2 indicates that, for sufficient low costs of information, the equilibrium will be characterized by fluctuations in the price and quantity of investment goods. The obvious next step is to characterize the properties of these fluctuations in terms of the information they convey regarding the underlying state of future demand θ.

Proposition 3 *When $c < \bar{c}$, the information content of prices (as captured by $\Pr(\theta_h \mid p_1)$) is given by :*

$$\Pr\left(\theta_h \mid p^h\right) = \left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}}\right) \quad \textit{and} \quad \Pr\left(\theta_h \mid p^l\right) = \left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}}\right),$$

where

$$1 > \Pr\left(\theta_h \mid p^h\right) > \mu \quad \textit{and} \quad \mu > \Pr\left(\theta_h \mid p^l\right) > 0.$$

Proposition 3 indicates that prices are informative but nevertheless noisy. In particular, the high price p^h indicates that the state θ_h is more likely to arise than what is expected based only on priors, but it does not indicate for sure that the high state of future demand will arise. Similarly, the low price indicates that the low state of future demand is more likely to arise than what is expected based on priors, but p^l does not indicate θ_l for sure. At a first pass, the observation of a noisy equilibrium appears natural given there are noise traders in the system. However, this type of intuition is somewhat misleading. In effect, it can be seen that the properties laid out by Propositions 1 and 2 are entirely invariant to the amount of noise traders in the system, that is, the extent of price (and aggregate investment) fluctuations and their informativeness are independent of the measure of noise traders. As we will later make precise (after we show how to construct an equilibrium), the presence of noise in the equilibrium price should not be thought as being driven by the noise traders since the amount of noise will remain intact even as we take a limit whereby the importance of noise traders is set to zero. In fact, the noise in the price system should be viewed as an equilibrium response to balance the incentive for information acquisition with the information revelation in prices. To see this, note that Propositions 2 and 3 are essentially derived from the two individual rationality constraints (evaluated at the equilibrium) given below by ($\star$) and ($\star\star$). The equation given by ($\star$) is the condition associated with being indifferent between acquiring or not information. The equation given by ($\star\star$) corresponds to the condition, for an uninformed investor, of being indifferent between acquiring or not the investment good. Both of these conditions have to be associated with indifferences in equilibrium since there cannot exist equilibria where everyone gathers information, or where all the uninformed want the good.

$$\Pr\left(\theta_h \mid p_1\right)\left(\theta_h - \frac{s+d}{s}p_1\right) = c \tag{$\star$}$$

$$\Pr\left(\theta_h \mid p_1\right)\left(\theta_h - \frac{s+d}{s}p_1\right) + \left(1 - \Pr\left(\theta_h \mid p_1\right)\right)\left(\theta_l - \frac{s+d}{s}p_1\right) = 0. \tag{$\star\star$}$$

Simple manipulations of these two equations allows one to derive the price support given in Proposition 2 as well as the information content of prices given in Proposition 3. Hence, the properties given by Proposition 2 and 3 should be viewed as equilibrium properties meant to balance the amount of information revealed by prices with the incentives for information gathering. However, before going into more detail regarding such intuition and its relationship with the Grossman-Stiglitz (1981) paradox, it is best to discuss how an entire equilibrium can be constructed. In particular, this will allow us to highlight the important role of noise traders in supporting the equilibrium but their very limited role in influencing the price process.

A complete description of the equilibrium requires us to specify the joint distribution of $\tilde{x}_i$, $\tilde{z}_i$ and $\tilde{p}_1$ conditional on all realizations of $\tilde{\theta}$ and $\tilde{k}$. As pointed out in Propositions 2 and 3, in equilibrium the most relevant aggregate features of the model are determined uniquely. However, this is not the case for the more detailed aspects of the model. In fact, the model does not uniquely determine the entire joint distribution of the endogenous variables (i.e. there is more than one way to support the outcomes described in Propositions 2 and 3). Hence, our objective in what follows will be to build one such joint distribution in a manner that both illustrates the equilibrium forces at play and indicates the extent to which other equilibrium joint distributions can be constructed. To this end, let us begin by assuming that the information gathering decision $\tilde{z}_i$ is simply an i.i.d random variable which generates a mass N of informed traders in all situations (N is therefore also the probability of $z_i = 1$). For now, let us assume only that $N < \bar{k}$. Given this distribution for $\tilde{z}_i$, it is rather trivial to derive the equilibrium distribution of $\tilde{x}_i$. First, conditional on becoming informed $z_i = 1$ (and conditional on being at an equilibrium price), the individual rationality constraints imply that x_i must be equal to 1 when $\tilde{\theta} = \theta_h$ and must equal 0 when $\tilde{\theta} = \theta_l$, since otherwise it would not be optimal to gather information. This distribution is given by (3.2).

$$\Pr(x_i = 1 \mid \theta, k, p_1) = \begin{cases} 1 & \text{if} \quad \theta = \theta_h \\ 0 & \text{if} \quad \theta = \theta_l. \end{cases} \tag{3.2}$$

The distribution of $\tilde{x}_i$ for the uninformed ($z_i = 0$) is also easy to derive since it directly follows from the market clearing conditions and the anonymity requirement which induces randomness in the allocation. In particular, the probability that $x_i = 1$ is given as follows for p_1 equal to either p^h or p^l:

$$\Pr(x_i = 1 \mid \theta, k, p_1) = \begin{cases} \frac{s^{-1}p_1 - N - \bar{k}}{1-N} & \text{if} \quad (\theta, k) = (\theta_h, \bar{k}) \\ \frac{s^{-1}p_1 - N}{1-N} & \text{if} \quad (\theta, k) = (\theta_h, 0) \\ \frac{s^{-1}p_1 - \bar{k}}{1-N} & \text{if} \quad (\theta, k) = (\theta_l, \bar{k}) \\ \frac{s^{-1}p_1}{1-N} & \text{if} \quad (\theta, k) = (\theta_l, 0). \end{cases} \tag{3.3}$$

The one object that remains to be constructed in order to complete the description of the equilibrium is the distribution of the price conditional on the realizations of both $\tilde{\theta}$ and $\tilde{k}$. Recall that we denote this conditional distribution by $\delta(p_1 \mid \theta, k)$, where $p_1 \in \left\{p^l, p^h\right\}$. There are two sets of equilibrium requirements that must be satisfied by this conditional distribution. First, it must generate an information content of prices that is consistent with that specified by Proposition 3. This gives rise to the two following conditions, which are simple and direct applications of Bayes' rule. In the following, we use the shorthand $\delta_{\theta,k}$ to denote $\delta(p^h \mid \theta, k)$.

$$\Pr\left(\theta_h \mid p^h\right) = \frac{\mu\left(\rho\delta_{\theta_h,\bar{k}} + (1-\rho)\delta_{\theta_h,0}\right)}{\mu\left(\rho\delta_{\theta_h,\bar{k}} + (1-\rho)\delta_{\theta_h,0}\right) + (1-\mu)\left(\rho\delta_{\theta_l,\bar{k}} + (1-\rho)\delta_{\theta_l,0}\right)}, \tag{3.4}$$

$$\Pr\left(\theta_h \mid p^l\right)$$
$$= \frac{\mu\left(\rho\left(1-\delta_{\theta_h,\bar{k}}\right) + (1-\rho)\left(1-\delta_{\theta_h,0}\right)\right)}{\mu\left(\rho\left(1-\delta_{\theta_h,\bar{k}}\right) + (1-\rho)\left(1-\delta_{\theta_h,0}\right)\right) + (1-\mu)\left(\rho\left(1-\delta_{\theta_l,\bar{k}}\right) + (1-\rho)\left(1-\delta_{\theta_l,0}\right)\right)}. \tag{3.5}$$

There are two other conditions that must be satisfied in equilibrium and these also restrict the conditional distribution of prices δ. These two conditions are the ones implied by the requirement that the realizations of $\tilde{x}_i$ be uninformative. [8] This requirement, which corresponds to $E[\tilde{\theta} \mid p_1] = E[\tilde{\theta} \mid p_1, x_i]$ or equivalently to $\Pr[\theta \mid p_1, x_i] = \Pr[\theta \mid p_1]$, can be written (using Bayes rule and the distribution for $\tilde{x}_i$ given in (3.3)) as follows.

