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Paul Beaudry; Michel Poitevin

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# Competitive Screening in Financial Markets when Borrowers can Recontract

PAUL BEAUDRY

*University of British Columbia, Boston University, C.R.D.E. and N.B.E.R.*

and

MICHEL POITEVIN

*Université de Montréal and C.R.D.E.*

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This paper examines how the possibility of recontracting affects the financing of projects when an entrepreneur is privately informed about the distribution of returns. We consider a game where an entrepreneur solicits initial financing for a project from competing uninformed financiers. Once the project is undertaken, but before its returns are realized, the entrepreneur can solicit additional financial contracts from competing financiers. It is assumed that these financiers can observe all previously signed contracts and that the seniority of claims is respected in the case of bankruptcy; however, the entrepreneur is never committed not to sell junior claims to competing financiers. The main results of the paper are that (1) the equilibrium is characterized by separation but nevertheless the modalities of financing depend critically on the market's priors about the project's riskiness, in particular, the amount of collateral posted by the entrepreneur varies with the market's prior perceptions about the project, (2) when the market is optimistic about the project, there exists a unique equilibrium outcome, it is separating, but the standard incentive-compatibility constraints are not binding, (3) even if the market is very pessimistic about a project's chances of success, there always exists an equilibrium in which a good project receives sufficient financing, that is, the market does not collapse due to a Lemons effect, (4) the entrepreneur's inability to commit not to recontract may be considered Pareto improving in certain situations. We discuss how the results of the paper may help explain observed financial flows.

## 1. INTRODUCTION

In their seminal paper on competitive screening, Rothschild and Stiglitz (1976) show that uninformed competitors can use different contractual packages in order to try to separate low- and high-risk individuals. When a pure-strategy separating equilibrium exists, the high-risk individuals are provided with a full-insurance contract while the low-risk individuals must bear some risk to satisfy the self-selection constraint; however, the possibility of attracting only the low-risk individuals to risky contracts depends crucially on the assumption that individuals are committed not to enter into any further contractual relationships. In the absence of such a commitment, the predicted separating equilibrium could not hold since low-risk individuals receive and accept contract offers that incite the forward-looking high-risk individuals to mimic and then solicit new contracts that would completely diversify their residual risk. Although it may be possible to link the validity of a contract to the absence of further contractual agreements, as is implicitly assumed by Rothschild and Stiglitz, in practice this may be very difficult or undesirable to enforce.

Consequently, it appears necessary to explore how the impossibility of limiting recontracting may affect our understanding of the functioning of markets with adverse selection.

The object of this paper is to examine the determination of competitive contracts when asymmetrically informed agents cannot commit to not soliciting new contractual offers once the initial contract has been signed. We imbed our analysis in the context of a project-financing game of which the insurance market is a special case. In particular, the examination of project financing allows us to address the additional issues of whether negative-value projects may be financed in competitive markets, and whether the availability of collateral can attenuate the adverse-selection problem when recontracting is possible.

In order to determine the implications of recontracting, we examine a game where uninformed financiers are allowed to bid for contingent claims on a project after an informed entrepreneur has entered into one or many previous financial agreements. It is assumed that all previously signed contracts are observable, and that the seniority of claims will be respected in the event of bankruptcy; however, throughout the analysis we assume that the entrepreneur cannot be directly restricted to not soliciting additional offers of financial contracts.<sup>1</sup> One important feature of our modelling strategy is that we allow for a potentially infinite number of rounds of recontracting. That is, at any point in the recontracting process informed agents can always solicit additional offers from uninformed agents. If only a finite number of rounds of recontracting were in fact permitted, then players could use the last stage of the game to effectively commit to contracts that would be vulnerable to our previous criticism of separating equilibria.

Our analysis covers two cases which differ by the value of the high-risk project. In the case where both projects have a positive value, the entrepreneur always obtains financing for the project although the low-risk entrepreneur remains incompletely diversified. The possibility of recontracting implies that even though final financial positions are separating, the low-risk entrepreneur is in fact subsidizing the high-risk type. As priors become more pessimistic the subsidization cost paid by the low-risk entrepreneur increases and he reduces his reliance on external financing by investing a progressively greater fraction of his wealth into the project in the form of collateral. This subsidization across types leads to a "Lemons premium" associated with external financing although the space of contracts allows for separation. In the case where the high-risk project has negative value and priors are pessimistic, the high-risk type drops out of the market and foregoes his project. As for the low-risk type, there is always an equilibrium in which he has enough financing to undertake his project and not be mimicked by the high-risk type, that is, the market does not necessarily collapse due to the Lemons effect.

Before describing the setup of the paper, let us briefly relate it to the literature on multiple contract purchases initiated by Jaynes (1978) and examined by Hellwig (1988) and Gale (1991). In fact, under certain conditions, our characterization of equilibrium outcomes is identical to that discussed by Jaynes (1978) even though the approaches are substantially different. For example, our analysis assumes that all past contracts are observable while Jaynes, and Hellwig's formalization of Jaynes, focuses on the strategic incentives that firms may have to conceal information about existing contractual relationships. Therefore, our approach can be considered as demonstrating the appropriateness of the Jaynes outcome as a description of competitive environments where information on contract purchases cannot be withheld. The relationship with these papers is discussed further in Section 3.<sup>2</sup>

1. This assumption will be discussed in Section 5.

2. Our analysis also parallels the literature on renegotiation with adverse selection since previously signed contracts are binding (for example, see Hart and Tirole (1988), Hosios and Peters (1988), Dewatripont (1989),

The remaining sections of the paper are structured as follows. In Section 2 we present the game setup. In Section 3 we analyse equilibrium behaviour for situations where all projects have a positive value. In Section 4 we extend the analysis of Section 3 to the case where the high-risk project has a negative value. In Section 5 we discuss our assumption that entrepreneurs cannot be restricted from selling junior claims. Section 6 discusses how the model provides insight about observed firms' financial decisions, that is, the choice of external vs. internal financing. Concluding comments are given in Section 7. All proofs are relegated to the Appendix.

## 2. THE SCREENING-RECONTRACTING GAME

Consider an environment where a risk-averse entrepreneur has a risky project which requires an initial investment of  $B$ . This project results in gross revenues of either  $\bar{R}_1$  or  $\bar{R}_2$  with  $\bar{R}_1 > \bar{R}_2$ . The probability that the revenues of the project be equal to  $\bar{R}_1$  is denoted by  $P(t)$ , where  $t$  represents the entrepreneur's private information. We consider the case where  $t$  can take two values  $H$  or  $L$ , with  $P(H) < P(L)$ . A project with  $t = L$  is referred to as a low-risk project. The entrepreneur has collateralizable wealth  $W$ . We do not impose any restriction on the relative size of  $W$  and  $B$ ; we assume however that the project is sufficiently risky that the entrepreneur is unwilling to finance the project solely with his own funds even if  $W$  is greater than  $B$ . Consequently, the entrepreneur will want to obtain outside financing in order to (1) gather sufficient funds to cover the fixed cost  $B$  if his wealth is smaller than  $B$  and (2) diversify the risk associated with the project.

The entrepreneur is assumed to have access to a market of financiers with which he can exchange contingent claims to the project. In exchange of a loan  $b$ , contingent-contractual payments are of the type  $\{\bar{s}_1, \bar{s}_2\}$ , where  $\bar{s}_i$  is the contracted payment from the entrepreneur to the financier when gross revenues  $\bar{R}_i$  are realized. The entrepreneur can solicit contingent claims from financiers sequentially; however, once his project has been undertaken the entrepreneur is never committed to not solicit additional claims (recontract) from other financiers. The expectation by initial financiers of future recontracting is likely to affect the nature of their financial offers to the entrepreneur. The object of this paper is to assess the implications of this type of recontracting on equilibrium financing.

The sequence in which the entrepreneur can meet financiers and exchange claims is made explicit by the following series of events.

- (0) Nature reveals the value of  $t$  to the entrepreneur.
- (1.1) The entrepreneur decides whether to solicit financial offers from  $J$  financiers. If he does not solicit any offer, the game ends.
- (1.2) If he does solicit financial offers,  $J$  financiers simultaneously announce the terms of initial contract offers with  $J > 2$ . Each financier is allowed to offer two contracts.<sup>3</sup>
- (1.3) The entrepreneur chooses one of the contracts or refuses them all. If he does not have enough financing or if he prefers not to undertake the project, he

Laffont and Tirole (1990), Dewatripont and Maskin (1990) and Beaudry and Poitevin (1993). The paper distinguishes itself from the renegotiation literature by (1) allowing "recontracting" instead of renegotiation, which makes it possible to examine situations where the informed party has the bargaining power even though competing uninformed parties make the offers, and (2) by allowing for an infinite number of rounds of recontracting before any realization of uncertainty.

3. Imposing that financiers must offer contracts is without loss of generality since the trivial contract consisting of a loan  $b=0$  and zero contingent claims is always an option.

refuses all offers and the game ends. Accepting an offer signals the entrepreneur's intention to undertake his project.<sup>4</sup>

- (2.1) If the project is undertaken, the entrepreneur decides, before the project's returns are realized, whether to solicit further financial offers from financiers.
- (2.2) If the entrepreneur solicits new offers, the entrepreneur provides financiers with a summary of previously accepted contracts. We refer to such a summary as an updated financial statement. Then a new cohort of  $J$  financiers propose incremental agreements.
- (2.3) The entrepreneur chooses whether to accept one of these new offers or reject them all.
- (3.1 . . .) Regardless of whether he has previously solicited new offers, the entrepreneur can again decide to solicit further financial agreements. It is assumed that the entrepreneur has an infinite number of these occasions to solicit more offers before the returns on the project are realized. For example, the round  $n$  of recontracting consists of steps  $n.1$  through  $n.3$  which are just the repetition of steps 2.1 through 2.3.

The sequence of play in the game has been chosen in the hope of capturing the effect of recontracting in a competitive environment. The recontracting phase of the game consists of repeated possibilities for the entrepreneur to solicit financial arrangements from different financiers. Most important in this sequencing is the inclusion of a potentially infinite number of rounds of recontracting. This is meant to capture the idea that a financier can never be sure that an entrepreneur will not try to further diversify his risk by selling junior claims on the project.<sup>5</sup>

Before defining payoffs for this game, we must describe more precisely the nature of financial contracts. Let financiers be indexed by the round in which they make an offer. For example,  $n_j$ ,  $j = 1, \dots, J$ , represents one of the  $J$  financiers making offers in round  $n$  of contracting. A financial offer from financier  $n_j$  consists of a loan  $b_{n_j}$  to the entrepreneur, and promised payments to the financier contingent on the realization of returns on the project. The payment to financier  $n_j$  in state  $i$  is denoted by  $\bar{s}_i^{n_j}$  for  $i = 1, 2$ . Given these definitions, the return to the entrepreneur in state  $i$  is  $\bar{R}_i - B + W + \sum_{n_j} b_{n_j} - \sum_{n_j} \bar{s}_i^{n_j}$ . The return to financier  $n_j$  in state  $i$  is  $\bar{s}_i^{n_j} - b_{n_j}$ .

