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# The commitment value of contracts under dynamic renegotiation

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and

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*We examine why different renegotiation processes can lead to opposite results regarding the commitment value of third-party contracts in the presence of asymmetric information. Our main result is that a contract loses all strategic value if renegotiation is allowed during the production stage rather than only before production begins. This result casts serious doubt on the relevance of previous findings which emphasize how contracts can have commitment value even in the presence of renegotiation. Our analysis can also be used to understand the differences between many of the results in the renegotiation literature.*

## 1. Introduction

■ Commitment plays a central role in many strategic situations. In multistage games, players like to commit *ex ante* to *ex post* distortions in order to increase their power over opponents. One way to precommit is through the use of a contract that changes a player's *ex post* incentives. There are numerous examples of this behavior in both the industrial organization and finance literature, in which precommitting through the use of a contract gives credibility to the *ex post* threat of being aggressive. (For example, see Aghion and Bolton, 1987; Brander and Lewis, 1986; Brander and Poitevin, 1992; Fershtman and Judd, 1987; Sklivas, 1987.) However, the ability of contracts to act as precommitment devices is often eliminated when such contracts can be renegotiated *ex post*, that is, once the rival has decided on his action (Katz, 1991; Schelling, 1960).

Recently, Dewatripont (1988) and Caillaud, Jullien, and Picard (1990) have argued that if contracts are signed in a situation involving asymmetric information, renegotiation does not eliminate the ability of contracts to act as precommitment devices. The basic intuition is that renegotiation cannot freely eliminate *ex post* distortions because the incentive-compatibility constraints remain important at the time of renegotiation. Therefore a player may sign a contract with a third party with whom there is an informational problem and use the incentive constraints to precommit to *ex post* distortions, thereby gaining a strategic advantage over his rival.

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The object of this article is to show that the commitment value of contracts depends not only on the possibility of renegotiation, but also on the specific nature of the renegotiation process. In particular, we examine the commitment value of contracts for both the case in which renegotiation is allowed only before production begins, and the case in which renegotiation is allowed during the production stage. Our main result is that, even in the presence of asymmetric information, contracts cannot generally be used as precommitment devices when renegotiation is allowed during the production stage.

The specific situation we consider is one in which a duopolist's manager has the opportunity to sign a contract with an agent (an employee or, equivalently, a union) in order to gain a strategic advantage over his rival. The initial contract is offered to the employee before production costs are privately revealed to the manager. A contract is represented by a mechanism specifying a transfer to be paid to the employee contingent on the output that the manager orders him to produce. If no renegotiation is possible, it is obvious that such a contract can improve on the manager's strategic position by requiring the employee to overproduce. Moreover, if the manager can secretly offer to renegotiate the contract before its implementation but after the state of nature has been revealed, we show that the contract still allows the manager to gain a strategic advantage. This result mirrors that found by Dewatripont (1988) and Caillaud, Jullien, and Picard (1990), except that in our game it is the informed party who proposes the renegotiation. This result also echoes Maskin and Tirole's (1992) finding that renegotiation does not eliminate distortions in an informed-principal relationship.

However, if the implementation of the output specified in the contract takes time, the manager has the opportunity to renegotiate the contract while output is being produced. We therefore examine in detail the case in which the manager can offer the employee new contracts during the production stage. We show that, when renegotiation can occur while output is being produced, the manager cannot use the contract to gain a strategic advantage over his rival. Any attempt to use the contract to precommit to a large output would be renegotiated away during the production stage. This renegotiation is actually anticipated by the rival firm, and therefore the contract has no precommitment value.

The stark difference between the two results highlights the importance of determining when renegotiation is likely to occur before assessing whether contracts can have commitment value. The discrepancy arises mainly because the decision to accept renegotiation depends on the specificity of the alternative imposed if renegotiation is refused. Renegotiation allows parties to attain a Pareto-improving outcome, and when it is allowed only before the contract is implemented, it is easy to write a contract in such a way that many different outcomes can follow a refused renegotiation proposal. For example, when a contract specifies a whole menu of future choices, renegotiation can be refused because the parties do not agree on which outcome will arise under the status quo contract. In other words, the parties cannot agree on what constitutes a Pareto-improving contract. However, as time elapses, many choices may disappear, making the likely outcome under the status quo contract more precise and therefore making it much easier for parties to agree on what constitutes a Pareto-improving renegotiation. In this article, the fact that production takes time and that renegotiation is allowed during the production stage forces the set of alternatives to shrink over time. The knowledge that renegotiation is inevitable at future stages causes the whole renegotiation game to unravel in such a way that no distortions can be upheld in equilibrium. Thus, recognizing that the execution of contracts often has a dynamic aspect, and that this leaves the door open for renegotiation on increasingly precise alternatives, implies that contracts are unlikely to have an important commitment value.

The article is structured as follows. In Section 2, we present the duopolistic environment considered. In Section 3, we analyze the equilibrium outcomes for a base case in which renegotiation is allowed only before production. In Section 4, which is the main section of the article, we examine the precommitment value of the contract when renegotiation

is allowed during the contract's implementation stage. Section 5 relates our results to the more general literature on renegotiation. Section 6 concludes. All proofs are found in the Appendix.