$$\frac{N}{\bar{k}} = \frac{\delta_{\theta_l,\bar{k}}\,\rho}{\rho\delta_{\theta_l,\bar{k}} + (1-\rho)\delta_{\theta_l,0}} - \frac{\delta_{\theta_h,\bar{k}}\,\rho}{\rho\delta_{\theta_h,\bar{k}} + (1-\rho)\delta_{\theta_h,0}}, \tag{3.6}$$

[8] Note that the realizations of $\tilde{z}_i$ are uninformative by the fact the $\tilde{z}_i$ is i.i.d..

$$\frac{N}{\bar{k}} = \frac{\left(1-\delta_{\theta_l,\bar{k}}\right)\rho}{\rho\left(1-\delta_{\theta_l,\bar{k}}\right)+(1-\rho)\left(1-\delta_{\theta_l,0}\right)} - \frac{\left(1-\delta_{\theta_h,\bar{k}}\right)\rho}{\rho\left(1-\delta_{\theta_h,\bar{k}}\right)+(1-\rho)\left(1-\delta_{\theta_h,0}\right)}. \tag{3.7}$$

In the Appendix, we show that the δs that solve equations (3.4) to (3.7) are always between 0 and 1 and that these solutions are consistent with (3.2) and (3.3). Hence, the above construction provides an equilibrium joint distribution for $\tilde{z}_1$, $\tilde{x}_i$ and $\tilde{p}_1$ which, to summarize, is given by (1) an i.i.d distribution for $\tilde{z}_i$ where $\Pr(z_i = 1) \in (0, \bar{k})$, (2) a conditional distribution for $\tilde{x}_i$ (conditional on p_1, z_i, θ, k) given by (3.2) if $z_i = 1$ and given by (3.3) if $z_i = 0$, and (3) a conditional distribution of prices $\delta(p_1 \mid \theta, k)$ given by the solution to equations (3.4) to (3.7). This joint distribution satisfies the equilibrium requirement since by construction the market always clears, the allocations optimize the agents decision problem conditional on prices (since the indifferences ($\star$) and ($\star\star$) are satisfied), and the realizations of $\tilde{z}_i$ and $\tilde{x}_i$ by construction do not contain information regarding $\tilde{\theta}$ beyond that contained in the realization of $\tilde{p}_1$. As mentioned previously, this equilibrium is not unique since the distribution of $\tilde{z}_i$ is not uniquely pinned down. Nevertheless, the set of equilibria generates a unique joint distribution for the price, the aggregate level of investment and the fundamental $\tilde{\theta}$.

Given the above description of an equilibrium, it is now possible to discuss the micro-behavior that supports the market outcome described in Proposition 2 and Proposition 3. First there is the information acquisition decision. Since prices are such that agents are indifferent between whether or not to obtain information, they are satisfied with the randomness in z_i. Second, there is the outcomes for the uninformed investor. Just as in the benchmark case without information acquisition, uninformed investors need to move in and out of the market to accommodate the (unobserved) behavior of noise traders. However, now the uninformed investors must also take into account the (unobserved) aggregate behavior of informed individuals. In equilibrium, when the price is high, an uninformed individual does not know whether it is high because of a large demand by informed traders or a large demand by the uninformed. In effect, at the equilibrium price, the uninformed trader is indifferent between receiving or not the good; hence he is acceptant of the fact that he does not receive the good with probability 1. Moreover, upon realizing whether or not he is served by the market, he remains satisfied with the outcome since the realization has not revealed any additional information. In particular, not being served by the market is interpreted by an uninformed agent as reflecting either the presence of many noise traders or many informed individuals. Finally, informed agents are satisfied with the allocation of goods since they receive the good only when they strictly want it, which corresponds to the situations where $\tilde{\theta} = \theta_h$.

In order to further help comprehend the nature of the equilibrium, Figure 2 plots the demand for and supply of capital in period 1. Recall that in the presence of asymmetric information, demand itself depends on the actual realization of the market price, hence we have plotted investment demand conditional on the equilibrium price $\tilde{p}_1 = p^h$. The two figures correspond to the case where noise traders are in and out of the market, respectively.

The interesting feature is that the equilibrium price is insensitive to the specific behavior of the different individuals in the economy. The critical assumption behind this result is that individual investment decisions are discrete, which assures that informed investors remain "informationally small" and gives rise to an elastic demand. It is worth noting that the same would be true if investors were risk neutral and investment decisions bounded. In particular, with risk neutral investors, it is easy to see that there can be no private incentives to purchase information unless individual investors are "small". Investment demand would still be flat (as depicted in Figure 2), but a necessary condition for an equilibrium to exist is that risk neutral investors be prevented from taking over the market. In effect, with risk neutral investors, individual action spaces must be bounded. For otherwise, informed investors would take over the entire market when the state of demand is high, in which case they would affect prices, thus making all private information public and eliminating the private incentives to acquire information. This is of course the well-known paradox discussed by Grossman and Stiglitz (1980), which would arise in the present setting if allocations where neither bounded nor discrete.

However, in our setting the Grossman and Stiglitz non-existence problem does not arise due to the discreteness of the action space and due to the fact that we allow for equilibrium randomness in the price process. In particular, note that in the equilibrium of our model market prices do take into account the private incentives to purchase costly information. Mechanically, this can be seen by noting that the indifference condition ($\star$) for private information acquisition has been explicitly used to calculate market prices. This is in contrast with the standard *rational expectations equilibrium* model, as described in Grossman and Stiglitz (1980), where market prices are determined as a function of the mass of informed agents without reference to the value of private information. Ex ante, every agent takes the mapping from the measure of informed agents to the price as given and decides whether to acquire information. An equilibrium is then a fixed point where the measure of agents who purchase information gives raise to a price which justifies those information acquisition decisions in the first place. In the absence of noise traders, then, the competitive equilibrium breaks down because prices do not maintain the private incentives to acquire information and informed agents become "informationally large". In effect, an informed agent can see her own private information reflected in the prices and at the same time believe that she can have no effect on those prices. The present model escapes these two problems by the fact that, when there are discrete decisions, informed agents always remain informationally small and thereby the market can aggregate information imperfectly in the sense that prices remain noisy even as the importance of noise traders goes to zero (as will be shown below); hence, this resolves the fundamental conflict between information revelation and information acquisition.

The equilibrium behavior described by Propositions 2 and 3 provides instances which can be interpreted as herding behavior within a market context. It is in this sense that we believe our analysis provides a link between the herding literature and the rational expectations literature. In particular, in equilibrium, there are realizations where the price p_1 is high as a result of the mass behavior of uninformed individuals, that is, investment demand is high even though neither informed agents or noise traders are contributing to the demand. In such cases, the price is high because many uninformed agents are acquiring the investment good, and many uninformed agents are acquiring the good because they believe that the high price is a signal of high future demand (θ_h). This, to us, has the flavor of herding. Moreover, it should be emphasized that such an outcome is not a multiple-equilibrium type result. In particular, in a static sense, it is not an equilibrium in our model to have high demand just because other uninformed individuals have a high demand. It is only an equilibrium realization because it is part of a larger stochastic equilibrium in which individuals sometimes make errors. Furthermore, the occurrence of herds in our model, as given for example by $\Pr(p^h \mid \theta_l))$, is not indeterminate as would often be the case in situations of multiple equilibria. Instead, it is uniquely pinned down by equilibrium conditions as indicated in Proposition 3. In this sense, the endogenous equilibrium randomness in prices that arises in our model is somewhat similar in structure to that associated with mixed strategies in games.

At this point, certain natural questions arise. How does such equilibrium come about? Where does this equilibrium randomness come from? Our analysis in this section does not give direct answers to these questions, in the same sense that, our analysis in the previous section did not give an explicit explanation to how, at constant prices, uninformed agents moved in and out of the market to accommodate the needs of noise traders. As mentioned previously, our analysis throughout the paper is strictly in the equilibrium tradition as we characterize the properties of outcomes that appear stable in the sense defined by the equilibrium. This has the advantage that our results are not dependent on a particular and very precise description of an order of play (as would be the case with a game-theoretic analysis), and our analysis takes into account potential feedbacks between price and actions. However, it has the clear disadvantage of not offering an explicit non-cooperative description of the price determination process. We view these advantages and disadvantages as suggesting a need to pursue both avenues as means of understanding market behavior. Hence, we view the contribution of this paper as highlighting how certain insights about herding found in a sequential context may carry over to a simultaneous context if the action space is discrete (or bounded).