It is clear that the return to each agent in each state only depends on the net claim he has on the project in that state. Therefore, to simplify notation we redefine all variables in terms of net payments. For state  $i = 1, 2$ , define  $R_i := \bar{R}_i - B$  as the net return on the project, and  $s_i^{n_j} := \bar{s}_i^{n_j} - b_{n_j}$  as the net payment to financier  $n_j$ . In state  $i$ , the return to the entrepreneur can now be written as  $R_i + W - \sum_{n_j} s_i^{n_j}$ , and the return to financier  $n_j$  as  $s_i^{n_j}$ . Let us also define the entrepreneur's net claims position  $S_i$  as the sum of all  $s_i^{n_j}$  contracted with different financiers, that is,  $S_i = \sum_{n_j} s_i^{n_j}$ .

We assume that all loans are reimbursed in the order in which they were made. This amounts to assuming that the seniority of loans can always be enforced. Moreover, without loss of generality, we can restrict attention to situations where the entrepreneur does not go bankrupt, that is, all claims are paid as promised. Formally, bankruptcy occurs when financial contracts are such that  $R_i + W < S_i$ , that is, in state  $i$ , net revenues and wealth

4. Implicit in this formulation is that it is verifiable whether the project has been undertaken or not, and therefore financial offers are made contingent on the project being undertaken.

5. Gale and Stiglitz (1989) analyze a game with one round of recontracting. In their model a firm can issue equity twice. They adopt a signalling approach and encounter severe problems of multiple equilibria. In particular, when there is no uncertainty resolved between the issues, as is assumed here, the standard separating outcome is always an equilibrium outcome of their game.

cannot cover promised payments. Now suppose signed contracts were such that the entrepreneur was bankrupt in state  $i$ . Because contracts are complete and the seniority of claims can be enforced, the marginal financier could foresee this possibility and write the contract so as to reflect the expected payment. Consequently, eliminating the possibility of bankruptcy is without loss of generality; this is not to say however that all financing is riskless, since in general  $s_1^i \neq s_2^i$ , but rather that all state-contingent claims are chosen such that they will always be paid as promised. Finally, we say that a project may be financed with riskless claims when  $R_2 + W \geq 0$ , since the entrepreneur can finance the fixed cost using his personal wealth and a riskless loan.<sup>6</sup>

There are three additional features of the game that need to be discussed. First, we assume that different financiers compete at different rounds of the recontracting process, and that any financier knows only the entrepreneur's updated financial statement when deciding to propose offers, that is, he cannot observe any rejected past offers. Therefore, a past contract proposal becomes observable only if it is chosen by the entrepreneur, in which case it becomes known through future financial statements. Both these assumptions are incorporated into the game to limit the financier's capacity to use trigger strategies in order to support collusion. This modelling assumption reflects the perception that competitive financial markets are populated by a great number of financiers, and therefore, enforcing collusion by having all financiers observing past rejected proposals is virtually impossible. This assumption is meant to incorporate into the model some sort of anonymity that characterizes competitive markets. More formally, making past rejected contracts unobservable to current financiers has the same kind of effect as restricting attention to Markov strategies. In this case the updated financial statement would represent the state variable upon which financiers' strategies could depend.<sup>7</sup>

Second, to ease presentation we assume that financiers cannot offer contracts that would result in the entrepreneur being "overinsured," that is, situations where  $R_2 - S_2 > R_1 - S_1$ .

Third, the entrepreneur's payoffs are not discounted for the time taken by the solicitation of new offers in the recontracting process. This is obviously a simplifying assumption but is made as a first step to understand the effect of recontracting on the determination of contracts in markets with adverse selection.

Before proceeding to the equilibrium analysis, we formally describe the players' histories, strategies, and payoffs for this game. Let  $\mathcal{J}_n^s$  denote the entrepreneur's information set when he solicits an offer in round  $n.1$ , and let  $\mathcal{J}_n^a$  denote the entrepreneur's information set when he makes his acceptance decision in round  $n.3$ . The entrepreneur's information set contains the complete past history of the game. As for financiers, their information set is assumed to contain only the entrepreneur's updated financial statement. In particular, it does not include past rejected proposals. Consequently, for  $n = 1, \dots, \infty$ , let  $\mathcal{J}_n$  denote financier  $n_j$ 's history when solicited to offer contracts in round  $n.2$ . The information set of financiers  $1_j$  is  $\{0, 0\}$  since the entrepreneur has no financing yet. The players' strategies and payoffs can now be specified.

A strategy  $\sigma_t$  for an entrepreneur with private information  $t$  is a sequence of functions  $\sigma_t = (\sigma_{t1}, \sigma_{t2}, \dots, \sigma_{tm})$  with  $\sigma_m = (\sigma_m^s, \sigma_m^a)$  such that, for all rounds  $n$ ,  $\sigma_m^s$  maps  $\mathcal{J}_n^s$  into  $\{\text{not solicit}, \text{solicit}\}$ , and  $\sigma_m^a$  maps  $\mathcal{J}_n^a$  into  $\{0, n_1^1, n_1^2, n_2^1, n_2^2, \dots, n_J^1, n_J^2\}$ , where 0 indicates the entrepreneur's decision to refuse all of the financiers' latest proposals, and  $n_j^k$  indicates the decision to accept financier  $n_j$ 's  $k$ -th proposal ( $k = 1, 2$ ).

6. The entrepreneur can always issue a riskless bond of size  $\bar{R}_2$  since this is the lowest possible return on the project.

7. See Fudenberg and Tirole (1991) for a presentation of Markov strategies and equilibria.

A strategy  $\omega_{n_j}$  for financier  $n_j$  is a function such that, for all  $n_j$ ,  $\omega_{n_j}$  maps  $\mathcal{J}_n$  into a subset of  $\mathbf{R}^4$ , where  $\omega_{n_j}(\mathcal{J}_n)$  indicates the proposal by financier  $n_j$  of two contingent claims of the type  $\{s_{1L}^{n_j}, s_{2L}^{n_j}, s_{1H}^{n_j}, s_{2H}^{n_j}\}$  when the entrepreneur solicits offers given he has financial position  $\mathcal{J}_n$  at the beginning of round  $n$ . As discussed above, we want to eliminate the possibility of overinsurance, and therefore  $\omega_{n_j}$  is restricted to belong to the set of offers such that, on acceptance, the entrepreneur always has a net position  $R_1 - S_1 \geq R_2 - S_2$ .

Beliefs  $\mu_{n_j}$  for a financier  $n_j$  are represented by a function such that, for all  $n_j$ ,  $\mu_{n_j}$  maps  $\mathcal{J}_n$  into  $[0, 1]$ , where  $\mu_{n_j}(\mathcal{J}_n)$  indicates financier  $n_j$ 's probability assessment that the entrepreneur has a project  $t = L$ .

The entrepreneur's payoff is given by the expected utility of revenues net of all contracted contingent claims payments. Therefore, if a type  $t$  entrepreneur undertakes the project and acquires a net claims position of  $\{S_1, S_2\}$  with  $R_t + W \geq S_t$ , then his expected utility is given by

$$U(S_1, S_2, t) := P(t)u(R_1 + W - S_1) + (1 - P(t))u(R_2 + W - S_2),$$

where  $u(\cdot)$  is a state-independent utility function defined on  $\mathbf{R}$  with  $u' > 0$ ,  $u'' < 0$ . If the entrepreneur does not undertake the project, he receives the payoff  $u(W)$ ; if he never stops soliciting offers, he receives a payoff smaller than  $u(W)$ .

Financier  $n_j$ 's payoff corresponds to the expected value of any net claim on the entrepreneur's project. It is given by

$$V(s_1^{n_j}, s_2^{n_j}, t) := P(t)s_1^{n_j} + (1 - P(t))s_2^{n_j}.$$

Each financier's reservation utility is  $V = 0$ .<sup>8</sup>

Given these strategies and beliefs, a Perfect Bayesian Equilibrium (PBE) for this game is defined as a tuple  $(\sigma_L, \sigma_H, \omega_{n_j}, \mu_{n_j})$  for  $n = 1, \dots, \infty$ , and  $j = 1, \dots, J$ , such that strategies are sequentially rational given beliefs, and beliefs satisfy Bayes rule when applicable.<sup>9</sup>

### 3. EQUILIBRIUM ANALYSIS WHEN BOTH PROJECTS HAVE A POSITIVE VALUE

In this section we determine the equilibrium financial contracts that the entrepreneur obtains when both types of project have positive value, that is,  $P(t)R_1 + (1 - P(t))R_2 \geq 0$  for  $t = H, L$ . The key step in solving for equilibrium contracts is to understand the outcome of the recontracting rounds for any initial net claims position. Therefore, before proceeding to the characterization of the equilibrium outcome of the game, we begin by examining only the recontracting rounds.

#### 3.1. Analysis of the recontracting subgame

Let us define a continuation game that begins after the project has been undertaken as a recontracting subgame.<sup>10</sup> A recontracting subgame starts at a given round  $\hat{n} \geq 2$  with a

8. This definition of payoffs reveals that the setup we consider is almost identical to that analyzed by Rothschild and Stiglitz (1976) except that we are interpreting the variables as related to the financing of a project instead of that of insuring an individual, and therefore we must consider the role of collateralizable wealth and the possibility that a project may have a negative value, that is,  $P(t)R_1 + (1 - P(t))R_2 < 0$ .

9. See Fudenberg and Tirole (1991) for a precise definition of a PBE.

10. This game is not a proper subgame of our game since it does not start at a single node; we take however the beliefs as given which makes it possible to find the equilibrium of the continuation game.

given net claims position  $\hat{S} := \{\hat{S}_1, \hat{S}_2\}$ , such that  $R_i + W - \hat{S}_i \geq 0$ . We assume that financiers  $\hat{n}_j$  know  $\hat{S}$  and that they have been solicited for bids on junior claims. Financiers hold prior beliefs  $\hat{\mu}$  that the entrepreneur is of type  $L$ . On the basis of these beliefs, the solicited financiers compete by offering contracts to the entrepreneur and all the remaining stages of the original game follow without change.