## 2. The model

■ Two firms compete by setting quantities in a homogeneous-good duopoly. The market inverse demand curve is given by  $P(q_1 + q_2)$ , where  $q_i$  is firm  $i$ 's output for  $i = 1, 2$ . We assume that  $P' < 0$  and  $P'' \leq 0$ . These two firms are run by owner-managers who are maximizing profits. Firm 1 has access to a state-contingent technology denoted by  $t$ . The state of this technology can take one of two values, either  $L$  or  $H$ . Technology  $H$  represents a high-productivity technology (low cost), whereas technology  $L$  is a low-productivity one (high cost). Firm 2 has access to a single technology. We assume that firm 1's technology eventually becomes known to the manager but not to other parties. The prior probability that  $t = H$  is  $\mu_0$ . Each firm's technology can produce a maximum of  $Q$  units, where  $Q$  is large enough that  $P(Q) < 0$ .

The manager of firm 1 offers a contract to an employee to undertake production. This contract can serve two purposes. First, it determines the terms of employment for the employee. Second, by giving appropriate incentives, the contract can potentially be used by the manager of firm 1 as a precommitment device to gain a strategic advantage over firm 2. The manager of firm 1 takes both of these roles into account when designing the contract.

An employment contract is represented by a function  $\Omega(\cdot)$  that specifies the wage  $w$  paid to the employee when  $q_1$  units have been produced, that is,  $w \triangleq \Omega(q_1)$ . A contract  $\Omega(\cdot)$  belongs to the set  $\mathcal{M}$  of all functions  $\Omega(\cdot)$  such that  $\Omega : [0, Q] \rightarrow \mathbb{R}_+$ .<sup>1</sup> We purposely do not invoke the revelation principle in our specification of the contract space because in the presence of renegotiation it is not directly applicable.

After having signed the contract, both firms start producing. After the production stage has taken place, all units are produced at cost  $C^1(q_1, t)$  for firm 1 and  $C^2(q_2)$  for firm 2. The units are then marketed by both firms and the price is set according to  $P(q_1 + q_2)$  such that the market clears.

For a given outcome  $\{q_1, w, q_2\}$ , firm 1's payoff function is given by  $U(q_1, w, q_2, t) = P(q_1 + q_2)q_1 - C^1(q_1, t) - w$  and firm 2's payoff function is given by  $\pi^2(q_1, q_2) = P(q_1 + q_2)q_2 - C^2(q_2)$ . We assume that  $C_i^i > 0$ ,  $C_{ii}^i > 0$ , because this implies that there is a well-defined solution to firm  $i$ 's profit maximization for any  $q_j \geq 0$  and that the reaction functions are downward sloping in  $(q_1, q_2)$ -space.<sup>2</sup> Moreover, we assume that  $C_1^1(q_1, H) < C_1^1(q_1, L)$  so that the high-productivity technology has a lower marginal cost than the low-productivity one. Finally, we denote firm 1's reservation value by  $U(0)$ .

The employee's payoff is given by  $V(q_1, w) = w - e(q_1)$ , where  $e(q_1)$  represents the employee's nonmonetary cost of effort to produce  $q_1$  units of the good. It is assumed that  $e(\cdot) \geq 0$ ,  $e' \geq 0$  and  $e'' \geq 0$ . The employee's reservation value is denoted by  $V(0)$ .

Within the context of this model, we want to address the question of whether firm 1 can use the public disclosure of its employment contract to gain a strategic advantage over firm 2 even when secret renegotiations are allowed after the disclosure. We choose to study the commitment value of contracts when it is the informed party (the manager) who initiates all renegotiation proposals. We adopt this assumption for two reasons. First, by allowing the informed player to offer the proposals, out-of-equilibrium offers can be met by quite arbitrary beliefs. This is likely to favor the strategic role of contracts. Therefore,

<sup>1</sup> We could allow for more general contracts that include arbitrary messages from the manager to the worker without changing the results of the article.

<sup>2</sup> These assumptions are standard in the industrial organization literature.

by choosing this order of play we do not bias the game in favor of finding no strategic role for contracts. Second, the literature on renegotiation has mainly examined cases in which it is the uninformed party who proposes renegotiations. Hence, this article will also permit us to identify the similarities and differences between these two approaches.

### 3. Equilibrium contracts with static renegotiation

■ In this section, we examine a base case for which the contract offered by firm 1 is publicly disclosed, but where the manager cannot commit not to secretly renegotiate the contract *ex post*. In particular, we assume that the manager can propose a renegotiation to the employee only after he has learned the state of technology but before the production stage starts. The manager therefore has only one opportunity to renegotiate the contract, and we refer to this type of renegotiation as static. This type of renegotiation is similar to that adopted by Dewatripont (1988) and Caillaud, Jullien, and Picard (1990), with the exception that they examined the case in which it is the uninformed agent who makes the renegotiation proposals. We shall show that their results on the commitment value of contracts also hold when the informed agent offers the renegotiation. The sequence of moves with *ex ante* (static) renegotiation is as follows:

- 1) The manager of firm 1 offers a contract  $\Omega(q_1) \in \mathcal{M}$  to the employee. The employee either accepts or rejects the contract. If he rejects the contract, the game ends and both firm 1 and the employee receive their reservation value. If he accepts the contract, he is hired. The contract is disclosed to firm 2.
- 2) Nature reveals the type of technology (or costs) to the manager of firm 1.
- 3) Firm 2 decides on its production level  $q_2$ , and simultaneously firm 1 can propose a renegotiation  $\tilde{\Omega}(q_1)$  to the employee. If the employee accepts the proposed renegotiation, it replaces the old contract, which becomes obsolete. If the proposal is rejected, the old contract remains in force.
- 4) The production stage then starts in each firm. The manager of firm 1 orders the employee to produce a certain level of output. Neither firm can observe its rival's production.
- 5) At the end of the production stage, both firms market their units at price  $P(q_1 + q_2)$ . Profits are realized and the wage payment specified in the contract is paid.