We now turn to examining some additional properties of the equilibria. As mentioned previously, the unique equilibrium outcome characterized in Propositions 2 and 3 can be supported in a set of ways. A natural question at this point is therefore whether the equilibrium outcome can be supported in a "sensible" way independent of the amount of noise imposed on the market. The next proposition provides an affirmative answer to this question.

Proposition 4 *For* $0 < c < \bar{c}$*, there always exists an equilibrium with the property that* $\delta\left(p^h \mid \theta_h, \bar{k}\right) = 1$ *and where , as* $\rho \to 0$*, the sequence of equilibria indexed by* ρ *converges* *to an equilibrium where* $\delta(\cdot \mid \theta_h, k)$ *dominates* $\delta(\cdot \mid \theta_l, k)$ *in the first-order stochastic sense, for* $k \in \{0, \bar{k}\}$*, and* $\delta(\cdot \mid \theta, \bar{k})$ *dominates* $\delta(\cdot \mid \theta, 0)$ *in the first-order stochastic sense, for* $\theta \in \{\theta_l, \theta_h\}$*.*

Proposition 4 identifies the existence of an equilibrium with interesting properties regarding the distribution δ even when noise trading is essentially eliminated from the model. In particular, the proposition shows that there always exists an equilibrium with the property that, whenever both informed investors and noise traders are in the market, the price is high (i.e., $\tilde{p}_1 = p^h$ with probability one); that is, there always exists an equilibrium where maximum demand always leads to the high price. Moreover, the proposition characterizes a sequence of equilibria indexed by ρ and its limit as ρ approaches zero. Recall that from Proposition 3 we know that as noise trading in the economy becomes negligible, the price system does not become perfectly informative since the $\Pr(\theta \mid p_1)$ is independent of ρ. The additional insight from Proposition 4 is that, as noise trading becomes negligible, the conditional distribution on prices δ can always be constructed to have the property that

$$\delta\left(p^h \mid \theta_h, \bar{k}\right) \geq \delta\left(p^h \mid \theta_l, \bar{k}\right) \geq \delta\left(p^h \mid \theta_l, 0\right), \tag{3.8}$$

$$\delta\left(p^h \mid \theta_h, \bar{k}\right) \geq \delta\left(p^h \mid \theta_h, 0\right) \geq \delta\left(p^h \mid \theta_l, 0\right). \tag{3.9}$$

In words, even when noise trading is small there always exists an equilibrium with the property that higher market prices signal higher levels of noise traders in the market for any given state of demand and higher future demand for any given level of noise traders in the market. It is also worth noting that this equilibrium has the property that $\delta\left(p^h \mid \theta_l, 0\right) > 0$ for $c \in (0, \bar{c})$ and $\rho \in (0, 1/2)$. Thus, it is possible that all investors participating in the market are uninformed even though the future demand is low. When this happens, aggregate investment is being entirely driven by the mass behavior of uninformed investors. This event has a distinct flavor of herding in that the "reason" why each of the uninformed is in the market is because the price is high. But it is precisely the behavior of the uninformed that pushes the price up!

Up to now, we have assumed that individual investment decisions are discrete and equal to either 0 or 1, which may seem a rather restrictive assumption. Therefore, we want to briefly discuss how our results extend to the case where individual investments are no longer restricted to being discrete but instead are assumed to be continuous and bounded, that is, the case with $x_i \in [0, b]$, where b is maximum level of investment for an individual. The interesting fact about this case is that our previous results carry over almost unchanged. In particular, to account for this modification, Propositions 2 and 3 need to be restated only to the extent of replacing the cost of information gathering c in the determination of prices and probabilities with the ratio of c/b. Moreover, it can be easily verified that one can construct an equilibrium to support the unique outcome described by such a modification of Propositions 2 and 3 (when c is replaced by c/b) using the same type of allocation rules we previously presented. The one minor change is that informed investors now receive an allocation b and therefore their aggregate demand is now given by Nb, instead of N, and uninformed individuals now receive either an allocation b or 0. The one insightful observation that is implied by this case relates to the limits as this bound b goes to 0 or infinity. For example, as the bound goes to infinity (which acts as if information gathering costs go to zero), the analogs to Proposition 2 and 3 would imply that the equilibrium tends to a fully revealing outcome. In contrast, when the bound declines towards 0, the equilibrium converges towards the non-revealing equilibrium described in the benchmark case. Hence, in an extended version of our model which allows for continuous but bounded decisions, we get the interesting result that the extent to which a market is informationally efficient depends precisely on the extent to which individual agents can take large positions in the market.

Allocative Errors

We now argue that the possibility of large allocative errors is inherent to the market equilibrium, and that the key property of the distribution of allocative errors is that, in order to achieve the balance between the private incentives to acquire information and its revelation, the price system trades off the probability of making an allocative error and its size. Recall, from Definition 2, that an allocative error takes place whenever there is a difference between the equilibrium level of aggregate investment and that associated with the first-best, which would arise in the full-information economy. [9]

From Proposition 3 and the definition of $p^f(\theta)$, the magnitudes of allocative errors are given by

$$s^{-1}p_1 - s^{-1}p^f(\theta)$$

$$= \begin{cases} -\left(\frac{1}{2} - \frac{1}{2}\sqrt{1-\frac{4c}{\theta_h-\theta_l}}\right)\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right) & \text{when } \theta=\theta_h \text{ and } p_1 = p^h \\ \left(\frac{1}{2} + \frac{1}{2}\sqrt{1-\frac{4c}{\theta_h-\theta_l}}\right)\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right) & \text{when } \theta=\theta_l \text{ and } p_1 = p^h \\ -\left(\frac{1}{2} + \frac{1}{2}\sqrt{1-\frac{4c}{\theta_h-\theta_l}}\right)\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right) & \text{when } \theta=\theta_h \text{ and } p_1 = p^l \\ \left(\frac{1}{2} - \frac{1}{2}\sqrt{1-\frac{4c}{\theta_h-\theta_l}}\right)\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right) & \text{when } \theta=\theta_l \text{ and } p_1 = p^l. \end{cases} \tag{3.10}$$

There are four types of errors corresponding to the four possible realizations of $\left(\tilde{\theta}, \tilde{p}_1\right)$. The nature of these errors is better understood in terms of the comparative statics of changes in the cost of information, for $c \in (0, \bar{c})$. Note first that the errors associated with $\left(\theta_h, p^h\right)$ and $\left(\theta_l, p^l\right)$ are both declining in size as the cost of information declines. Indeed, $s^{-1}p^h - s^{-1}p^f(\theta_h)$ and $s^{-1}p^l - s^{-1}p^f(\theta_l)$ are simply the natural errors associated with incomplete information, very much the analog of those errors found in the benchmark economy without information. What is interesting, however, is the presence of the other two errors, $s^{-1}p^h - s^{-1}p^f(\theta_l)$ and $s^{-1}p^l - s^{-1}p^f(\theta_h)$, and their dependence on the cost of information. As the cost of information becomes smaller, the size of each of these two errors increases in absolute value. Furthermore,

[9] Note that our main results would remain valid if the supply of capital were perfectly inelastic, in which case changes in prices would not induce changes in aggregate investment. Nevertheless, we wish to emphasize the implications of our model for understanding aggregate investment fluctuations.

their size is always larger in absolute value than the maximum error when information acquisition is not allowed (see equation (3.1) above).

Of course, one has to inquire about the probability distribution of the errors as well. The conditional probability of allocative errors given the state of demand $\tilde{\theta}$ is derived in the Appendix. Its main properties are illustrated in Figure 3, which shows the probability and the size of each of the four types of errors for a given $c \in (0, \bar{c})$. In addition, the figure shows the two errors in the benchmark economy in which no individual has access to information, where $e_l \equiv \mu \left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right)$ is the benchmark error which takes place when $\tilde{\theta} = \theta_l$ and $e_h \equiv -(1-\mu)\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right)$ is the benchmark error which takes place when $\tilde{\theta} = \theta_h$. Note that, the conditional probability of each of these two errors, given θ, is equal to 1. The arrows in the figure indicate how the size of each error moves as the cost of information declines below $\bar{c}$, where we use $e(p_1, \theta)$ to denote the allocation error $s^{-1}p_1 - s^{-1}p^f(\theta)$.