For any recontracting subgame, Lemma 1 specifies the minimum level of utility that a given type can achieve from any net claims position. In particular, it specifies what minimum level of utility a type  $t$  can achieve in equilibrium if he obtains the equilibrium net claims position of type  $t'$  and then solicits more offers. Define  $S'_t := \{S'_{1t}, S'_{2t}\}$  as type  $t$ 's equilibrium outcome in the subgame.

**Lemma 1.** *For any net claims position  $\tilde{S}$  on the equilibrium path of a recontracting subgame leading to the outcome  $\{S'_L, S'_H\}$ , the following inequality must hold for  $t = H, L$ .*

$$U(S'_t, t) \geq U(\Phi_H(\tilde{S}), t) \quad (3.1)$$

where  $\Phi_H(\tilde{S}) := \arg \{ \max_S U(S, H) \text{ s.t. } V(S, H) \geq V(\tilde{S}, H) \}$ .

The interpretation of constraint (3.1) is that, if a net claims position  $\tilde{S}$  is attained along the equilibrium path, each type of entrepreneur must have at least as much utility in equilibrium as he would have if he decided to reach  $\tilde{S}$ , solicit more offers and then have financiers make competing incremental offers on the basis of their most pessimistic beliefs. These pessimistic beliefs are equivalent to attributing a probability of one that the entrepreneur is of type  $H$ . For these beliefs, competition between financiers always guarantees that financiers earn zero profit, and therefore that the contract  $\Phi_H(\tilde{S}) - \tilde{S}$  is offered.<sup>11</sup> Consequently, soliciting more offers from  $\tilde{S}$  yields at least  $U(\Phi_H(\tilde{S}), t)$ .

Constraint (3.1) can be used to derive a “recontracting-induced incentive-compatibility” constraint. The recontracting-induced incentive-compatibility constraint is  $U(S'_t, t) \geq U(\Phi_H(S'_t), t)$ . This constraint is more stringent than the usual incentive-compatibility constraint which can be written as  $U(S'_t, t) \geq U(S'_t, t)$ , and therefore recontracting necessarily restricts equilibrium outcomes.

The next lemma uses constraint (3.1) to characterize the equilibrium outcome for a type  $L$  entrepreneur in any recontracting subgame. Denote by  $V^P(s_1^n, s_2^n, \hat{\mu})$  the average profit of financier  $n_j$  having beliefs  $\hat{\mu}$ , that is,

$$V^P(s_1^n, s_2^n, \hat{\mu}) := \hat{\mu} V(s_1^n, s_2^n, L) + (1 - \hat{\mu}) V(s_1^n, s_2^n, H).$$

**Lemma 2.** *For any prior beliefs ( $0 \leq \hat{\mu} \leq 1$ ) and any initial net claims position  $\hat{S}$ , the outcome  $\{S'_L, S'_H\}$  can be supported as an equilibrium of the recontracting subgame only if  $S'_L$  is the solution to the following maximization problem:*

$$S'_L := \arg \left\{ \max_{S_L} U(S_L, L) \quad \text{s.t.} \quad \begin{array}{l} V(S_L, L) \geq V(\hat{S}, L) \\ V^P(S_L, \hat{\mu}) \geq V^P(\hat{S}, \hat{\mu}) \end{array} \right\}.$$

Lemma 2 states that, in any recontracting subgame, a type  $L$  entrepreneur obtains his preferred financial position within the set of claims for which (1) he is not subsidized

11. The possibility of positive profits is eliminated by the assumption that financiers' histories are restricted to the entrepreneur's financial statements, and therefore, no collusion is possible as future financiers cannot effectively learn about the defection of a predecessor.



and (2) financiers have non-negative profits evaluated at pooling odds. The reasons for these two constraints are directly related to Lemma 1. First, if type  $L$  is subsidized, then it must be the case that the high-risk entrepreneur is subsidizing him, otherwise some financier would be making expected negative profits; however, the fact that type  $H$  is subsidizing type  $L$  implies that the type  $H$  entrepreneur is obtaining an equilibrium level of utility lower than  $U(\Phi_H(\hat{S}), H)$ , which obviously contradicts Lemma 1. Second, if type  $L$  were to obtain a position  $S'_L$  such that, evaluated at pooling odds, it renders negative profits to the financiers, it would have to be the case that  $V(S'_H, H) > V(S'_L, H)$ , otherwise some financiers would again be making expected negative profits; this implies however a payoff to a type  $H$  entrepreneur lower than that predicted by Lemma 1 when the type  $H$  entrepreneur obtains the net claims position  $S'_L$  and then solicits further offers. Type  $H$  would then prefer to mimic type  $L$  and solicit further offers rather than follow his equilibrium strategy.

Intuitively, the reason that the type  $L$  entrepreneur can obtain additional financing only at pooling odds is that he can never convince financiers that he has a low-risk project. In the presence of recontracting, the low-risk entrepreneur has lost the possibility of separating himself and obtaining financing at fair odds. In a one-stage game, the low-risk entrepreneur can generally obtain financing at fair odds if his equilibrium net claims position leaves him with sufficient residual risk (to satisfy the incentive-compatibility constraint of type  $H$ ). In contrast, within the recontracting framework, the signalling value of accepting a net claims position with sufficient residual risk is eliminated since the entrepreneur is never committed not to further diversify his risk. In light of Lemma 2, it is now straightforward to characterize type  $H$ 's equilibrium outcome.

**Lemma 3.** *For any prior beliefs ( $0 \leq \hat{\mu} \leq 1$ ) and any initial net claims position  $\hat{S}$ , the outcome  $\{S'_L, S'_H\}$  can be supported as an equilibrium of a recontracting subgame only if  $S'_H = \Phi_H(S'_L)$ .*

In order to understand Lemma 3, it is helpful to recognize that any financing offered and accepted by the low-risk type will also be accepted by the high-risk type. The high-risk type has an incentive to mimic the low-risk type since this allows him to obtain a financial position on the lowest possible iso-profit curve  $V(\cdot, H)$  for the financiers; however, the high-risk type does not content himself with the net claims position he obtains from mimicking the low-risk type. Once he has obtained the net claims position  $S'_L$ , he is then ready to reveal that the project has high risk and thereby obtain further financing at fair odds for his type. It is important to note that it is only when the high-risk type reveals himself that separation will actually be achieved.

Corollary 1 combines Lemmas 2 and 3 in order to make explicit how Lemma 1 is almost sufficient for describing the effects of recontracting. Recall from Lemma 1 that constraint (3.1) has to be satisfied for any contractual position along the equilibrium path. Corollary 1 indicates that there are two specifications of constraint (3.1) that are relevant for the determination of equilibrium outcomes in a recontracting subgame. These two specifications correspond to constraints on the minimum utility levels that can be obtained by the type  $H$  entrepreneur if he solicits offers from either the initial contractual position  $\hat{S}$  or from entrepreneur  $L$ 's equilibrium position  $S'_L$ . In effect, Corollary 1 indicates that it is only these two constraints, plus the zero-profit condition for the financiers, which completely summarize how recontracting limits type  $H$ 's contractual possibilities.

**Corollary 1.** *For any beliefs ( $0 \leq \hat{\mu} \leq 1$ ) and any initial net claims position  $\hat{S}$ , the outcome  $\{S'_L, S'_H\}$  can be supported as an equilibrium of the recontracting subgame only if it is the solution to the following maximization problem.*

$$\{S'_L, S'_H\} := \arg \left\{ \begin{array}{ll} \max_{S_L, S_H} U(S_L, L) & \text{s.t. } \hat{\mu} V(S_L, L) + (1 - \hat{\mu}) V(S_H, H) \geq V^P(\hat{S}, \hat{\mu}) \\ & U(S_H, H) \geq U(\Phi_H(S_L), H) \\ & U(S_H, H) \geq U(\Phi_H(\hat{S}), H) \end{array} \right\}.$$

As summarized in Corollary 1, Lemmas 1, 2 and 3 provide a means of characterizing the equilibrium outcome for any recontracting subgame as the solution to a maximization problem. We can now use this characterization to examine the equilibrium outcomes of the whole game.

### 3.2. Analysis of the complete game

Proposition 1 uses Corollary 1 to characterize the unique equilibrium outcome of the game. The statement of the proposition indicates that the equilibrium outcome can be written as the solution to a sequence of maximization problems.

**Proposition 1.** *There exists a unique PBE outcome of the game  $S^* = \{S_H^*, S_L^*\}$  described by:*

$$S_L^* = \arg \left\{ \begin{array}{ll} \max_{S_L} U(S_L, L) & \text{s.t. } \mu_0 V(S_L, L) + (1 - \mu_0) V(S_L, H) \geq 0 \\ & R_2 + W - S_{2L} \geq 0 \end{array} \right\},$$

$$S_H^* = \arg \{ \max_{S_H} U(S_H, H) \text{ s.t. } V(S_H, H) \geq V(S_L^*, H) \}.$$

The exact description of the outcome defined in Proposition 1 depends on the wealth of the entrepreneur. Therefore it is useful to distinguish two cases. In the first case, the entrepreneur has sufficient wealth to finance the project by issuing only riskless claims, that is,  $R_2 + W \geq 0$ . This case is of particular interest since it can be compared with the analysis of project financing of Leland and Pyle (1977) and of the insurance market of Rothschild and Stiglitz (1976), Wilson (1977) and Riley (1979). In the second case, the entrepreneur's wealth is not sufficient to finance the project only with riskless claims, that is,  $R_2 + W < 0$ . This case allows us to examine both the role of collateral, and whether both projects always obtain financing. Before examining these two cases, it is helpful to note that a constraint equivalent to the constraint  $U(S_H, H) \geq U(\Phi_H(\hat{S}), H)$  in Corollary 1 is redundant in Proposition 1 because of the assumption that the entrepreneur prefers to abandon the project rather than finance it solely with his own funds, that is,  $u(W) > U(0, 0, t)$ .