The strategy of firm 1 consists in offering a contract  $\Omega(q_1)$  at stage 1 and a contract  $\tilde{\Omega}(q_1)$  at stage 3, and ordering a production level  $q_1$  at stage 4. The employee must accept or reject the two contract offers. His beliefs are revised following the renegotiation proposal. Firm 2 makes its production decision in stage 3 conditional on the initial contract offer and its expectation of the resolution of the game.

We use the concept of perfect Bayesian equilibrium (PBE) to solve this game. In a PBE, strategies must be consistent given a specified set of beliefs, and beliefs must be updated with Bayes' rule whenever possible.<sup>3</sup> An outcome of the game can be represented by the tuple  $\{q_L, w_L, q_H, w_H, q_2\}$ , where  $q_t$  and  $w_t$  represent respectively the output produced and the wage paid by firm 1 when its technology is  $t$ .<sup>4</sup> We now examine the perfect Bayesian equilibrium outcomes of this game.

<sup>3</sup> See Fudenberg and Tirole (1991) for a precise definition of perfect Bayesian equilibrium.

<sup>4</sup> The interpretation of  $q_t$  is that the outstanding contract leads the manager of type  $t$  to stop production after  $q_t$  units have been produced.

**Proposition 1.** There exists a PBE outcome of the *ex ante* (static) renegotiation game which satisfies the following conditions:

$$\{q_L^s, w_L^s, q_H^s, w_H^s, q_2^s\} = \left\{ \begin{array}{l} \max_{\substack{q_L, w_L, \\ q_H, w_H, \\ q_2}} \mu_0 U(q_H, w_H, q_2, H) + (1 - \mu_0) U(q_L, w_L, q_2, L) \\ \text{subject to} \\ \text{(a) } U^s(t) \geq U(q_{t'}, w_{t'}, q_2, t) \text{ for } t, t' = L, H; \\ \text{(b) } \mu_0 V(q_H, w_H) + (1 - \mu_0) V(q_L, w_L) \geq V(0); \\ \text{(c) } q_2 = \operatorname{argmax}_q \mu_0 \pi^2(q_H, q) + (1 - \mu_0) \pi^2(q_L, q); \\ \text{(d) } \{q_L, q_H, w_L, w_H\} = \\ \operatorname{arg} \left\{ \begin{array}{l} \max_{\substack{q'_L, w'_L, \\ q'_H, w'_H}} \mu_0 U(q'_H, w'_H, q_2, H) + (1 - \mu_0) U(q'_L, w'_L, q_2, L) \\ \text{subject to} \\ \text{(i) } V(q'_H, w'_H) \geq V(q_H, w_H) \\ \text{(ii) } V(q'_L, w'_L) \geq V(q_L, w_L) \\ \text{(iii) } U(q'_L, w'_L, q_2, L) \geq U(q'_H, w'_H, q_2, L) \\ \text{(iv) } U(q'_H, w'_H, q_2, H) \geq U(q'_L, w'_L, q_2, H) \end{array} \right\} \end{array} \right\},$$

where  $U^s(t) = U(q_t^s, w_t^s, q_2^s, t)$  is type  $t$ 's equilibrium payoff.

Proposition 1 characterizes the equilibrium outcome that confers the maximum expected payoff to firm 1 within the set of PBE outcomes. This equilibrium outcome is characterized by the solution to the maximization problem. Constraint (a) represents standard incentive-compatibility constraints that the outcome must satisfy, constraint (b) is the employee's participation constraint, constraint (c) specifies firm 2's optimal response, and constraint (d) represents the restrictions imposed by the renegotiation process. The optimal contract from the point of view of firm 1 is the contract that induces an outcome that maximizes its expected utility conditional on satisfying the four constraints.

It is important to note that constraint (d) is not binding at the optimum but is nevertheless included in the maximization problem to explicitly illustrate the effects of renegotiation on out-of-equilibrium initial contract offers. To understand constraint (d), consider an initial contract that leads to the outcome  $\{q_L, w_L, q_H, w_H\}$  if not renegotiated, and a renegotiation proposal that leads to a different outcome  $\{q'_L, w'_L, q'_H, w'_H\}$  if accepted. Such a proposal would always be accepted by the employee when the outcome  $\{q'_L, w'_L, q'_H, w'_H\}$  satisfies constraints (i) and (ii), because in these circumstances the employee can never lose, regardless of his beliefs. Constraints (iii) and (iv) represent standard incentive-compatibility constraints that must be satisfied for the proposed outcome to actually be implemented. Such an outcome  $\{q'_L, w'_L, q'_H, w'_H\}$  must not exist for the initial contract to survive renegotiation and be implemented. Furthermore, by an appropriate choice of out-of-equilibrium beliefs (which we discuss below), this condition is also sufficient for a contract to be robust to renegotiation. This explains why constraint (d) captures the effect of renegotiation.