While the likelihood of large allocative errors, that is, $s^{-1}p^h - s^{-1}p^f(\theta_l)$ and $s^{-1}p^l - s^{-1}p^f(\theta_h)$, approaches zero as c becomes negligible, it remains positive so long as $c > 0$ and the size of each of these two errors increases as c declines, approaching the maximum feasible error $\left|\left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right)\right|$ as c vanishes. In Figure 3, $e_l^f = -e_h^f = \left(s^{-1}p^f(\theta_h) - s^{-1}p^f(\theta_l)\right)$ denote the maximum feasible errors associated with θ_l and θ_h. These results reflect the fact that, in order to achieve the balance between the private incentives to acquire information and the revelation of information, the price system trades off the probability of making an allocative error and its size. As the cost of information vanishes the probability of an error becomes negligible, but the magnitude of aggregate errors must increase so that private information has positive value.

4 Conclusion

In this paper, we have examined the equilibrium determination of prices and aggregate investment in a market where information is costly to acquire and individual decisions are discrete. We have shown why, in such a setting, non-fundamental randomness in the market price (and thereby in the aggregate level of investment) necessarily arises as an equilibrium phenomenon.

In particular, we have shown that this randomness acts as a balancing mechanism that allows prices to convey information while maintaining the private incentives to acquire information. Moreover, we have argued that this phenomenon is conceptually akin to herding type behavior which has previously been shown to arise in environments with sequential interactions and discrete decisions. Hence, we view our analysis as providing a bridge between the herding literature and the rational expectations literature; with our bridge indicating that herding type behavior in markets may intrinsically be linked to discreteness or boundedness of individual level decisions.

A Appendix

Proof of Proposition 1

Our assumptions on $p^f(\theta)$ and $\bar{k}$ guarantee that it is not profitable for all agents to invest and there is profit to be made by investing if no one else does. For a symmetric, interior solution, then, individuals must be indifferent between investing and staying out of the market. This, together with market clearing (see equation (2.5) in the main text) implies the indifference condition

$$\mu\left(\theta_h - \frac{s+d}{s}p_1\right) + (1-\mu)\left(\theta_l - \frac{s+d}{s}p_1\right) = 0.$$

Using the definition of $p^f(\theta)$, it follows that $p_1 = p^n$ is the unique price that makes investors indifferent. Then, $\delta(p^n \mid k) = 1$, for $k \in \{0, \bar{k}\}$, since supp[δ] is a singleton. In addition, market clearing requires the adding-up condition $p^n = s\left(k + \Pr(x_i = 1 \mid \theta, k, p^n)\right)$ to hold for $k \in \{0, \bar{k}\}$ and $\theta \in \{\theta_l, \theta_h\}$. ■

Proof of Proposition 2 and Proposition 3

We begin by identifying necessary and sufficient conditions for existence of an equilibrium. To this end, let us assume momentarily that, conditional on p_1, $\tilde{z}_i$ is an i.i.d. random variable which generates a mass $N(p_1)$ of informed agents. This assumption will be relaxed below. Lemma 1 and Lemma 2 characterize necessary equilibrium conditions whenever either $N(p_1) > 0$ for all $p_1 \in$ supp[δ] or $N(p_1) = 0$ for all $p_1 \in$ supp[δ].

Lemma 1 *If $N(p_1) > 0$ for all $p_1 \in supp[\delta]$, then $supp[\delta] = \left\{p^l, p^h\right\}$ and*

$$p_1 = \Pr(\theta_h \mid p_1)\, p^f(\theta_h) + \left(1 - \Pr(\theta_h \mid p_1)\right) p^f(\theta_l), \quad \textit{for } p_1 \in \left\{p^l, p^h\right\}$$

where

$$\Pr\left(\theta_h \mid p^h\right) = \left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}}\right) \quad \textit{and} \quad \Pr\left(\theta_h \mid p^l\right) = \left(\frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4c}{\theta_h - \theta_l}}\right).$$

Proof: Note first that $N(p_1) < 1$, for otherwise p_1 would reveal $\tilde{\theta}$ in which case purchasing information cannot be optimal. Then suppose that $N(p_1) \in (0,1)$ for all $p_1 \in \text{supp}[\delta]$. In this case it must be that all individuals are indifferent between purchasing information and remaining uninformed. For information to have value, it must also be the case that the informed invest if and only if they learn that $\tilde{\theta} = \theta_h$. Therefore, Condition **(b)** in Definition 3 implies that

$$\Pr(\theta_h \mid p_1)\left(\theta_h - \frac{s+d}{s} p_1\right) = c. \tag{$\star$}$$

In addition, uninformed agents must be indifferent between investing and not investing at each equilibrium price, for otherwise the price would reveal the behavior of the informed and thus the value of $\tilde{\theta}$, in which case information would have no value. Thus, Condition **(a)** in Definition 3 implies that

$$\Pr(\theta_h \mid p_1)\left(\theta_h - \frac{s+d}{s} p_1\right) + \left(1 - \Pr(\theta_h \mid p_1)\right)\left(\theta_l - \frac{s+d}{s} p_1\right) = 0. \tag{$\star\star$}$$

It is then straightforward to verify, using the definition of $p^f(\theta)$, that p^l, p^h, $\Pr\left(\theta_h \mid p^h\right)$ and $\Pr\left(\theta_h \mid p^l\right)$ are the unique solution to the system of equations formed by equations ($\star$) and ($\star\star$). ■

Lemma 2 *If $N(p_1) = 0$ for all $p_1 \in$ supp$[\delta]$, then supp$[\delta] = p^n = \mu\, p^f(\theta_h) + (1-\mu)\, p^f(\theta_l)$. Furthermore, p^n can be supported as part of an equilibrium if and only if $c \geq \bar{c} = \mu(1 - \mu)(\theta_h - \theta_l)$. This equilibrium replicates that characterized in Proposition 1.*

Proof: Suppose that $N(p_1) = 0$ for all $p_1 \in \text{supp}[\delta]$. That $\text{supp}[\delta] = p^n$ follows from the argument given in Proposition 1. In addition, for no one to desire to purchase information, Condition **(b)** in Definition 3 requires that

$$\mu\left(\theta_h - \frac{s+d}{s} p^n\right) \leq c.$$

Substitution for p^n into this inequality reveals that remaining uninformed is in fact optimal if and only if $c \geq \bar{c}$. An equilibrium can then be constructed as shown in Proposition 1. ■

Next, under the assumption that $\tilde{z}_i$ is i.i.d. we show that it is necessarily the case that $N(p_1) > 0$ for all $p_1 \in$ supp$[\delta]$ if $c < \bar{c}$, and that $N(p_1) = 0$ for all $p_1 \in$ supp$[\delta]$ if $c \geq \bar{c}$. To that end, the following two lemmas are useful.

Lemma 3 *For* $p_1 \in \left\{p^l, p^h\right\}$, $\Pr(\theta_h \mid p_1)(1 - \Pr(\theta_h \mid p_1)) < \mu(1-\mu)$ *if and only if* $c < \bar{c}$.

Proof: This lemma follows immediately from Lemma 1 once one notes that, for $p_1 \in \left\{p^l, p^h\right\}$, $\Pr(\theta_h \mid p_1)(1 - \Pr(\theta_h \mid p_1)) = c/(\theta_h - \theta_l)$. ■

Lemma 4 *For each* $p_1 \in$ *supp*$[\delta]$, $N(p_1) > 0$ *if and only if* $\Pr(\theta_h \mid p_1)(1 - \Pr(\theta_h \mid p_1)) < \mu(1-\mu)$.