When  $R_2 + W \geq 0$ , the entrepreneur can finance the project with riskless claims and therefore the second constraint in the definition of  $S_L^*$  is never binding. The whole game is then essentially a special case of a recontracting subgame where  $\hat{S} = \{0, 0\}$  and  $\hat{\mu} = \mu_0$ . Therefore, when  $R_2 + W \geq 0$ , the unique equilibrium outcome of Proposition 1 has the entrepreneur with a low-risk project acquiring the net claims position corresponding to his preferred financial position among the set of all contracts with non-negative pooling profits. As for the high-risk entrepreneur, he always manages to completely diversify his risk and not to pledge any of his wealth as collateral into the project. In fact, his equilibrium financial position can be interpreted as a fixed-wage managerial contract, or equivalently

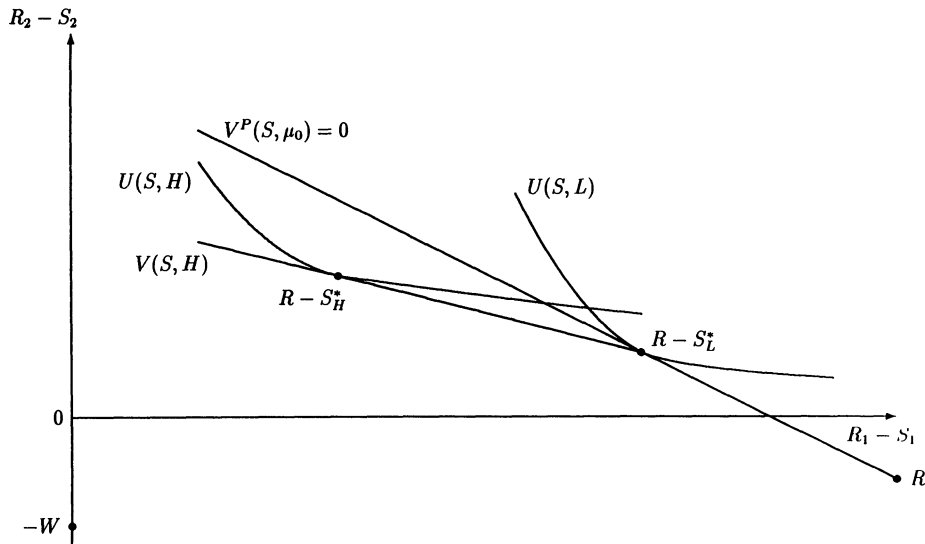


FIGURE 1

Equilibrium outcomes: Personal financing is possible

as a pure equity contract in which financiers bear all the risk. The configuration for this equilibrium outcome is given by Figure 1.

Note that when the entrepreneur can finance his project with riskless claims, financial securities are used only for the purpose of risk diversification. If pooling odds are relatively bad, the low-risk entrepreneur places a substantial fraction of his personal wealth into the project in the form of collateral; that is, if state 2 occurs, the entrepreneur must use some of his wealth to fulfill his financial obligations. This is done at the expense of risk diversification. The low-risk entrepreneur faces the trade-off of either reducing his collateral exposure and thus financing at pooling odds, or increasing his stake in the firm and thus reducing the amount of risk diversification. When pooling odds are unfavourable this trade-off goes towards increased collateral and reduced risk diversification. When pooling odds are attractive, the trade-off goes the other way. Consequently, the degree of risk diversification is positively related to priors regarding the likelihood of a low-risk project. This result contrasts sharply with previous models of capital-raising games with adverse selection (for example, see Leland and Pyle (1977)) in which the separating equilibrium is independent of priors.

When  $R_2 + W < 0$ , the entrepreneur does not have the option of undertaking the project only with riskless claims. In this case, the equilibrium outcome can take two possible configurations depending on whether the financing constraint,  $R_2 + W - S_{2L} \geq 0$ , is binding at the optimum or not. In particular, if type  $L$ 's tangency with the zero-profit pooling line,  $V^P(\cdot, \mu_0) = 0$ , is at a point where  $R_2 + W - S_{2L} \geq 0$ , this constraint is not binding. The equilibrium outcome is then virtually identical to that of the case when  $R_2 + W \geq 0$ . In contrast, if this tangency occurs at a point such that  $R_2 + W - S_{2L} < 0$ , that is, if at this point of tangency the entrepreneur still would not have enough financing to realize the project, then the financing constraint is binding at the optimum. In this case, the type  $L$  entrepreneur obtains a corner-solution financing arrangement. This possibility is illustrated in Figure 2.

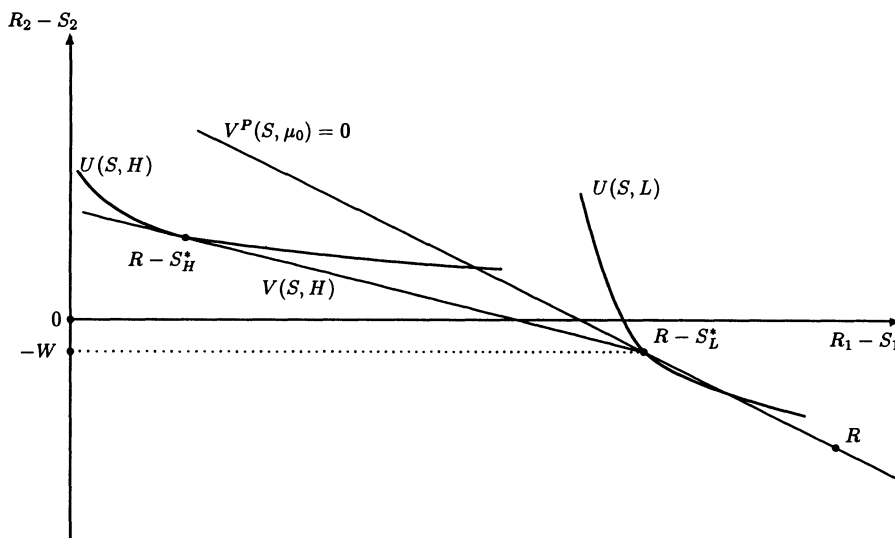


FIGURE 2

Equilibrium outcomes: Personal financing is impossible and beliefs are pessimistic

In Figure 2, the entrepreneur with a low-risk project invests all his collateralizable wealth into the project, while the entrepreneur with a high-risk project still manages to completely diversify his risk and not associate any of his wealth with the project. Since the type  $L$  entrepreneur loses all wealth in state 2, his financing arrangement is essentially equivalent to a debt contract for which he pledges his entire wealth as collateral. As for the type  $H$  entrepreneur, his financial arrangement corresponds, as before, to a fixed-wage managerial contract (or pure equity contract) since he does not maintain any risky position in the project (all the equity has been sold). Consequently, when priors are pessimistic, a low-risk project will be undertaken with the maximal amount of collateral possible.

The above characterization implies that both types of project can always secure enough financing to undertake the project. This is not surprising given that it is common knowledge that both types of project have a positive value; however, the financing arrangement is such that the amount of diversification depends on the priors of financiers. As priors become more pessimistic, the cross-subsidization cost paid by the low-risk entrepreneur increases and he gradually increases his collateral in the project at the expense of risk diversification. At one point, he might place himself in a position where he may lose his entire collateralizable wealth if the bad state occurs. On the other hand, for constant priors, if the fraction of total wealth that is collateralizable decreases, the constraint  $R_2 + W - S_{2L} \geq 0$  is more likely to bind, and therefore, the low-risk type is made worse off while the high-risk type is better off.<sup>12</sup>

We now describe informally equilibrium strategies and beliefs leaving the formal description for the Appendix. Although the equilibrium outcome is unique, there are many paths leading to it. For instance, there is some arbitrariness in the choice of the initial offers. The following description corresponds to the strategies and beliefs stated in the proof of Proposition 1. They have the property that no recontracting actually occurs in

12. This suggests that the low-risk type may want to postpone his project in order to increase his collateral. This possibility is studied in Beaudry, Boyer and Poitevin (1993).

equilibrium, that is, no junior claims are ever solicited in equilibrium. Let us begin by considering behaviour along the equilibrium path. First, both types of entrepreneur solicit initial financing for their respective project. Financiers  $1_j, j=1, \dots, J$  offer the contracts  $\{S_L^*, S_H^*\}$ . The entrepreneur accepts from financier 1 the offer  $S_L^*$  if he is of type  $L$ , and accepts  $S_H^*$  if he is of type  $H$ . Finally, regardless of type, the entrepreneur undertakes the project and does not solicit further offers.

In the case of a deviation by the entrepreneur, equilibrium play is as follows. If the entrepreneur does not solicit initial financing, he prefers not to undertake the project even if  $W \geq B$ . The entrepreneur then gets  $u(W)$  which is easily proven to be less than  $U(S_L^*, t)$  under our assumptions. This deviation is therefore not profitable. For any deviation regarding the acceptance decision of initially offered contracts, the entrepreneur is believed to be of type  $H$  if he solicits more offers. Hence, he is offered  $s^* = S_H^* - S_L^*$  if he initially accepted  $S_L^*$  and is offered  $s^* = \{0, 0\}$  if he accepted  $S_H^*$ . If he continues to solicit more offers, he is re-offered these same contracts indefinitely. Therefore, he does not find it advantageous to deviate.

In order to describe the play following deviations by financier  $1_j$ , let us first consider deviations that consist in withdrawing contract offers relative to those prescribed along the equilibrium path. If financier  $1_j \neq 1_1$  withdraws either one or both of his contract offers, the strategies of the other players are unchanged. If financier  $1_1$  withdraws either of his offers, the entrepreneur regardless of type chooses his preferred contract among the set offered by financier  $1_2$  and then undertakes the project. In both cases, financiers do not gain by withdrawing a contract.

Let us now consider deviating offers by financier  $1_j \neq 1_1$ . Any deviation by such a financier that is accepted by at least one type of entrepreneur is expected to be followed by offers yielding outcomes such as those described in Corollary 1. Therefore, when type  $L$  expects to gain from accepting the deviation, type  $H$  will gain as well in subsequent rounds of recontracting, thus inducing both types to accept the deviating offer. Given the characterization of type  $L$ 's equilibrium outcome, this deviating offer must then confer negative profits to the financier. When only type  $H$  expects to gain from the deviation, it trivially confers negative profits to the deviating financier. Therefore, any deviation by financier  $1_j \neq 1_1$  cannot be profitable. Since the equilibrium play in recontracting subgames not reached in equilibrium is virtually identical to that described above, deviations by financiers  $n_j > 2_j$  would not be profitable also.