It is also important to note that the outcome described in Proposition 1 is not the unique equilibrium outcome of the game because, in general, there are several different equilibrium outcomes that can arise after the renegotiation of a specific initial contract offer. However, the outcome described in Proposition 1 is the unique equilibrium outcome that survives Cho and Kreps' (1987) intuitive criterion. This fact is not proven here because it is essentially a corollary of Proposition 7 in Maskin and Tirole (1992). In effect, Maskin and Tirole analyze a static renegotiation process identical to ours and show that any initial contract supporting an outcome satisfying constraint (d) remains the unique equilibrium contract following the application of the intuitive criterion to the renegotiation stage. Therefore, it is clear that the outcome in Proposition 1 is the unique outcome that survives the intuitive criterion, because the criterion forces a nonrenegotiated outcome after the principal initially offers the menu  $\{q_L^s, w_L^s\}, \{q_H^s, w_H^s\}$ .

The equilibrium outcome  $\{q_L^s, w_L^s, q_H^s, w_H^s, q_2^s\}$  has the property that when the state of technology is  $L$ , firm 1 produces at a level that is an optimal response given firm 2's output; that is, there is no overproduction when the state is  $L$ . However, in state  $H$ , firm 1 overproduces relative to its optimal response to firm 2's output. It is this overproduction that gives firm 1 its strategic advantage. State  $H$ 's distortion in production is not renegotiated away at the renegotiation stage because type  $L$ 's incentive-compatibility constraint is strictly binding. Any Pareto-improving renegotiation for type  $H$  violates type  $L$ 's incentive constraint and is therefore rejected by the employee in the belief that it was offered by a type- $L$  manager.<sup>5</sup> Note that renegotiation is proposed *before* the worker actually knows the quantity that would actually be produced if renegotiation were rejected. For this reason, the incentive-compatibility constraint plays an important role at the renegotiation stage.

One equilibrium play of the game is as follows. At stage 1, a contract  $\Omega^s(q_1)$  is offered and is accepted by the employee. This contract specifies  $w_H^s = \Omega^s(q_H^s)$ ,  $w_L^s = \Omega^s(q_L^s)$ , and a very high wage for all other levels of production. In the renegotiation stage, the manager reoffers the same contract, which is rejected, and firm 2 chooses the level of output that is its best response to firm 1's production (constraint (c)). In stage 4, the manager of type  $t$  orders the output level  $q_t^s$ . By constraint (a), it is incentive compatible for the manager of firm  $t$  to order the production level  $q_t^s$  because the wage specified by  $\Omega^s(q_1)$  is very high for every level of production other than  $q_t^s$ . Out of equilibrium, a renegotiation that satisfies conditions (i)–(iv) is believed to be offered by either type (prior beliefs) and is accepted; any offer that decreases the employee's payoff on type  $t$ 's outcome is believed to come from type  $t$  and is therefore rejected.

The importance of studying this equilibrium is that it demonstrates how firm 1 can use a contract to precommit to a high output. It is obvious that, even with renegotiation, the contract  $\Omega^s(q_1)$  has commitment value compared to the case in which the contract is not disclosed to firm 2. This can be illustrated by the following argument. Suppose that the initial contract was not publicly disclosed and that it sustained overproduction to induce firm 2 to reduce its output. If firm 2 effectively reduces its output below its expected Cournot level, firm 1 can increase its expected profit by offering an *ex post* efficient contract because the deviation to this new contract would be undetected by firm 2. Firm 2 should then rationally anticipate such deviation by firm 1 and consequently the initial contract cannot induce it by reducing its output. In this case, firm 1 produces at its *ex post* conditional Cournot output level, which we denote  $q_t^c$ , and firm 2 produces its best response to the pair  $\{q_L^c, q_H^c\}$ , which is its Cournot output level and denoted  $q_2^c$ . Therefore, if the initial contract is not observable (or disclosed) to firm 2, it cannot have any strategic value.

Throughout the article, we shall repeatedly make use of the Cournot mapping that defines  $q_t^c$  as a function of  $q_2$ . This mapping is given by the level of production that

<sup>5</sup> This explains why there is no overproduction in type  $L$ 's production level: it can always be renegotiated away without violating type  $H$ 's incentive-compatibility constraint.

maximizes  $U(q_1, w_1, q_2, t)$  with respect to  $q_1$  and  $w_1$  subject to  $V(q_1, w_1) \geq V(0)$ . It is clear that  $q_1^c$  depends on  $q_2$  but that it is independent of  $V(0)$ .

Corollary 1 states that contracts have commitment value in the static-renegotiation game, where  $U^c = \mu_0 U(q_H^c, w^c, q_2^c, H) + (1 - \mu_0)U(q_L^c, w^c, q_2^c, L)$  is firm 1's expected payoff when the contract is not observable, and  $U^s = \mu_0 U^s(H) + (1 - \mu_0)U^s(L)$  is firm 1's expected equilibrium payoff when the contract is observable.

*Corollary 1.* The equilibrium contract  $\Omega^s(q_1)$  has commitment value, that is,  $U^c < U^s$ .