Proof: To prove sufficiency, suppose that $N(p_1) = 0$ and simply note that Condition **(c)** in Definition 4 implies that $\Pr(\theta_h \mid p_1) = \mu$. To prove necessity, suppose first that $N(p_1) > 0$ for $p_1 = p^h$. Using Bayes' rule,

$$\Pr\left(\theta_h \mid p^h\right) = \frac{\mu \Pr\left(p^h \mid \theta_h\right)}{\mu \Pr(p^h \mid \theta_h) + (1-\mu)\Pr(p^h \mid \theta_l)} > \frac{1}{2}, \tag{A.1}$$

and straightforward manipulation then shows that for (A.1) to hold with $\Pr\left(p^h \mid \theta_l\right) \geq 0$ it must be that $\Pr\left(p^h \mid \theta_h\right) \geq \Pr\left(p^h \mid \theta_l\right)$, with equality if and only if $\Pr\left(p^h \mid \theta_h\right) = 0$. For $\Pr\left(p^h\right) > 0$, the inequality must therefore be strict. Next, using Bayes' rule,

$$\Pr\left(\theta_h \mid p^h\right)\left(1 - \Pr\left(\theta_h \mid p^h\right)\right) = \mu(1-\mu)\left(\frac{\Pr\left(p^h \mid \theta_h\right)\Pr\left(p^h \mid \theta_l\right)}{\left(\mu \Pr(p^h \mid \theta_h) + (1-\mu)\Pr(p^h \mid \theta_l)\right)^2}\right), \tag{A.2}$$

and, to derive a contradiction, note that $\Pr\left(\theta_h \mid p^h\right)\left(1 - \Pr\left(\theta_h \mid p^h\right)\right) \geq \mu(1-\mu)$ if and only if the term in parentheses on the right-hand side of equation (A.2) is larger than 1. Straightforward manipulation shows that this can only happen if $\Pr\left(p^h \mid \theta_h\right) \leq \Pr\left(p^h \mid \theta_l\right)$, which contradicts the fact that $\Pr\left(\theta_h \mid p^h\right) > 1/2$. Therefore, if $N\left(p^h\right) > 0$, then $\Pr\left(\theta_h \mid p^h\right)\left(1 - \Pr\left(\theta_h \mid p^h\right)\right) < \mu(1-\mu)$. But from Lemma 1, $\Pr\left(\theta_h \mid p^l\right) = 1 - \Pr\left(\theta_h \mid p^h\right)$. Hence the same must be true at $p_1 = p^l$. This concludes the proof since p^l and p^h exhaust the list of possible prices in supp$[\delta]$ when $N(p_1) > 0$. ■

Lemma 3 and Lemma 4 together imply that an equilibrium such that $N(p_1) > 0$ for all $p_1 \in \text{supp}[\delta]$ may exist only if $c < \bar{c}$. Next, note that $\text{supp}[\delta] = \{p_1, p^n\}$ with $p_1 \neq p^n$ cannot be part of an equilibrium, since the fact that p^n is uninformative implies that p_1 must also be uninformative. It remains to verify that $\text{supp}[\delta] = \left\{p^l, p^h, p^n\right\}$ cannot be part of an equilibrium. But this also follows from Lemma 3 and Lemma 4. Furthermore, Lemma 2 does describe an equilibrium whenever the inequality stated therein is satisfied, which happens if and only if $c \geq \bar{c}$.

Our so far maintained assumption that $\tilde{z}_i$ is i.i.d. implies that $N(p_1)$ is common knowledge. We relax this assumption now. In this case, $N(p_1)$ may be random conditional on p_1. For that to be the case, note that it must be that all realizations of $N(p_1)$ at each p_1 are strictly less than 1, and that ex ante individual rationality (i.e., Conditions **(a)** and **(b)** in Definition 3) still requires that Conditions ($\star$) and ($\star\star$) above must hold at p_1; and, in particular, Lemma 3 and Lemma 4 apply. Putting our previous results together, we have shown that (1) if $c \geq \bar{c}$, then the equilibrium characterized in Proposition 2 is the unique equilibrium, and (2) if $0 < c < \bar{c}$, then $\text{supp}[\delta] = \left\{p^l, p^h\right\}$, and p^l, p^h, $\Pr\left(\theta_h \mid p^l\right)$ and $\Pr\left(\theta_h \mid p^h\right)$ are uniquely determined by Conditions ($\star$) and ($\star\star$) above. It is now immediate to prove Proposition 3.

Proof of Proposition 3

Lemma 1 gives $\Pr\left(\theta_h \mid p^h\right)$ and $\Pr\left(\theta_h \mid p^l\right)$ and Lemma 3 and Lemma 4 imply that $\Pr\left(\theta_h \mid p^h\right) > \mu > \Pr\left(\theta_h \mid p^l\right)$. It only remains to be shown that these values are in fact probabilities. Simple manipulations show that a necessary and sufficient condition for this to be the case is that $c < \bar{c}$. This concludes the proof of Proposition 3. ■

Next, to construct an equilibrium we will show that we can support the unique equilibrium outcome given above by p^l, p^h, $\Pr\left(\theta_h \mid p^l\right)$ and $\Pr\left(\theta_h \mid p^h\right)$. To that end, we will restrict our search to the set of equilibria which satisfy the following two conditions, in addition to all other equilibrium conditions: (1) $\tilde{z}_i$ is an i.i.d. random variable conditional on p_1, and (2) the randomization $\tilde{x}_i$ is uninformative. The first restriction implies that information acquisition decisions generate a mass $N(p_1)$ of informed traders. Therefore, $N(p_1)$ is not random conditional on p_1. This immediately implies that the ex post individual rationality condition **(b.1)** in Definition

3 is satisfied. The second extra restriction amounts to requiring that $\Pr[\theta \mid p_1, x_i] = \Pr[\theta \mid p_1]$. Using the probability distribution characterized by equation (3.3) in the main text, this condition can be written as $N/\bar{k} = \Pr[\bar{k} \mid p_1, \theta_l] - \Pr[\bar{k} \mid p_1, \theta_h]$, for $p_1 \in \left\{p^l, p^h\right\}$, which using Bayes' rule delivers equations (3.6) and (3.7) in the main text.

Therefore, as explained in the main text, equations (3.4)–(3.7) must also be satisfied in equilibrium. Equations (3.4)–(3.5) apply Bayes' rule to the conditional probabilities derived in Lemma 2, both at $p_1 = p^l$ and $p_1 = p^h$. Equations (3.6) and (3.7) are simply the necessary Condition **(a.1)** in Definition 3 at $p_1 = p^l$ and $p_1 = p^h$, respectively, where we have used Bayes' rule to express them in terms of δ. Together, they form a system of 4 equations into 6 unknowns: $N\left(p^l\right)$, $N\left(p^h\right)$, $\delta\left(p^h \mid \theta_h, \bar{k}\right)$, $\delta\left(p^h \mid \theta_h, 0\right)$, $\delta\left(p^h \mid \theta_l, \bar{k}\right)$, $\delta\left(p^h \mid \theta_l, 0\right)$. Furthermore, note that δ must also be consistent with the adding-up conditions associated with equation (3.3) in the main text, in that the following statements must hold true in equilibrium: for each $p_1 \in \left\{p^l, p^h\right\}$,

(1) if $\delta\left(p_1 \mid \theta_h, \bar{k}\right) > 0$, then $s^{-1}p_1 > N\left(p_1\right) + \bar{k}$,

(2) if $\delta\left(p_1 \mid \theta_h, 0\right) > 0$, then $s^{-1}p_1 > N\left(p_1\right)$ and $1 - s^{-1}p_1 > \bar{k}$,

(3) if $\delta\left(p_1 \mid \theta_l, \bar{k}\right) > 0$, then $s^{-1}p_1 > \bar{k}$ and $1 - s^{-1}p_1 > N\left(p_1\right)$,

(4) if $\delta\left(p_1 \mid \theta_l, 0\right) > 0$, then $1 - s^{-1}p_1 > N\left(p_1\right) + \bar{k}$,

(5) if $\delta\left(p_1 \mid \theta_h, \bar{k}\right) < 1$, then $1 - s^{-1}p_1 > \min\left\{N\left(p_1\right), \bar{k}\right\}$,

(6) if $\delta\left(p_1 \mid \theta_h, 0\right) < 1$, then $1 - s^{-1}p_1 > N\left(p_1\right)$ or $s^{-1}p_1 > \bar{k}$,

(7) if $\delta\left(p_1 \mid \theta_l, \bar{k}\right) < 1$, then $1 - s^{-1}p_1 > \bar{k}$ or $s^{-1}p_1 > N\left(p_1\right)$,

(8) if $\delta\left(p_1 \mid \theta_l, 0\right) < 1$, then $s^{-1}p_1 > \min\left\{N\left(p_1\right), \bar{k}\right\}$.

Verifying that there exists an equilibrium then amounts to showing that there exists a solution to the system given by (3.4)–(3.7) such that $\delta_{\theta,k} \in [0, 1]$, for all θ and k, and $N\left(p_1\right)/\rho\bar{k} > 0$ for $p_1 \in \left\{p^l, p^h\right\}$. Any such solution will immediately satisfy the previous list of conditional statements since it must be such that $N\left(p_1\right) < \rho\bar{k} < (1+\rho)\bar{k} < \min\left\{s^{-1}p_1, 1 - s^{-1}p_1\right\}$, for $p_1 \in \left\{p^l, p^h\right\}$ and $\rho \in (0, 1)$, where the first inequality follows from inspection of (3.6) and (3.7) and the last one follows once one notes that $p^f\left(\theta_l\right) < p^l < p^h < p^f\left(\theta_h\right)$ for any $c \in (0, \bar{c})$, since we have assumed that $0 < (1+\rho)\,\bar{k} < \min\left\{s^{-1}p^f\left(\theta_l\right), 1 - s^{-1}p^f\left(\theta_h\right)\right\}$.