The inefficiency characterizing type  $L$ 's equilibrium outcome is sustained because soliciting any further financing is interpreted as a signal that the project is of high risk. For example, even after  $S_L^*$  has been agreed to, which, in equilibrium, is only taken by the type  $L$  entrepreneur, any further soliciting results in the financier inferring that the project is of type  $H$ . At first glance, these beliefs may appear contradictory: why are financiers "changing their mind" about the type that holds  $S_L^*$ ? Formally, financiers do not change their mind since they are not required to move or update beliefs between the time the entrepreneur chooses a contract and solicits further offers; however, even if we allowed financiers to update beliefs after the entrepreneur chooses a contract, we believe that it is internally consistent to allow for a switching support of beliefs off the equilibrium path, that is, where Bayes rule does not apply.<sup>13</sup> In fact, most equilibrium concepts allow for a switching support of beliefs (including Perfect Bayesian Equilibrium), and in many

13. A formal argument in favor of allowing switching supports is presented in Noldeke and van Damme (1990).

cases, eliminating this possibility leads to non-existence problems.<sup>14</sup> In the light of these considerations, the beliefs underlying Proposition 1 seem quite reasonable.

It is of interest to note that although we restrict our attention to pure-strategy equilibria, most of our results do not change if the entrepreneur is allowed to play mixed strategies. In particular, Lemma 1 has to hold for any net claims position attained with positive probability along the equilibrium path.

In summary, the results derived from Proposition 1 are that (1) when both projects have a positive value an (pure-strategy) equilibrium outcome always exists and is unique, (2) the outcome is separating, with the entrepreneur of type  $L$  never completely diversifying his risk, (3) the equilibrium configuration depends on priors and the available amount of wealth ( $R_2 + W$ ), (4) the standard incentive-compatibility constraint is not binding. In particular, these results imply that the low-risk type varies his collateral with priors with the consequence that the degree of diversification is positively correlated with priors. This contrasts with standard screening models where, when a separating equilibrium exists, the modalities of financing do not change when priors change.

### 3.3. *The implication of recontracting for equilibrium contracts*

We now briefly discuss the efficiency effects of allowing the entrepreneur to recontract. When recontracting is excluded, the analysis of our screening game encounters the same non-existence problem as uncovered by Rothschild and Stiglitz (1976). Therefore, we compare the equilibrium outcome of Proposition 1 with the outcomes that Wilson (1977) and Riley (1979) have suggested as relevant characterizations of the competitive determination of contracts in the presence of adverse selection.<sup>15</sup> Recall that the difference between the Riley and Wilson outcomes results from differences in the allowed reactions of uninformed players following out-of-equilibrium contract offers. In the setup analysed by Wilson, contracts that make negative profits following an out-of-equilibrium offer can be withdrawn. In contrast, in the Riley setup, players can react to out-of-equilibrium offers by offering new contracts. When the Rothschild and Stiglitz non-existence problem arises, the Wilson outcome corresponds to a pooling configuration while the Riley outcome corresponds to a separating configuration. When the non-existence problem does not arise, both the Riley and the Wilson outcomes correspond to the separating-equilibrium outcome discussed in Rothschild and Stiglitz.<sup>16</sup>

In most strategic situations, the elimination of a possibility to commit is welfare worsening;<sup>17</sup> in the presence of adverse selection and competition, however, this is not necessarily the case. Whenever the parameters of the model are such that the non-existence problem arises in the Rothschild and Stiglitz game, the recontracting-proof outcome always dominates both the Riley outcome and the Wilson outcome. Note that, in such cases, the Wilson outcome dominates the Riley outcome and therefore we need only consider why

14. Madrigal, Tan and da Costa Werlang (1987) and Noldeke and van Damme (1990) present a game of incomplete information in which the unique Nash equilibrium is supported by a changing support of beliefs.

15. Although the original work by Wilson and Riley did not adopt standard equilibrium concepts, recent work has presented modified screening games in which the Wilson and Riley outcomes are PBE outcomes. For example, Desruelle (1988) offers a formalization of the Wilson outcome, while Engers and Fernandez (1987) offer a formalization of the Riley outcome. Nevertheless, these modified games still exclude the possibility of recontracting, and are therefore good points of comparison with our game.

16. It is interesting to note that the non-existence problem encountered by Rothschild and Stiglitz does not arise when dynamic elements are introduced, either along the lines discussed in Wilson and Riley, or in the explicit dynamic setting analyzed here.

17. See Dewatripont and Maskin (1990) for a discussion of this effect in renegotiation problems.

the recontracting-proof outcome dominates the Wilson outcome. Under this parameterization the entrepreneur with a low-risk project attains the same net financial position in both the recontracting-proof outcome and in the Wilson outcome, that is, he receives his preferred pooling contract. It is the entrepreneur with the high-risk project that is better off with the recontracting-proof outcome than with the Wilson outcome. The type *H* entrepreneur attains higher utility by the fact that he can always profitably recontract from type *L*'s contract. Thus the possibility of recontracting eliminates all distortions on type *H*'s equilibrium outcome at no cost to the financier. This is why recontracting improves welfare. In the cases where the non-existence problem does not arise, the recontracting-proof outcome does not dominate the separating equilibrium (which is also the Riley and Wilson outcome), since the type *L* is worse off, while the type *H* is better off. Intuitively, the reason why non-commitment can be Pareto improving is that, in the presence of adverse selection and commitment, the market equilibrium may be inefficient since it does not necessarily allow for cross-subsidization. In fact, cross-subsidization is similar to a public good where, in the presence of commitment, the benefits of the public good can be totally appropriated by a non-provider (since the non-provider can skim the market). This public good is therefore never provided. In contrast, without a commitment not to recontract, a financier cannot "steal" only the good risk since the bad risk have an advantage to mimic good risk and recontract at a future date.

It is important to note that the equilibrium configuration described in Proposition 1 (when  $R_2 + W \geq 0$ ) was first derived by Jaynes (1978) in a closely related (but non-game-theoretic) analysis of multiple purchases of insurance contracts.<sup>18</sup> In a relatively critical appraisal of Jaynes' work, Hellwig (1988) proposes a game tree in which the "Jaynes" outcome is supported as a Nash equilibrium. Hellwig's analysis suggests that this outcome can only be supported when information on purchases of contracts can be strategically withheld from other contractors. In particular, the equilibrium requires that uninformed contractors share information on all purchases along the equilibrium path, but in the case of a deviation, that they behave collusively and not share information with the deviating contractor. Hellwig therefore concludes that the relevance of the "Jaynes" outcome as describing competitive contractual markets is dubious. In contrast, Proposition 1 demonstrates that the "Jaynes" outcome can be supported even when new contractors are perfectly informed about all previous contractual arrangements, which seems to be quite reasonable for well-organized financial markets. Our analysis can therefore be considered as providing a game-theoretic foundation to the "Jaynes" outcome that arises in a competitive environment and does not depend on the concealment of previous transactions.

Beaudry and Poitevin (1993) show that the "Jaynes" outcome can also arise in a situation where a single contract is renegotiated over time and where all the renegotiation proposals are made by the informed player. This suggests that it is not the identity of the proposer (informed vs. uninformed) that is important for obtaining this outcome, but rather the possibility of recontracting.

#### 4. EQUILIBRIUM ANALYSIS WHEN THE HIGH-RISK PROJECT HAS NEGATIVE VALUE

In the above section we show that when both projects have a positive value, sufficient financing is always obtained for each type. The equilibrium is such that the high-risk

18. Gale (1991) also discusses this equilibrium configuration in a model of repeated financing. Asheim and Nielsen (1991) characterize an outcome similar to Jaynes' using cooperative game theory concepts.

project is being implicitly subsidized by the low-risk project, even though the final outcome is separating. This cross-subsidization between types makes it important to examine the case of the high-risk project having a negative value. In this section, we assume that the high-risk project is non-profitable, and we investigate whether investment is optimal in the sense that only the low-risk project receives financing; specifically we examine whether the presence of negative-value projects causes the market to collapse as in Akerlof's (1970) famous Lemons problem. We therefore consider the worst possible case in which the average project has negative value, while the low-risk project has a positive value.

**Proposition 2.** *Suppose that (1)  $V(R_1, R_2, L) > 0$ , and (2)  $V^P(R_1, R_2, \mu_0) < 0$ . Then, independently of priors  $\mu_0$ , there always exists an equilibrium in which the low-risk project is financed, with the contractual outcome for type  $L$  given by the solution to the following maximization problem:*

$$S_L^* = \arg \left\{ \begin{array}{ll} \max_{S_L} U(S_L, L) & \text{s.t. } V(S_L, H) \geq V(R_1, R_2, H) \\ & V(S_L, L) \geq 0 \\ & R_2 + W - S_{2L} \geq 0 \end{array} \right\}.$$

This proposition shows that Akerlof's conjecture about markets breaking down does not necessarily arise in our setting even though the average value of a project may become arbitrarily negative. The presence of negative-value projects does not cause the market to collapse since, when the contracting space is rich enough, it is possible for the low-risk type to separate from the high-risk project even though the latter is not undertaken. In order to understand this possibility, suppose the high-risk type does not undertake his project. In this case, the high-risk type must be at least indifferent between not undertaking his project and undertaking it and giving all the returns to a financier, that is, setting  $\{S_1, S_2\} = \{R_1, R_2\}$ . Therefore, by Lemma 1, the low-risk type's contract cannot be on an iso-profit curve  $V(\cdot, H)$  of lower value than  $V(R_1, R_2, H)$ , that is,  $V(S_L^*, H) \geq V(R_1, R_2, H)$ . Otherwise the high-risk type would prefer to mimic type  $L$  and solicit offers rather than forego his project. When the low-risk type has some collateral  $W > 0$  to invest in his project there are always contracts on the curve  $V(\cdot, H) = V(R_1, R_2, H)$  that satisfy the financing constraint, the low-risk type's and the financier's individual-rationality constraints. It is then always possible for the low-risk type to secure enough financing for his project regardless of the priors or how bad is the high-risk project. In this case, investment is optimal as only the profitable low-risk projects are undertaken.

Figure 3 illustrates an equilibrium where priors are pessimistic and only the low-risk project is undertaken. In this example, both entrepreneurs solicit financial offers. Financiers offer the unique contract  $S_L^*$ . This contract is taken only by the low-risk entrepreneur and after the project is undertaken, the entrepreneur does not solicit more offers. If he did, financiers would believe the entrepreneur is of type  $H$  and would therefore offer the contract  $\{0, 0\} - S_L^*$ . This explains why the high-risk entrepreneur does not accept the offer  $S_L^*$ , undertake the project and solicit more offers, since then he would not be better off than by refraining from undertaking the project.