A result similar to Corollary 1 was derived in models in which the uninformed agent makes the renegotiation proposals. Dewatripont (1988) and Caillaud, Jullien, and Picard (1990) have shown that, even in the presence of renegotiation, incentive constraints are such that a party can use a constraint to precommit. Incentive constraints play a similar role in our equilibrium outcome even though renegotiations are proposed by the informed agent. Therefore, with renegotiation occurring before the production stage, contracts can have a commitment value regardless of which party proposes renegotiation.

#### 4. Equilibrium contracts with a dynamic renegotiation process

■ In this section, we argue that the specification of the renegotiation process is important in assessing the strategic value of contracts. One criticism of the literature that finds a strategic value for contracts is that secret (undisclosed to firm 2) renegotiation could arise after the initial contract offer, but before the state of technology becomes known to the manager, and that such renegotiation would remove all strategic value of the contract. There are at least two objections to this criticism. First, Caillaud, Jullien, and Picard (1990) have shown that this criticism is unfounded when the contract has the additional purpose of solving an agency problem. Second, such secret renegotiation may not always be feasible; for example, the true game may be one in which the agent is hired after the state of technology becomes known to the manager. Therefore, to fully understand the potential commitment value of contracts, it is useful to study the situation in which renegotiations are allowed only after the state of technology has become known. If we can show that a more general renegotiation process eliminates all strategic value of contracts, a much stronger point can be made against suggesting that asymmetric information confers strategic value to contracts even in the presence of renegotiation.

When production takes place over time, it seems unreasonable to assume that parties can commit to not renegotiate during the production stage. In this section therefore, we allow renegotiation to occur while production is being undertaken. We qualify this type of renegotiation as dynamic. We assume that during the production stage a total of  $Q$  units can be produced and we consider the production stage to be divided into  $n > 0$  subperiods. Therefore, in each subperiod a level of output equal to  $Q/n$  is produced. We shall be examining the property of the set of equilibria as  $n$  becomes arbitrarily large.

At the beginning of a subperiod, the manager can decide whether production should continue under the existing contract, and he can propose a contract renegotiation to the worker. A renegotiation proposal  $\tilde{\Omega}(q_1)$  specifies wage payments conditional on all remaining possible levels of output. If the proposed renegotiation is refused, the terms of the relationship are determined by the last contract agreed upon, and the employer's decision regarding current production is implemented. If the renegotiation is accepted, the new contract replaces the old one, and the current production decision is given by this new contract.

The game has the following sequence of moves, where  $\mathcal{M}_j$  is the set of functions that specify a wage contingent on all output levels of at least  $(j - 1)Q/n$  (with  $\mathcal{M}_0 = \mathcal{M}$ ), and where  $jQ/n$  is the produced output after  $j$  subperiods of production.<sup>6</sup>

<sup>6</sup> The sequencing of moves has been chosen so that under symmetric information, *ex post* inefficiencies cannot be supported in equilibrium and therefore any distortion that may arise in this scenario necessarily results from the informational asymmetry. In other words, the game has been set in such a way that indifferences and multiple equilibria cannot be used to support distortions as in Fernandez and Glazer (1991).



- 1) The manager of firm 1 offers a contract  $\Omega_0(q_1) \in \mathcal{M}$  to the employee. The employee either accepts or rejects the contract. If he rejects the contract, the game ends and both firm 1 and the employee receive their reservation value. If the employee accepts the contract, he is hired. The contract is disclosed to firm 2.
- 2) Nature reveals the type of technology (or costs) to the manager of firm 1.
- 3) Firm 2 decides on its production level  $q_2$ , and the production stage starts in firm 1. In each subperiod  $j = 1, \dots, n$ , we have the following sequence of moves.
  - j.1) First, the manager must decide whether to produce in subperiod  $j$  under the existing contract, and whether to propose a renegotiation  $\Omega_j(q_1) \in \mathcal{M}_j$ .
  - j.2) If the employee refuses the renegotiation, the previously agreed upon contract is in force and production continues according to the decision made in j.1. If the employee accepts the proposed renegotiation  $\Omega_j(q_1)$ , it becomes the outstanding contract, and the manager can decide whether to produce according to the new contract.
- 4) At the end of the production stage, firms market their units at price  $P(q_1 + q_2)$ . Profits are realized and the wage payment specified in the contract is paid.

The strategies of the players are more complicated in this dynamic-renegotiation case than they were in the previous case and it is worthwhile to describe them more formally.

For  $j = 1, \dots, n$ , define by  $\mathcal{H}_j$  the complete past history of the game that arises before the  $j$ th subperiod, and denote by  $\mathcal{H}_j^+$  the history of the game that includes both  $\mathcal{H}_j$  and the decisions made by the employer at the beginning of the  $j$ th subperiod. Define  $\mathcal{H}_0 = \emptyset$  and  $\mathcal{H}_0^+ = \{\Omega_0\}$ . Note that these histories contain all past contract proposals by the manager and past acceptance or rejection decisions of the employee.