Lemma 5 *Suppose that* $0 < c < \bar{c}$. *Then,*

$$\Pr\left(p^h\right) = \frac{\Pr\left(\theta_h \mid p^h\right) + \mu - 1}{2\Pr\left(\theta_h \mid p^h\right) - 1} = 1 - \Pr\left(p^l\right),$$

where $\Pr\left(\theta_h \mid p^h\right)$ *is given by Lemma 1.*

Proof: Noting that the denominator of equation (3.4) is equal to $\Pr\left(p^h\right)$, whereas the denominator of (3.5) is equal to $\Pr\left(p^l\right)$, one can write (3.4) and (3.5) as

$$\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right) = \mu\left(\rho\delta_{\theta_h,\bar{k}} + (1-\rho)\delta_{\theta_h,0}\right)$$

$$\Pr\left(p^l\right)\Pr\left(\theta_h \mid p^l\right) = \mu - \mu\left(\rho\delta_{\theta_h,\bar{k}} + (1-\rho)\delta_{\theta_h,0}\right).$$

To verify the proposition, use these two equations to solve for $\Pr\left(p^h\right)$ as a function of $\Pr\left(\theta_h \mid p^h\right)$ and μ, noting that $\Pr\left(p^h\right) + \Pr\left(p^l\right) = 1$ and the symmetry result $\Pr\left(\theta_h \mid p^l\right) = 1 - \Pr\left(\theta_h \mid p^h\right)$. ■

Lemma 6 *The system of equations (3.4)–(3.7) is equivalent to*

$$\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu} = \rho\delta_{\theta_h,\bar{k}} + (1-\rho)\delta_{\theta_h,0} \tag{A.3}$$

$$\frac{\Pr\left(p^h\right)\left(1 - \Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu} = \rho\delta_{\theta_l,\bar{k}} + (1-\rho)\delta_{\theta_l,0} \tag{A.4}$$

$$\frac{N\left(p^h\right)}{\rho\bar{k}} = \left(\frac{(1-\mu)\delta_{\theta_l,\bar{k}}}{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}\right) - \left(\frac{\mu\delta_{\theta_h,\bar{k}}}{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}\right) \tag{A.5}$$

$$\frac{N\left(p^l\right)}{\rho\bar{k}} = \left(\frac{(1-\mu)\left(1-\delta_{\theta_l,\bar{k}}\right)}{(1-\mu) - \Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}\right) - \left(\frac{\mu\left(1-\delta_{\theta_h,\bar{k}}\right)}{\mu - \Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}\right). \tag{A.6}$$

Proof: Equation (A.3) follows immediately from equation (3.4) by noting that its denominator is equal to $\Pr\left(p^h\right)$. To derive equation (A.4), use (3.4) again to write $1-\Pr\left(\theta_h \mid p^h\right)$. Equations

(A.5) and (A.6) follow from using the (A.3) and (A.4) to write equations (3.6) and (3.7) as a linear system into the unknowns $\delta_{\theta_h,\bar{k}}$ and $\delta_{\theta_l,\bar{k}}$. ■

Existence of a solution to the system of linear equations (A.3)–(A.6) is not an issue. The system has multiple solutions. However, showing that there exists an equilibrium still requires that there is a solution to (A.3)–(A.6) which is consistent with an equilibrium. This is done in Lemma 7.

Lemma 7 *There exist $\delta_{\theta,k} \in [0,1]$, for all θ and k, and $N(p_1)/\rho\bar{k} > 0$ for $p_1 \in \left\{p^l, p^h\right\}$ which solve (A.3)–(A.6).*

Proof: Note first that it follows from (A.3) and (A.4) that $\delta_{\theta_h,0} \in [0,1]$ if and only if

$$\delta_{\theta_h,\bar{k}} \leq \frac{1}{\rho}\left(\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right) \tag{A.7}$$

and

$$1-\delta_{\theta_h,\bar{k}} \leq \frac{1}{\rho}\left(1-\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right) \tag{A.8}$$

and $\delta_{\theta_l,0} \in [0,1]$ if and only if

$$\delta_{\theta_l,\bar{k}} \leq \frac{1}{\rho}\left(\frac{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu}\right) \tag{A.9}$$

and

$$1-\delta_{\theta_l,\bar{k}} \leq \frac{1}{\rho}\left(1-\frac{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu}\right). \tag{A.10}$$

Letting

$$\delta_{\theta_l,\bar{k}} = \frac{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu} - (1-\rho)\alpha, \tag{A.11}$$

with

$$\alpha \in S = \left(0, \min\left\{ \left(\frac{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu}\right)\min\left\{\frac{1}{\rho}, \frac{1}{1-\rho}\right\}, \right.\right.$$
$$\left.\left. \left(1-\frac{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu}\right)\frac{1}{\rho}\right\}\right) \tag{A.12}$$

and noting that $\left(\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)/(1-\mu)\right) \in (0,1)$ for any $c \in (0,\bar{c})$, one can easily verify from (A.9)–(A.11) that, for any $\alpha \in S$, $\delta_{\theta_l,\bar{k}} \in (0,1)$ and $\delta_{\theta_l,0} \in [0,1]$ for $\rho \in (0,1)$ and $c \in (0,\bar{c})$. Next, substituting for $\delta_{\theta_l,\bar{k}}$ from (A.11) one can rewrite (A.5) and (A.6) as

$$\delta_{\theta_h,\bar{k}} = \left(\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right)$$
$$\times \left(1-(1-\rho)\alpha\left(\frac{1-\mu}{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}\right)-\frac{N\left(p^h\right)}{\rho\bar{k}}\right) \tag{A.13}$$

$$1-\delta_{\theta_h,\bar{k}} = \left(1-\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right)$$
$$\times \left(1-(1-\rho)\alpha\left(\frac{1-\mu}{(1-\mu)-\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}\right)-\frac{N\left(p^l\right)}{\rho\bar{k}}\right). \tag{A.14}$$

It follows from (A.7) and (A.8) that $\delta_{\theta_h,0} \in [0,1]$ so long as $N\left(p^h\right) > 0$ and $N\left(p^l\right) > 0$. Further, any $\delta_{\theta_h,\bar{k}}$ that solves (A.13) and (A.14) belongs to $[0,1]$ if

$$0 < \frac{N\left(p^h\right)}{\rho\bar{k}} < 1-(1-\rho)\alpha\left(\frac{1-\mu}{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}\right) \tag{A.15}$$

and

$$0 < \frac{N\left(p^l\right)}{\rho\bar{k}} < 1-(1-\rho)\alpha\left(\frac{1-\mu}{(1-\mu)-\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}\right). \tag{A.16}$$

Finally, for any value of $\delta_{\theta_h,\bar{k}}$ that solves (A.13) and (A.14) it must be the case that

$$\left(\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right)\frac{N\left(p^h\right)}{\rho\bar{k}}+\left(1-\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right)\frac{N\left(p^l\right)}{\rho\bar{k}}$$

$$= \left(\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right)(1-\rho)\,\alpha\left(\frac{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu}\right)^{-1}$$

$$+ \left(1-\frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)}{\mu}\right)(1-\rho)\,\alpha\left(1-\frac{\Pr\left(p^h\right)\left(1-\Pr\left(\theta_h \mid p^h\right)\right)}{1-\mu}\right)^{-1}. \quad \text{(A.17)}$$

It is straightforward to verify that, for any $\rho \in (0,1)$ and $c \in (0,\bar{c})$, one can always choose a value for $\alpha \in S$, $\left(N\left(p^h\right)/\rho\bar{k}\right) \in (0,1)$ and $\left(N\left(p^l\right)/\rho\bar{k}\right) \in (0,1)$ so that equations (A.15), (A.16) and (A.17) are satisfied. ■

This concludes the proof of Proposition 2. ■

Proof of Proposition 4

First note that the following is a solution to the system of equations (3.4)–(3.7), which can be readily verified by substitution into the equivalent system given by Lemma 6.