We should mention that in the case where the high-risk project has negative value, there also exist equilibria in which no project is financed. This equilibrium is supported by the beliefs that any initial solicitation is believed to come from a high-risk entrepreneur. In this case, investment is sub-optimal as no project is financed even though the low-risk project is profitable. The presence of these two types of equilibria underlines a coordination problem that may affect competitive markets with adverse selection. This coordination



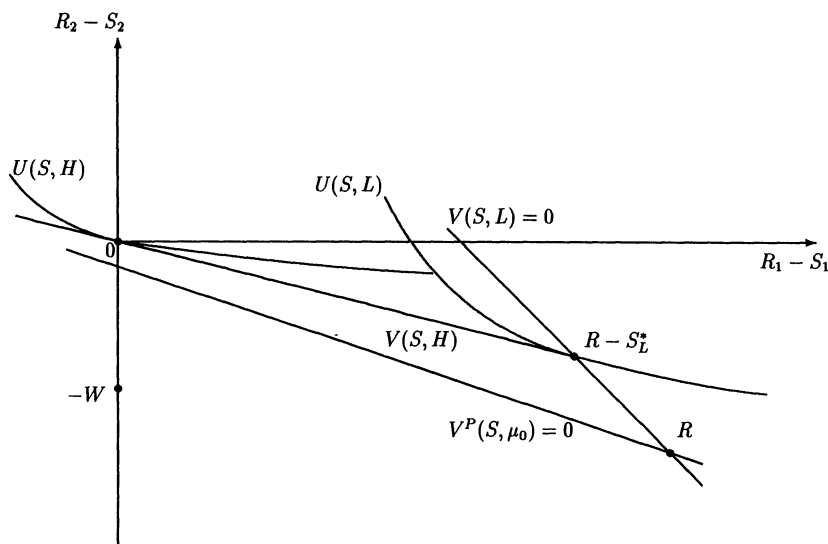


FIGURE 3

Equilibrium outcomes: High-risk project has negative value an beliefs are pessimistic

failure originates through the beliefs of financiers when initially solicited by an entrepreneur. Therefore, in this environment, the possible collapse of the financial market arises due to a coordination failure and not due directly to the asymmetric nature of information as in Akerlof's original analysis.

## 5. THE PROBLEM OF COMMITMENT

The central assumption underlying our analysis is that entrepreneurs cannot commit not to sell junior claims once the project is undertaken; it seems however reasonable to argue that a financier could include in a financial agreement a clause that limits the entrepreneur's right to sell such claims. Debt covenants may be considered such an example; the appropriate question to ask however is whether a financier would in fact want to enforce such a clause if it was included in the agreement. In the model presented in this paper, financiers have no interest in enforcing such a clause *ex post* since junior claims do not directly affect a senior claimant's returns. For a senior claimant, the sale of junior claims can only indicate to him that his claim may not be as valuable as he thought it would be, that is, the acquisition of a junior claim may reveal to the senior claimant that his client is of a given type, for example a type  $H$ , when he was in fact uncertain of the entrepreneur's type. But, given that previous contracts must be enforced, this new information cannot affect the senior claimant's returns. It is a typical example of a sunk investment. In summary, *ex ante*, a senior claimant would like to commit to enforcing a restriction on the sale of junior claims in order to make his offer unattractive to high-risk entrepreneurs. But, once he has acquired a claim on a high-risk project, the sale of junior claims by the entrepreneur leaves him just indifferent. Therefore, the equilibria described in this paper would generally remain valid (although multiplicity of equilibria would arise) if (1) financiers could write in their contracts restrictions on the future sales of claims, (2) the entrepreneur could ask for these restrictions to be waived once he has received new offers, and

(3) financiers could not bind themselves to enforcing these restrictions by the use of a third party.

## 6. EXPLAINING VARIATIONS IN EXTERNAL VS. INTERNAL FINANCING

In this section, we briefly indicate how the results of our recontracting analysis may be helpful in explaining changes in firms' financial decisions over the business cycle. Strictly speaking, our model applies to entrepreneurial firms; it may, however, be reasonable to consider our analysis as also applying to managerial firms, since managers are often assumed to be risk averse either with respect to funds under their control, or to stockholders' wealth. To highlight some observations about firms' financial decisions, Table 1 reports information on the sources and uses of funds by non-financial American corporations for the period 1948–1987.<sup>19</sup> There are at least two features in the table worth noticing. First, the reliance on internal funds used for investment increases in recessions. Second, financial slack, measured by the change in marketable securities, increases in booms. We argue that these facts cannot be easily explained by existing agency models of financial decisions but that the recontracting approach may provide an explanation.<sup>20</sup>

Agency models of financial decisions, starting with Jensen and Meckling (1976), have argued that, in the presence of informational asymmetries (whether it be moral hazard or adverse selection), firms would choose the financial structure that minimizes the agency costs generated by those asymmetries. Based on the idea that managers are likely to be better informed about the value of investments, Myers (1984) has developed the “pecking-order” theory of corporate financial structure. According to this theory, firms prefer to finance internally until depletion of their financial slack and then finance on external markets, with different instruments ranking differently. This would minimize the agency costs of financing since it reduces the demand for external finance. Even though this theory seems plausible, the data reported in Table 1 do not seem to be reconcilable with a strict “pecking-order” theory. On the one hand, if cash flow is more volatile than physical investment opportunities, then we should observe a greater reliance on external funds during recessions as firms' cash flow would not suffice to finance investments. On the other hand, if investment opportunities are more volatile than cash flow, then, during booms, firms should be depleting their stock of marketable securities since this would be the cheapest source of funds, that is, according to the “pecking-order” theory firms should exhaust their sources of internal financing before going to external financial markets. Obviously, these predictions seem at odds with the data of Table 1.

In contrast, the recontracting framework may provide an explanation of the data of Table 1. Recall from Proposition 1 that when priors are optimistic both types of entrepreneur rely mostly on external financing as entrepreneurs do not post their wealth as collateral on the project. When priors are pessimistic, good entrepreneurs invest all their wealth into the project, thus reducing their external financing.<sup>21</sup> These predictions seem more consistent with Table 1 since, in booms, when the expected productivity of investment is high, and therefore priors are optimistic, the cross-subsidization cost that good projects associate

19. These data are drawn from the Economic Report of the President (1990).

20. See Myers (1990) for an overview of why traditional models of financial decisions seem rather incompatible with observed decisions.

21. This argument can be made more formally. For any net payment  $\{S_1, S_2\}$ , the amount of risk capital provided internally by the low-risk entrepreneur is given by  $|\min\{0, \bar{R}_1 - B - S_{1L}, \bar{R}_2 - B - S_{2L}\}| = |\min\{0, \bar{R}_2 - B - S_{2L}\}|$ . It is easy to verify that  $dS_{2L}^*/d\mu_0 \leq 0$ , that is, type  $L$ 's reliance on internal financing decreases as the market's prior improves. Consequently, if there are enough good types in the population, the average reliance on internal funds will decrease in booms.

TABLE I

*Non-farm non-financial corporate business: sources and uses of funds†*

Year	Sources			Uses		
	Total (in billion \$)	Internal (in %)	External (in %)	Total (in billion \$)	Capital expenditures (in %)	Increase in financial assets (in %)
1948	29.4	67.0	33.3	25.6	80.9	19.5
*1949	20.5	97.6	2.4	18.4	81.0	19.0
1950	42.6	43.4	56.3	40.4	59.4	40.6
1951	36.9	56.4	43.6	37.9	80.7	19.5
1952	30.2	74.5	25.8	30.0	84.7	15.3
1953	28.6	78.0	21.7	28.5	91.9	8.1
*1954	29.8	81.9	18.1	28.1	82.9	17.4
1955	53.4	56.0	43.8	49.1	66.2	33.6
1956	45.1	66.7	33.5	41.1	90.5	9.7
1957	43.5	73.6	26.4	40.0	89.3	10.5
*1958	42.2	72.7	27.5	38.5	72.2	28.1
1959	56.6	64.3	35.7	52.1	72.9	27.3
*1960	48.2	74.5	25.7	41.8	90.4	9.3
1961	55.8	66.1	33.9	50.7	72.0	28.0
1962	60.6	71.3	28.7	56.2	77.9	22.2
1963	68.5	68.6	31.5	60.3	73.8	26.0
1964	74.2	70.5	29.5	64.9	77.2	23.0
1965	92.7	63.8	36.2	83.4	73.9	26.1
1966	99.0	63.9	36.1	92.0	81.8	18.2
1967	94.9	67.7	32.3	87.6	81.3	18.7
1968	114.0	57.7	42.4	106.2	71.0	29.0
1969	116.0	56.2	43.8	115.0	72.4	27.6
*1970	101.8	61.7	38.3	97.9	80.9	19.1
1971	127.4	58.6	41.4	121.8	69.9	30.1
1972	153.4	56.3	43.7	145.1	65.5	34.5
1973	215.2	25.0	56.4	189.7	62.7	37.3
1974	179.0	49.9	47.9	191.1	72.5	27.5
*1975	155.3	80.4	19.6	153.4	73.2	26.8
1976	214.6	66.2	33.8	210.4	74.6	25.4
1977	259.3	63.7	36.3	242.2	74.2	25.8
1978	314.1	58.0	42.0	324.7	66.8	33.2
1979	326.0	60.6	39.4	368.1	64.7	35.3
*1980	324.8	61.6	38.4	342.1	71.2	28.8
1981	375.8	63.7	36.3	383.6	74.7	25.3
*1982	298.5	81.2	18.8	303.5	84.5	15.5
1983	420.3	68.0	32.0	385.8	70.2	29.8
1984	492.6	68.3	31.7	502.7	73.7	26.3
1985	459.2	76.7	23.3	435.3	78.6	21.4
1986	492.2	72.6	27.4	454.3	73.0	27.0
1987	474.1	74.4	25.6	436.6	82.7	17.3

† Data taken from the *Board of Governors of the Federal Reserve System* (1990).

\* Trough of the recession.

with external financing is low and consequently most firms rely on external financing. This allows firms to accumulate financial slack. Inversely, in recessions, the "Lemons premium" is high and therefore good projects are financed mostly internally, thus reducing the stock of marketable securities. This leads firms with good projects to find themselves in a more financially fragile position in recessions. The recontracting approach can help explain observed financial flows since it is consistent with a time-varying Lemons premium that induces time-varying financial policy. In effect, a low Lemons premium in boom will lead

good firms to rely more heavily on external financing and even accumulate financial assets, while bad times will induce firms to rely mostly on internal funds.