A strategy  $\Omega_i$  for firm 1's manager with private information  $i$  is a sequence of functions  $\Omega_i = \{\Omega_0, \Omega_{i1}, z_{i1}, \dots, \Omega_{ij}, z_{ij}\}$  for  $j = 0, \dots, n$ , such that  $\Omega_0$  maps from  $\mathcal{H}_0$  into  $\mathcal{M}$ ,  $\Omega_{ij}$  maps from  $\mathcal{H}_j \cup \{i\}$  into  $\mathcal{M}_j \times \{0, 1\}$ , and  $z_{ij}$  maps from  $\mathcal{H}_{j+1}^+ \cup \{i\}$  into  $\{0, 1\}$ , where 0 means "stop production," and 1 means "continue production."<sup>7</sup> The decision about whether to continue production under the existing contract is included in  $\Omega_{ij}$ , and the decision of whether to pursue production under the renegotiated contract is given by  $z_{ij}$ .

A strategy  $\sigma$  for the employee is a sequence of functions  $\sigma = \{\sigma_0, \dots, \sigma_j\}$  for  $j = 0, \dots, n$  such that  $\sigma_j$  maps  $\mathcal{H}_j^+$  into  $\{0, 1\}$ , where 0 means "reject" the renegotiation proposal and 1 means "accept" it. The employee's beliefs is a sequence of functions  $\mu = \{\mu_0, \dots, \mu_j\}$  such that  $\mu_j$  maps  $\mathcal{H}_j^+$  into  $[0, 1]$ . The beliefs  $\mu_j$  represent the employee's subjective probability that the manager is of type  $H$ .

Firm 2's history is restricted to the initial contract proposal and acceptance decision, and is denoted  $\mathcal{H}^2$ . A strategy for firm 2 is a function  $\lambda$  which maps  $\mathcal{H}^2$  into an output decision  $q_2 \in \mathcal{R}_+$ .

Given these strategies and beliefs, a perfect Bayesian equilibrium for this game is defined as a tuple  $(\Omega_L, \Omega_H, \sigma, \mu, \lambda)$  such that strategies are sequentially rational given beliefs, and beliefs satisfy Bayes' rule when applicable.

We now characterize the PBE outcomes of this dynamic-renegotiation game. Viewing renegotiation offers as the result of communication between the manager and the employee, we are interested in characterizing equilibrium outcomes in the limit when this communication is almost costless. When there are few subperiods, communication does not occur often and firms can commit to producing many units between rounds of

<sup>7</sup> Note that  $\Omega_0$  is not contingent on the type, because it represents the initial contract offer that is made before the manager learns his type.

renegotiation. Our results are derived in terms of when communication can arise arbitrarily often; that is, when the number of subperiods grows, leaving constant the total output that can be produced. Our first result states that, as the manager can renegotiate more and more often, the implemented outputs  $q_t^d$  are not significantly larger than the *ex post* efficient levels  $q_t^c$ .

*Lemma 1.* For all  $\epsilon > 0$ ,  $\exists n^d(\epsilon)$  such that for  $n \geq n^d(\epsilon)$ , firm  $t$ 's output is  $q_t^d < q_t^c + \epsilon$  for  $t = L, H$ .

Lemma 1 states that no substantial overproduction can subsist with a dynamic-renegotiation process. This is the most important result of the article, and the following discussion conveys the main ideas behind the proof. In order to understand the result, it is easiest to suppose that overproduction does arise and then show why it would always be renegotiated away. In this discussion, we denote a candidate equilibrium outcome by  $\{\hat{q}_H, \hat{w}_H, \hat{q}_L, \hat{w}_L, \hat{q}_2\}$ .

Let us first suppose that along the equilibrium path  $\hat{q}_H > q_H^c$ , that is, there is overproduction in the high-productivity state. Let us also suppose that the required output  $\hat{q}_H = kQ/n$ , that is,  $\hat{q}_H$  is produced by the end of the  $k$ th subperiod with the manager ordering production to stop at the beginning of subperiod  $k + 1$ . The wage paid in this case is assumed to be determined by the contract prevalent at the beginning of the  $k$ th period (the case in which this assumption does not hold is treated in the Appendix). If the periods are small enough, there always exists an outcome  $\{\tilde{q}, \tilde{w}\}$ , with  $\tilde{q} = \hat{q}_H - Q/n$ , which could improve on both the firm's and the employee's equilibrium payoffs regardless of the employee's beliefs. Such a Pareto improvement exists because  $\hat{q}_H$  involves overproduction. The manager can make it credible that  $\{\tilde{q}, \tilde{w}\}$  will be implemented following an accepted renegotiation by proposing a contract at the beginning of subperiod  $k$  that specifies: (1) wage  $\tilde{w}$  if production is stopped now, and (2) a very high wage if production is continued into the future. Although this Pareto improvement looks attractive to the employee, he might still refuse this renegotiation proposal if he thought that the outcome following a rejection would be much better for him than the outcome  $\{\hat{q}_H, \hat{w}_H\}$  (which is the outcome that would have arisen in the absence of renegotiation). However, in any subgame that gives the employee an outcome preferable to  $\{\hat{q}_H, \hat{w}_H\}$ , the firm necessarily gets less than  $U(\hat{q}_H, \hat{w}_H, \hat{q}_2, H)$ . This is because at  $\{\hat{q}_H, \hat{w}_H\}$  there are no gains from further production regardless of the type of technology. Hence, refusing the renegotiation cannot lead to an outcome in which the firm is worse off than  $U(\hat{q}_H, \hat{w}_H, \hat{q}_2, H)$ , because the firm can always assure itself of the payoff associated with  $\{\hat{q}_H, \hat{w}_H\}$  by stopping production immediately after a refused renegotiation. Consequently, the renegotiation proposal is necessarily accepted, which upsets the original equilibrium and demonstrates that no significant overproduction can take place. Hence,  $\hat{q}_H$  cannot be significantly greater than  $q_H^c$  as  $n$  becomes large.