$$\delta_{\theta_h,\bar{k}} = 1,$$

$$\delta_{\theta_h,0} = \frac{\Pr\left(p^h\right)\Pr\left(\theta_h \mid p^h\right)-\mu\rho}{\mu(1-\rho)},$$

$$\delta_{\theta_l,\bar{k}} = \left(\frac{1-\Pr\left(\theta_h \mid p^h\right)}{\Pr\left(\theta_h \mid p^h\right)(1-\mu)^2}\right)\left(\Pr\left(p^h\right)\left(\Pr\left(\theta_h \mid p^h\right)-\mu\right)+\mu(1-\mu)\right),$$

$$\delta_{\theta_l,0} = \left(\frac{1-\Pr\left(\theta_h \mid p^h\right)}{(1-\mu)(1-\rho)}\right)\left(\Pr\left(p^h\right)-\rho\left(\frac{\Pr\left(p^h\right)\left(\Pr\left(\theta_h \mid p^h\right)-\mu\right)+\mu(1-\mu)}{\Pr\left(\theta_h \mid p^h\right)(1-\mu)}\right)\right),$$

$$\frac{N\left(p^h\right)}{\rho\bar{k}} = \frac{N\left(p^l\right)}{\rho\bar{k}} = \frac{\Pr\left(\theta_h \mid p^h\right)-\mu}{\Pr\left(\theta_h \mid p^h\right)(1-\mu)}.$$

To see that this is the unique solution to (3.4)–(3.7) with the property that $\delta_{\theta_h,\bar{k}} = 1$ and $N\left(p^h\right)/\rho\bar{k} = N\left(p^l\right)/\rho\bar{k}$, note that (A.3)–(A.6) together with these two restrictions form a linear system of 6 independent equations into 6 unknowns. Straightforward but tedious manipulation then shows that $0 < \rho \leq 1/2$ is a sufficient condition for $N\left(p^h\right)/\rho\bar{k} = N\left(p^h\right)/\rho\bar{k} \in (0,1)$ and $\delta_{\theta,k} \in [0,1]$ for all $\theta \in \{\theta_l, \theta_h\}$, $k \in \{0, \bar{k}\}$ and $c \in (0, \bar{c})$.

It is also straightforward to take limits as ρ approaches zero and to verify that the limiting equilibrium satisfies

$$\delta\left(p^h \mid \theta_h, k\right)\delta\left(p^l \mid \theta_l, k\right) - \delta\left(p^h \mid \theta_l, k\right)\delta\left(p^l \mid \theta_h, k\right) \geq 0, \qquad \text{for } k \in \{0, \bar{k}\} \tag{A.18}$$

$$\delta\left(p^h \mid \theta, \bar{k}\right)\delta\left(p^l \mid \theta, 0\right) - \delta\left(p^h \mid \theta, 0\right)\delta\left(p^l \mid \theta, \bar{k}\right) \geq 0, \qquad \text{for } \theta \in \{\theta_l, \theta_h\}. \tag{A.19}$$

It is well known (see, for instance, Milgrom (1981)) that (A.18) is necessary and sufficient for $\delta(\cdot \mid \theta_h, k)$ to dominate $\delta(\cdot \mid \theta_l, k)$ in the first-order stochastic sense, for $k \in \{0, \bar{k}\}$, and (A.19) is necessary and sufficient for $\delta(\cdot \mid \theta, \bar{k})$ to dominate $\delta(\cdot \mid \theta, 0)$ in the first-order stochastic sense, for $\theta \in \{\theta_l, \theta_h\}$. ■

The Distribution of Allocative Errors

The error size for each event (θ, p_1) is given by equation (3.10) in the main text and it follows immediately from Proposition 2 and the definition of $p^f(\theta)$. To find the conditional probability of an error given θ, simply note that

$$\Pr(p_1 \mid \theta) = \frac{\Pr(p_1)\Pr(\theta \mid p_1)}{\Pr(\theta)},$$

and substitute for $\Pr(p_1)$, from Lemma 5, to obtain

$$\Pr\left(p^h \mid \theta_h\right) = \left(\frac{\mu - \left(1 - \Pr\left(\theta_h \mid p^h\right)\right)}{\Pr(\theta_h \mid p^h) - \left(1 - \Pr(\theta_h \mid p^h)\right)}\right)\left(\frac{\Pr\left(\theta_h \mid p^h\right)}{\mu}\right)$$

and

$$\Pr\left(p^h \mid \theta_l\right) = \left(\frac{\mu - \left(1 - \Pr\left(\theta_h \mid p^h\right)\right)}{\Pr\left(\theta_h \mid p^h\right) - \left(1 - \Pr\left(\theta_h \mid p^h\right)\right)}\right)\left(\frac{1 - \Pr\left(\theta_h \mid p^h\right)}{1 - \mu}\right),$$

with $\Pr\left(p^l \mid \theta_h\right) = 1 - \Pr\left(p^h \mid \theta_h\right)$ and $\Pr\left(p^l \mid \theta_l\right) = 1 - \Pr\left(p^h \mid \theta_l\right)$.