The idea that a varying Lemons premium may help explain the time-series properties of firms' financial decisions is not new; we believe however that the recontracting framework provides a much clearer description of this phenomenon than previous models. For example, Gertler and Hubbard (1988) and Bernanke and Gertler (1989, 1990) explain a varying Lemons premium by a balance-sheet effect.<sup>22</sup> In these models, entrepreneurs invest all of their wealth in the project, and consequently, agency costs are a decreasing function of the entrepreneur's net worth. In recessions, net worth is low and it is therefore costly to finance investments on external markets, thereby causing some firms to reduce the size of their investment projects. This negative relationship between agency costs and collateralizable wealth is the crucial factor explaining the increased cost of external funds in downturns; however, despite the fact that investment decreases, their approach predicts that the ratio of internal to external funds decreases in recessions. Hence, the same criticism discussed for the "pecking-order" theory applies: these models cannot explain the greater reliance on internal funds in recessions as shown in Table 1.

## 7. CONCLUSION

The object of this paper is to examine how the possibility of recontracting affects our understanding of markets with adverse selection. Our main finding is that recontracting effectively forces low-risk projects to cross-subsidize high-risk projects even though the environment is competitive and that separation is achieved in equilibrium. One direct implication of this result is that the contractual terms offered by market participants depend on their prior beliefs. Therefore, recontracting provides a potential framework to examine markets that seem to simultaneously exhibit a Lemons-type problem and separation.

## APPENDIX

The following lemma provides a preliminary result that is used in subsequent proofs.

Define by  $S_t^n$  type  $t$ 's net claims position at the beginning of round  $n$  if all players have followed their equilibrium strategy since the beginning round of the recontracting subgame, that is,  $S_t^n$  is the net claims position on the equilibrium path of type  $t$ . Also define  $V^n(S, \mu) := \mu V(S, L) + (1 - \mu)V(S, H)$ .

**Lemma 0.** *Consider any recontracting subgame starting at a net claims position  $\hat{S}$  with financiers' beliefs  $\hat{\mu}$  in round  $\hat{n}$ . Then, for all  $n \geq m$*

$$V^n := \hat{\mu} V(S_L^n, L) + (1 - \hat{\mu}) V(S_H^n, H) \geq V^m.$$

*Proof.* Following  $\hat{S}$ , offers in  $\hat{n}.2$  and acceptance decisions in  $\hat{n}.3$  can lead to two possible equilibrium net claims positions in  $\hat{n} + 1$ ,  $\{S_H^{\hat{n}+1}, S_L^{\hat{n}+1}\}$  such that either:

- (i)  $S_L^{\hat{n}+1} = S_H^{\hat{n}+1}$ ; or
- (ii)  $S_L^{\hat{n}+1} \neq S_H^{\hat{n}+1}$ .

In both cases, financiers' individual rationality implies that  $V^{\hat{n}+1} \geq V^{\hat{n}}$ . Note that we do not necessarily have  $V(S_i^{\hat{n}+1}, t) \geq V(\hat{S}, t)$  since some cross-subsidization across types is possible if the same financier finances both types but with different contracts.

22. Stiglitz and Weiss (1981) also explains the varying Lemons premium by changes in the market's perception about the project; they cannot, however, explain the varying reliance on internal funds since this is not a choice variable in their model.

Now consider the beliefs in round  $(\hat{n}+1).2$  following an equilibrium solicitation decision in  $(\hat{n}+1).1$ . In case (i), beliefs  $\mu^{\hat{n}+1} \in \{0, \hat{\mu}, 1\}$ . If  $\mu^{\hat{n}+1} = \hat{\mu}$ , the previous argument can be used to show that  $V^{\hat{n}+2} \geq V^{\hat{n}+1}$ . If  $\mu^{\hat{n}+1} \in \{0, 1\}$ , then one type is soliciting and the other one is not. Individual rationality then implies that  $V(S_t^{\hat{n}+2}, t) \geq V(S_t^{\hat{n}+1}, t)$ , with  $t=L$  if  $\mu^{\hat{n}+1}=1$ , and  $t=H$  if  $\mu^{\hat{n}+1}=0$ . In that case the other type's equilibrium net claims position is left unchanged since its equilibrium strategy calls for it not to solicit. This implies that  $V^{\hat{n}+2} \geq V^{\hat{n}+1}$ . In case (ii), beliefs  $\mu^{\hat{n}+1} \in \{0, 1\}$  since separation was achieved by the offers of round  $\hat{n}$ . The argument made above for case (i) then implies that  $V^{\hat{n}+2} \geq V^{\hat{n}+1}$ .

It is clear that these arguments can be repeated to show that  $V^n \geq V^m$  for all  $n \geq m$ .  $\parallel$

*Proof of Lemma 1.* The proof proceeds in two steps. We first show that constraint (3.1) must be satisfied for  $\tilde{S} = \hat{S}$  (where  $\hat{S}$  represents the net claims position at the beginning of a recontracting subgame) and then we show that the same argument can be applied to any  $\tilde{S}$  on the equilibrium path.

*Step 1.* Let us denote a conjectured equilibrium outcome of a recontracting subgame by  $\{S'_L, S'_H\}$  and suppose that  $U(S'_H, H) < U(\Phi_H(\hat{S}), H)$ . In the first round of contract offers, there is at least one solicited financiers who knows that his contract offers will not be accepted (since there are more financiers than types of entrepreneurs) and therefore is making zero expected profits. Now consider the financier making an offer within the set:  $\{s/U(s+\hat{S}, H) > U(S'_H, H), V(s, H) > 0 \text{ and } V^P(S-\hat{S}, \hat{\mu}) > 0\}$ . The maintained assumption that  $U(S'_H, H) < U(\Phi_H(\hat{S}), H)$  guarantees this set to be non-empty (this is a direct consequent of there being strictly positive gains from trade between this financier and the entrepreneur of type  $H$ ). By construction of the set, if a contract within this set is offered, it is necessarily in the interest of type  $H$  to accept the offer since rejecting the offer could never lead to a better outcome. That rejecting an offer cannot lead to a better outcome is due to the fact that rejected offers are not observable by other financiers and therefore cannot influence the remaining play of the game. Consequently, it is in the interest of this financier to deviate and offer an element within the set which, again by construction, guarantees the deviating financier more profits than his equilibrium payoff regardless of whether it is also accepted by the type  $L$  entrepreneur.

To complete the proof that constraint (3.1) must be satisfied for  $\tilde{S} = \hat{S}$ , it is necessary to prove that  $U(S'_L, L) < U(\Phi_H(\hat{S}), H)$  is also impossible. This statement follows immediately from the above argument that  $U(S'_H, H) \geq U(\Phi_H(\hat{S}), H)$  combined with the standard incentive compatibility constraint  $U(S'_L, L) \geq U(S'_H, L)$ .

*Step 2.* Let us consider a net claims position  $\tilde{S}$  that can be obtained along the equilibrium path and let us consider the play of the game when the entrepreneur solicits further offers after obtaining  $\tilde{S}$  (this may be out-of-equilibrium play). Denote the induced outcome of this play by  $\{\tilde{S}_L, \tilde{S}_H\}$ , where  $\tilde{S}_t$  represents the outcome for type  $t$  if he is soliciting. From step 1, it must be the case that  $U(\tilde{S}_t, t) \geq U(\Phi_H(\tilde{S}), t)$ , for  $t=L$  or  $H$ . Consequently, it must be the case that  $U(S'_t, t) \geq U(\Phi_H(\tilde{S}), t)$  by the definition of an equilibrium.  $\parallel$

*Proof of Lemma 2.* There are three steps in the proof. First it will be shown that a type  $L$  entrepreneur must receive an equilibrium payoff at least as high as that defined by the maximization problem given in the statement of the proposition. Second it will be shown that the constraint  $V(S'_L, L) \geq V(\hat{S}, L)$  must be satisfied. Thirdly the relevance of the constraint  $V^P(S'_L, \hat{\mu}) \geq V^P(\hat{S}, \hat{\mu})$  is proven. Regrouping these results implies that a type  $L$  entrepreneur must receive exactly the level of utility defined by the maximum, which is attainable only through the position  $S'_L$ .

*Step 1.* Suppose that type  $L$ 's equilibrium payoff is less than that given by the maximization problem. Then, in the first round of contract offers, there is at least one financier who is making zero expected profits and who would gain by deviating and offering a contract with the set  $\{s/U(s+\hat{S}, L) > U(S'_L, L), V(s, L) > 0 \text{ and } V^P(s, \hat{\mu}) > 0\}$ . By offering such a contract, the financier is sure to gain since it will necessarily be accepted by type  $L$ , and even if it is accepted by type  $H$ , its expected profits remain positive. The type  $L$  is sure to accept the offer since rejecting can not lead to an outcome he prefers.

*Step 2.* Suppose that  $V(S'_L, L) < V(\hat{S}, L)$ . Then by Lemma 0 it must be the case that  $V(S'_H, H) > V(\hat{S}, H)$ . But if  $V(S'_H, H) > V(\hat{S}, H)$ , then  $U(S'_H, H) < U(\Phi_H(\hat{S}), H)$  which contradicts Lemma 1.

*Step 3.* Suppose  $V^P(S'_L, \hat{\mu}) < V^P(\hat{S}, \hat{\mu})$ . Then by Lemma 0 it must be the case that  $V(S'_H, H) > V(S'_L, H)$ . But if  $V(S'_H, H) > V(S'_L, H)$ , then  $U(S'_H, H) < U(\Phi_H(S'_L), H)$  which contradicts Lemma 1.  $\parallel$

*Proof of Lemma 3.* From Lemma 1, we know that  $U(S'_H, H) \geq U(\Phi_H(S'_L), H)$  and therefore if  $S'_H \neq \Phi_H(S'_L)$  it must be the case that  $V(S'_H, H) < V(S'_L, H)$ ; however, from Lemma 0 we know that  $\hat{\mu}V(S'_L, L) + (1-\hat{\mu})V(S'_H, H) \geq \hat{\mu}V(\hat{S}, L) + (1-\hat{\mu})V(\hat{S}, H)$ , and we know from Lemma 2 that  $V^P(S'_L, L) = V^P(\hat{S}, \hat{\mu})$ . Combining these two conditions, we have that  $V(S'_H, H) \geq V(S'_L, H)$ , which contradicts the premise. Consequently, it is impossible for  $U(S'_H, H) > U(\Phi_H(S'_L), H)$  and therefore  $S'_H$  must be equal to  $\Phi_H(S'_L)$ .  $\parallel$