The previous argument also applies for the case in which  $\hat{q}_L > q_H^c$ , but does not directly apply to the case in which  $q_L^c < \hat{q}_L < q_H^c$ . In this latter case, the worker could believe that the refusal of a renegotiation may lead to a better outcome for him than would  $\{\hat{q}_L, \hat{w}_L\}$ , because at  $\hat{q}_L < q_H^c$  there are still gains from further production if the manager turns out to be of type  $H$ . Nevertheless, the type- $L$  manager can always construct a renegotiation proposal that the worker will want to accept (this renegotiation proposal actually signals his type). There are two crucial elements to this renegotiation proposal. The first is simply the specification of a wage contingent on production stopping during this period. This wage needs to be set to create a Pareto improvement over  $\{\hat{q}_L, \hat{w}_L\}$  contingent on the technology being of type  $L$ . The second element corresponds to the wage associated with production  $q_H^c$ , that is,  $\tilde{\Omega}(q_H^c)$ . This wage must be set so that

$$U(\hat{q}_L - Q/n, \tilde{\Omega}(\hat{q}_L - Q/n), \hat{q}_2, H) < U(q_H^c, \tilde{\Omega}(q_H^c), \hat{q}_2, H) < U(\hat{q}_L, \hat{w}_L, \hat{q}_2, H).$$

For all other levels of production the wage can be set very high. The first inequality is an incentive constraint for the type- $H$  manager to ensure that he prefers the outcome  $\{q_H^c, \bar{\Omega}(q_H^c)\}$  to the outcome  $\{\hat{q}_L - Q/n, \bar{\Omega}(\hat{q}_L - Q/n)\}$ . The second inequality implies that the employee gets higher utility under the renegotiation offer than he would under the equilibrium contract if the manager is of type  $H$ . Therefore the worker will be ready to accept this contract because, regardless of his beliefs about the type of manager offering it, he is strictly better off than he would be in the equilibrium play of any subgame. For example, if the employee were to believe that this proposal was offered by a manager with technology  $H$ , he would want to accept it because it would guarantee him a payoff larger than any payoff he could possibly achieve in any subgame equilibrium that would follow a refusal. Although Lemma 1 does not cover the case of underproduction, similar arguments lead to the implication that underproduction would also be renegotiated away.

In the following proposition we construct equilibrium strategies and beliefs, and give firm 1's equilibrium payoff. Even though the equilibrium outcome is not unique, we specify bounds on the expected payoff that firm 1 can earn in any equilibrium.<sup>8</sup> Let us define  $U^d$  as firm 1's expected equilibrium payoff when dynamic renegotiation is possible, that is,  $U^d = \mu_0 U^d(H) + (1 - \mu_0) U^d(L)$ , where  $U^d(t)$  is type  $t$ 's equilibrium payoff.

**Proposition 2.** For all  $\epsilon > 0$ ,  $\exists n^d(\epsilon)$  such that for  $n \geq n^d(\epsilon)$ , firm 1's expected equilibrium payoff in the dynamic-renegotiation game is  $U^d \in [U^c, U^c + \epsilon]$ .

Proposition 2 states that, under dynamic renegotiation, firm 1's equilibrium payoff cannot be substantially higher than it is in the case in which contracts are unobservable. This is the direct consequence of Lemma 1, because overproduction was shown to be renegotiated away in the production stage. Therefore, the best that firm 1 can hope to do in the initial stage is to offer a contract specifying *ex post* efficient output levels (in the limit). This contract is accepted and not renegotiated at any time before or during the production stage. Any attempt to offer a contract that would induce  $q_t > q_t^c$  in the absence of renegotiation would not be credible. Firm 2 would expect any such contract to be renegotiated to  $q_t^c$  and therefore would produce accordingly. Consequently the contract has no commitment value.

When the renegotiation process is almost frictionless, firm 1 is virtually indifferent between disclosing or not the employment contract to firm 2. This contrasts sharply with Proposition 1 and the results of Dewatripont (1988) and Caillaud, Jullien, and Picard (1990). The difference arises because of the renegotiation process being assumed. When renegotiation can occur during the implementation phase of the contract, the set of alternatives that can be reached though renegotiation is reduced as production takes place over time; that is, once a given output has been produced, one cannot renegotiate to an outcome with less production. This form of commitment implies that overproduction will always be renegotiated away despite binding *ex ante* incentive-compatibility constraints. For example, once sufficient output has been produced, the *ex ante* incentive constraint becomes irrelevant because it involves choices over output levels that cannot be reached anymore. This changes the set of renegotiation proposals that will be accepted at a given point, and hence allows the manager to eliminate overproduction. This discussion implies that, with dynamic renegotiation, the role played by incentive-compatibility constraints is very different than it would be with static renegotiation. For example, when production has reached  $q_H^c$ , there are really no more relevant incentive constraints, and therefore overproduction cannot be sustained.