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Figure 1

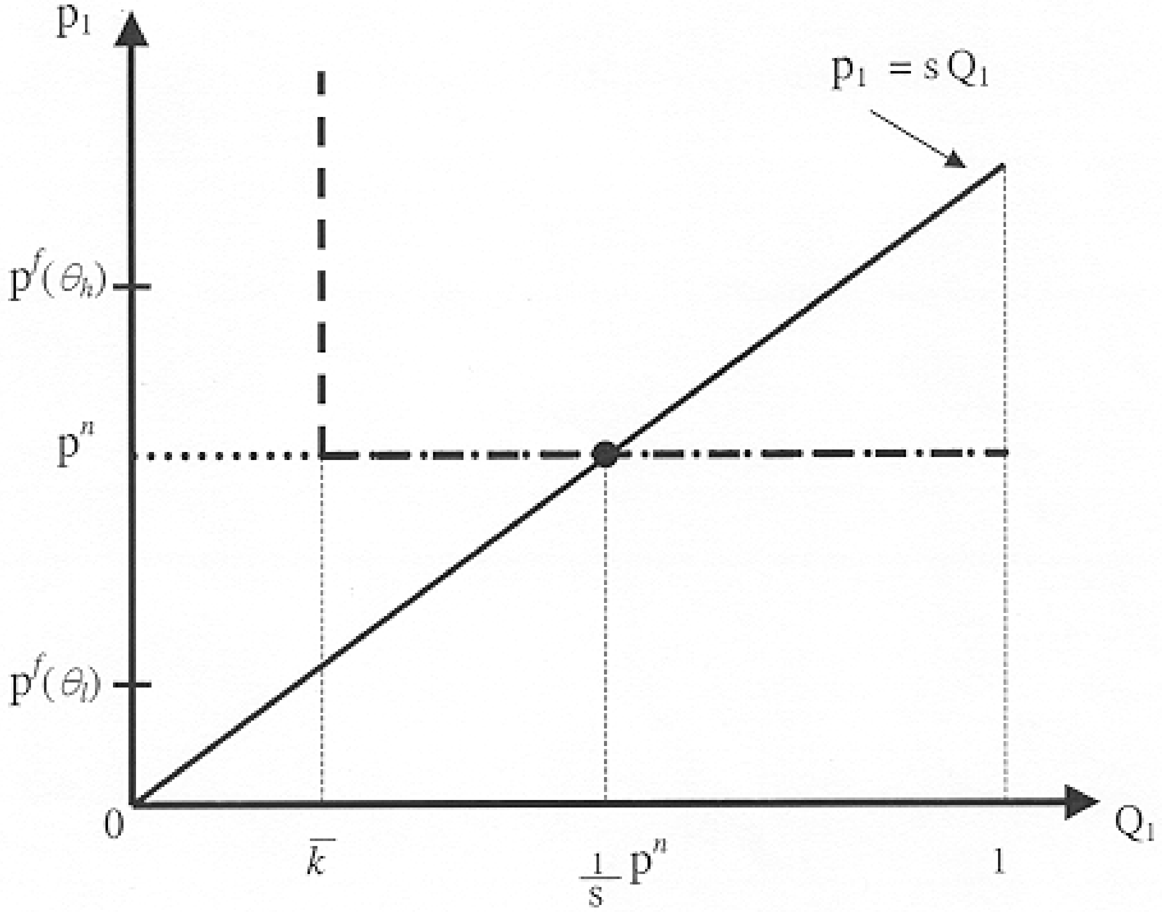
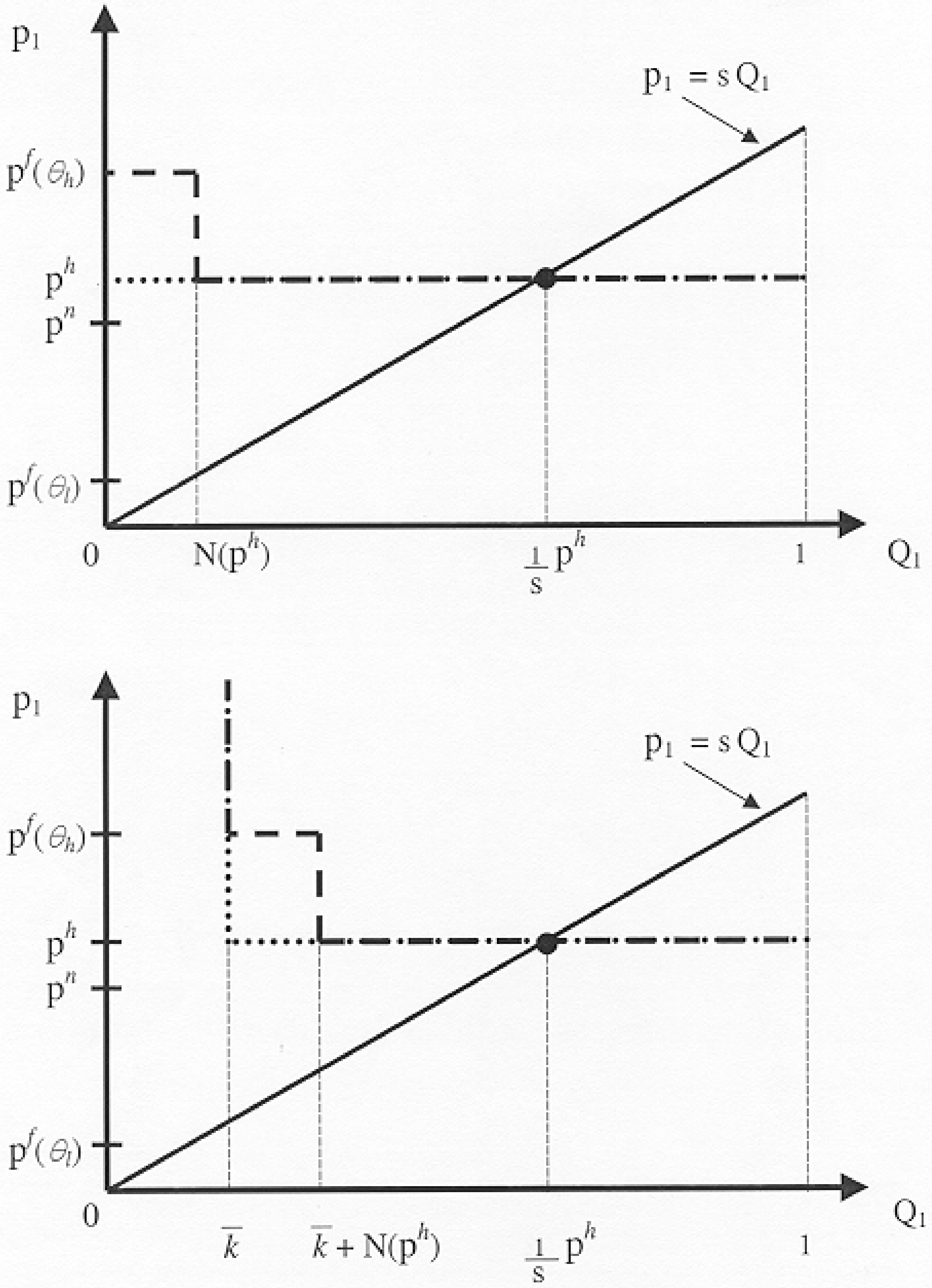
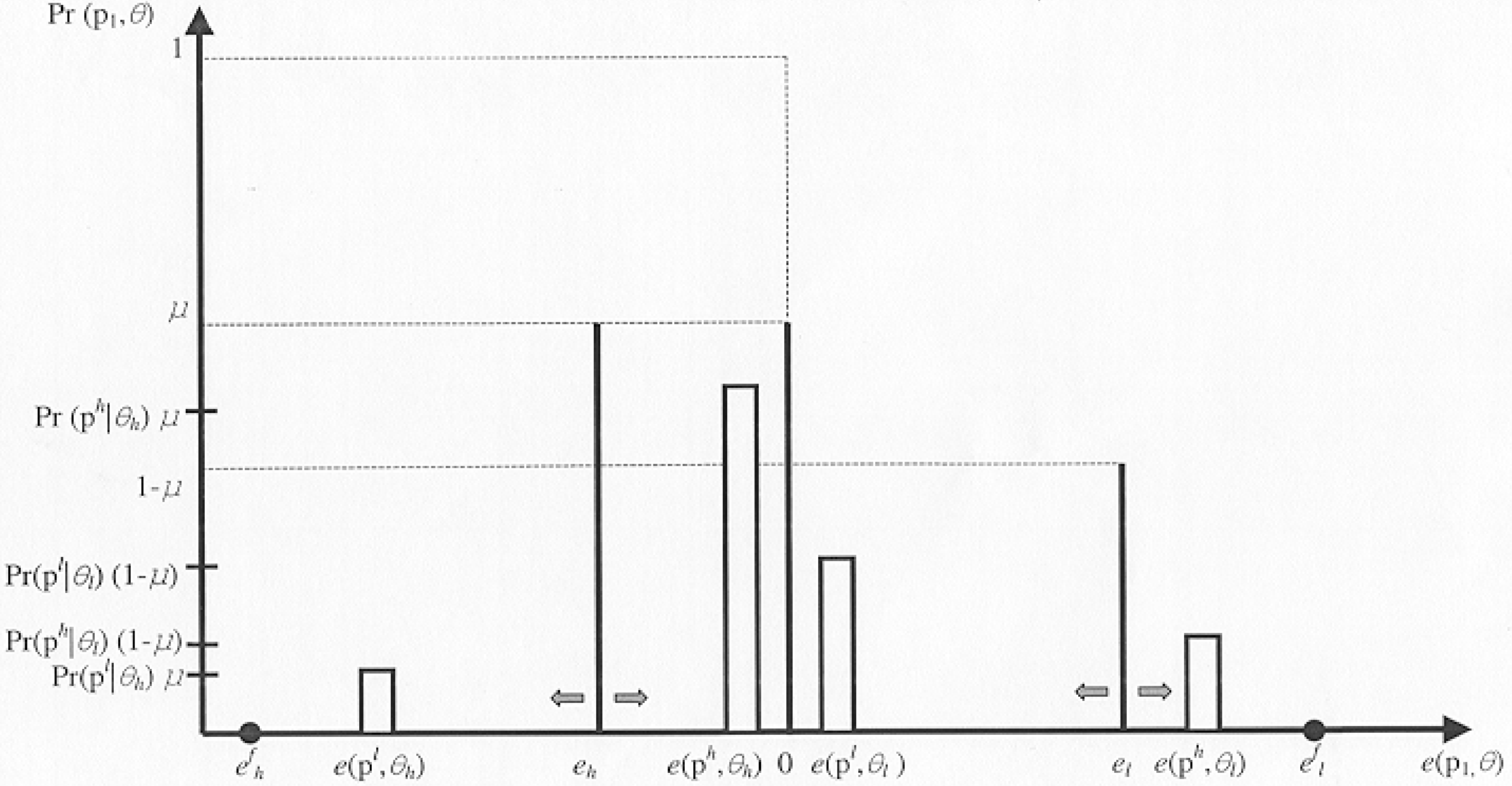


Figure 2



p1
p1 = s Q1
p^f(θh)
p^h
p^n
p^f(θl)
0
N(p^h)
(1/s) p^h
1
Q1
p1
p1 = s Q1
p^f(θh)
p^h
p^n
p^f(θl)
0
k̄
k̄ + N(p^h)
(1/s) p^h
1
Q1

Figure 3


Pr (p1, θ)
1
μ
Pr (p^h|θh) μ
1-μ
Pr(p^l|θl) (1-μ)
Pr(p^h|θl) (1-μ)
Pr(p^l|θh) μ
e^f_h
e(p^l, θh)
e_h
e(p^h, θh)
0
e(p^l, θl)
e_l
e(p^h, θl)
e^f_l
e(p1, θ)