*Proof of Corollary 1.* The only element that needs proof in Corollary 1 is showing that the stated maximization is equivalent to maximizing the same objective function subject to the constraint set  $\{V^P(S_L, \hat{\mu}) \geq V^P(\hat{S}, \hat{\mu}), V(S_L, L) \geq V(\hat{S}, L), U(S_H, H) \geq U(\Phi_H(S_L), H)\}$ , since this latter maximization is clearly a restatement of Lemmas 2 and 3. In order to see this equivalence, note that  $U(S_H, H) \geq U(\Phi_H(S_L), H)$  implies  $V(S_H, H) \leq V(S_L, H)$  and that  $U(S_H, H) \geq U(\Phi_H(\hat{S}), H)$  implies  $V(S_H, H) \leq V(\hat{S}, H)$ . Then note that (1) combining  $V(S_H, H) \leq V(S_L, H)$  with  $\hat{\mu}V(S_L, L) + (1 - \hat{\mu})V(S_H, H) \geq \hat{\mu}V(\hat{S}, L) + (1 - \hat{\mu})V(\hat{S}, H)$  implies  $V^P(S_L, \hat{\mu}) \geq V^P(\hat{S}, \hat{\mu})$ , and (2) combining  $V(S_H, H) \leq V(\hat{S}, H)$  with  $\hat{\mu}V(S_L, L) + (1 - \hat{\mu})V(S_H, H) \geq \hat{\mu}V(\hat{S}, L) + (1 - \hat{\mu})V(\hat{S}, H)$  implies  $V(S_L, L) \geq V(\hat{S}, L)$ . Therefore the maximization in the proposition is equivalent to maximizing the same objective function subject to the constraint set  $\{V^P(S_L, \hat{\mu}) \geq V^P(\hat{S}, \hat{\mu}), V(S_L, L) \geq V(\hat{S}, L), U(S_H, H) \geq U(\Phi_H(S_L), H), U(S_H, H) \geq U(\Phi_H(\hat{S}), H)\}$ . But then by Lemma 3, it is obvious that the constraint  $U(S_H, H) \geq U(\Phi_H(\hat{S}), H)$  is redundant in this maximization, which proves the necessary equivalence.  $\parallel$

*Proof of Proposition 1.* We first prove necessity by showing that each type's outcome must be the solution to the relevant maximization problem of the proposition. Then we show that there exist strategies and beliefs that support this outcome as a PBE outcome of the game.

*Necessity.* We first show that both types of entrepreneur must solicit financing in equilibrium. This gives us the financiers' equilibrium beliefs following the initial solicitation by the entrepreneur. We then use Corollary 1 to show that the solution to the maximization problems of the proposition effectively characterizes each type's equilibrium outcome of the complete game. Finally, we argue that  $S^*$  is unique.

*Step 1.* At its initial move, each type of entrepreneur can solicit or not financing. By Lemma 1, we know type  $H$  always solicits in equilibrium. His payoff is then greater or equal to  $U(\Phi_H(0, 0), H)$ . As for type  $L$ , suppose he does not solicit any financing. His payoff would therefore be  $u(W)$  since then he prefers not to undertake his project rather than finance it internally (if this is possible). But since type  $H$ 's project has positive value, it must be the case that  $U(\Phi_H(0, 0), L) > u(W)$ . Type  $L$  therefore solicits financing as well in the initial round. Since both types solicit financing in equilibrium, the beliefs  $\mu_j$ , for  $j = 1, \dots, J$  must be equal to  $\mu_0$ .

*Step 2.* Suppose that the solution to type  $L$ 's maximization problem is such that the financing constraint,  $R_2 + W - S_{2L} \geq 0$ , is not binding. In that case, the complete game is just a special case of a recontracting subgame characterized in Corollary 1 in which  $\hat{S} = \{0, 0\}$  and  $\hat{\mu} = \mu_0$ . Note that, in Corollary 1, when  $\hat{S} = \{0, 0\}$ , the constraint  $U(S_H, H) \geq U(\Phi_H(\hat{S}), H)$  is never binding at the maximum (because  $u(W) > U(0, 0, L)$ ). Moreover, by Corollary 1 we know that each type's equilibrium outcome must then be the solution to the maximization problem.

*Step 3.* Suppose that the solution to the maximization problem of the proposition is such that the financing constraint is binding. We first show that if, in equilibrium  $S_L^*$  is offered initially and accepted, then  $S^*$  must be the equilibrium outcome. We then show that  $S_L^*$  is in fact offered initially and accepted.

If the financing constraint is binding, then the outcome  $S^*$  corresponds to the solution of the maximization problem in Corollary 1 when  $\hat{S} = S_L^*$  and  $\hat{\mu} \leq \mu_0$ . Furthermore, we know that if type  $L$  solicits from  $S_L^*$  he cannot earn more than  $U(S_L^*, L)$  without some financier making negative expected profits. Therefore, upon solicitation from  $S_L^*$ , financiers must have the beliefs  $\mu^* \leq \mu_0$ . Hence  $S^*$  is the final outcome when  $S_L^*$  is offered and accepted.

We now determine whether  $S_L^*$  must in fact be offered and accepted in any equilibrium. From the maximization, it is obvious that the type  $L$  entrepreneur cannot accept another contract weakly preferred to  $S_L^*$  without a financier making negative profits (given the recontracting-induced incentive-compatibility constraint). Furthermore, if, in equilibrium, type  $L$ 's utility was lower than  $U(S_L^*, L)$ , using the same argument as in the proof of Lemma 2, it is possible to show that there is a financier in the initial round that can increase his profits by offering a deviating contract. Such a contract will exist as long as type  $L$ 's utility is lower than  $U(S_L^*, L)$ . Therefore,  $S_L^*$  must be offered and accepted.

*Step 4.* Given the curvature properties of the utility and profit functions, it is easy to show that  $\{S_H^*, S_L^*\}$  is unique.

*Sufficiency.* Sufficiency must be shown by constructing strategies and beliefs that support this outcome as a PBE outcome. These strategies and beliefs are now written formally.

Because the characterization of the equilibrium outcome of a subgame depends on the net claims position of the entrepreneur at the beginning of the subgame (see Corollary 1), we introduce some notation that will facilitate the writing of the strategies and beliefs. Denote by  $\bar{S}_n$ , the entrepreneur's net claims position at the beginning of round  $n$  ( $\bar{S}_1 = \{0, 0\}$ ), and by  $\bar{S}^j(\bar{S}_n)$  the net claims position induced if financier  $n_j$ 's  $k$ -th proposal

is accepted. Finally, denote by  $\Omega(\hat{S}) = \{\Omega_L(\hat{S}), \Omega_H(\hat{S})\}$  the mapping from entrepreneur  $t$ 's net claims position  $\hat{S}$  into two new net claims positions. This function is defined as follows.

$$\Omega(\hat{S}) := \arg \left\{ \begin{array}{ll} \max_{S_L, S_H} U(S_L, L) & \text{s.t. } \mu_0 V(S_L, L) + (1 - \mu_0) V(S_H, H) \geq V^P(\hat{S}, \mu_0) \\ & U(S_H, H) \geq \max \{U(\Phi_H(S_L), H), U(\Phi_H(\hat{S}), H)\} \\ & R_t + W - S_{tL} \geq 0 \end{array} \right\}$$

where  $\Phi_H(\hat{S}) := \arg \{ \max_S U(S, H) \text{ s.t. } V(S, H) \geq V(\hat{S}, H) \}$ .

In words, the function  $\Omega(\hat{S})$  takes an arbitrary net claims position  $\hat{S}$  into the two net claims positions characterized in Corollary 1; however, in the event that the position thereby defined would not satisfy  $R_t + W - S_{tL} \geq 0$  (would not permit the project to be realized), the mapping  $\Omega(\cdot)$  is defined to take this additional constraint into account and therefore always maps into net positions that would make the project realizable. Note that it can be shown that, for any  $\hat{S}$  there always exists a unique solution to this maximization problem and that therefore the function  $\Omega(\cdot)$  always exists.

We now construct the entrepreneur's strategy. For  $t = H, L$ , and all  $n = 1, \dots, \infty$ ,

$$\sigma_t = \begin{cases} \text{For all histories } \mathcal{H}_n^s, \\ \sigma_{tn} = \begin{cases} \text{not solicit} & \text{if } \bar{S}_n = \Omega_t(\bar{S}_n), \\ \text{solicit} & \text{otherwise.} \end{cases} \\ \text{For all histories } \mathcal{H}_n^a, \\ \sigma_{tn} = \begin{cases} 0 & \text{if } \max_{n_j^t} U(\Omega_t(\bar{S}^{tj}), t) < U(\Omega_t(\bar{S}_n), t), \\ n_j^{k*} = \arg \{ \max_{n_j^t} U(\Omega_t(\bar{S}^{tj}), t) \} & \text{otherwise (if indifferent, choose a contract from financier } n_1). \end{cases} \end{cases}$$

We now construct the financiers' strategies. For all  $n = 1, \dots, \infty$ ,  $j = 1, \dots, J$ , and for all financial statements  $\mathcal{H}_n$ ,

$$\omega_{nj} = \{\Omega_L(\mathcal{H}_n) - \mathcal{H}_n, \Omega_H(\mathcal{H}_n) - \mathcal{H}_n\}.$$

The financiers' beliefs are given as follows. For all  $n = 1, \dots, \infty$ , and  $j = 1, \dots, J$ ,

$$\mu_{nj} = \begin{cases} \mu_0 & \text{if } U(\mathcal{H}_n, L) < U(\Omega_L(\mathcal{H}_n), L), \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to verify that these strategies and beliefs, which are described verbally in the main text, do in fact form a PBE. The main element to notice is that the financiers, whether on or off the equilibrium path, always offer the two contracts defined by  $\Omega(\cdot) - \mathcal{H}_n$ . These offers are then followed by the entrepreneur accepting his preferred contract among the two and then stopping soliciting offers.  $\parallel$

*Proof of Proposition 2.* The following strategies and beliefs support the equilibrium outcome  $S_L^*$  in which only type  $L$  is financed.

Both types of entrepreneur solicit financing in round 1; upon solicitation, financiers keep their initial prior beliefs; they then offer  $S_L^*$ ; it is taken only by type  $L$  and there is no further solicitation. If an entrepreneur ever solicits further financing (in any subgame), he is taken to be a type  $H$  and is offered  $\Phi_H(S_L^*) - S_L^*$ . Therefore it is not in the interest of type  $H$  to accept the initial financing (he would obtain exactly his reservation utility). Moreover, by construction of  $S_L^*$ , the financiers cannot make an initial offer that is better for type  $L$  without attracting type  $H$  also and thereby making negative profits.

The characterization of  $S_L^*$  and the existence of the above strategies and beliefs are independent of financiers' prior beliefs  $\mu_0$ . This shows that there always exists an equilibrium in which type  $L$  is financed regardless of priors.  $\parallel$

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