<sup>8</sup> The nonuniqueness arises because there are many different wage distributions between the two states consistent with the employee's individual-rationality constraint and the firm's incentive-compatibility constraints. However, in all of these equilibrium outcomes, outputs are *ex post* efficient and the expected wage is constant. Therefore, firm 1 has the same expected payoff.

This argument shows that *ex post* commitment can reduce the value of *ex ante* commitment. This effect takes place through the role played by incentive-compatibility constraints at different stages of the game. If firm 1 cannot commit to a production level before renegotiating, incentive-compatibility constraints play a strong role and renegotiation has no bite. However, if firm 1 can commit *ex post* to a certain production level before renegotiating, incentive-compatibility constraints play a much weaker role and renegotiation can have an effect. In this case firm 1 cannot commit to some production distortions and the contract therefore has no commitment value. This shows that *ex post* commitment can conflict with *ex ante* objectives and can actually reduce the value of *ex ante* commitment to a contract.

## 5. General remarks

■ The contracting literature has arrived at very different results about the implications of renegotiation. On the one hand, models of renegotiation in which the informed party makes the offers have often found that renegotiation does not change the predictions related to non-renegotiation models. For example, Nosal (1991) examines the case of hidden information and Maskin and Tirole (1992) examine the case of adverse selection and both articles find that allowing renegotiations initiated by the informed party does not affect equilibrium allocations. On the other hand, the articles by Dewatripont (1989) (hidden information), Laffont and Tirole (1990) and Hart and Tirole (1988) (adverse selection) all find that allowing for renegotiation significantly changes equilibrium allocations, and generally reduces the degree of *ex post* distortions. In all of these articles, however, it is the uninformed party who proposes the renegotiation. Therefore, the effects of renegotiation on contractual outcomes appear to depend on which player proposes the renegotiations. If correct, this is obviously a disheartening result.<sup>9</sup>

The current article suggests that focusing on who proposes renegotiations may be a misguided way of regrouping results. In particular, we believe that it is more enlightening to classify the results found in the literature according to whether renegotiation is allowed only once before actions are taken, or whether it is allowed while actions are being taken. Once this distinction is recognized, a more interesting pattern of predictions seems to emerge. For example, in the previously mentioned literature, all of the models in which it is the *uninformed* party who proposes the renegotiations are also models in which renegotiation occurs as actions are being undertaken. The models in which it is the *informed* party who proposes the renegotiations are all cases in which renegotiation is allowed only before actions are taken. Therefore, the literature tends to indicate that results in which renegotiation has no effect on equilibrium outcomes may be the consequence of the assumption that all renegotiation proposals arise *before* any actions are undertaken. Our results confirm this interpretation, because renegotiation was shown to have drastically different implications depending on which renegotiation process was chosen, even though it was the informed party who was always assumed to offer the renegotiations.

To better understand why timing is probably the central element in renegotiation, first consider the trivial case in which a monopolist in a situation of adverse selection signs a contract with an agent and tries to renegotiate the contract before any actions are taken. It is obvious that allowing the monopolist to renegotiate his contract offer (which may comprise a menu of allocations) once it has been accepted by the agent does not have any effect on equilibrium allocations because no information has been revealed. In the case in which it is the monopolist who is informed (as in Maskin and Tirole (1992)), the reason why renegotiation is ineffective is less obvious because the original contract may in fact signal some information. However, by Myerson's (1983) inscrutable principle, we know

<sup>9</sup> The literature on moral hazard has come to a similar conclusion. See Ma (1994) for a discussion.

that an informed principal can generally propose an initial contract that does not reveal any information.<sup>10</sup> Therefore, even when it is the informed party who offers the initial contract, there is no reason for information to be revealed, and thus allowing an immediate renegotiation has no effect. In contrast, when renegotiation is allowed after some actions are undertaken (even if this only means that a particular outcome has been selected from a menu), there is always some element that has changed relative to the initial position and therefore there is scope for renegotiation.<sup>11</sup> This is why renegotiation in a dynamic setting almost always involves changes in the equilibrium allocations. Moreover, this change is usually in the direction of reducing *ex post* distortions.

It is also of potential interest to relate the results of this article to that of Noldeke and van Damme (1990) who study a “dynamic” version of Spence’s (1973) education model. In Noldeke and van Damme, an informed worker is offered employment contracts after different amounts of education have been undertaken. Their model is close in spirit to ours because contract offers occur during the time that investment in education arises. Noldeke and van Damme show that the unique equilibrium outcome that satisfies the never-a-weak-best-response criterion is the standard separating equilibrium. In fact, this result shows that allowing these multiple rounds of contract offers does not contribute to the reduction of *ex post* distortions. This result is in sharp contrast with our result on dynamic renegotiation. There is, however, one major difference between the two models that accounts for these results. In Noldeke and van Damme no explicit contract is initially signed between the informed and uninformed agents. This allows for more severe punishments of out-of-equilibrium offers, which in turn allows for an *ex ante* better outcome to be sustained. When an explicit agreement is signed *ex ante*, any party can always at least enforce it if a deviation occurs. This is what gives force to renegotiation as opposed to multiple rounds of contract offers.

## 6. Conclusion

■ This article studies the commitment value of contracts when renegotiation is possible. Contrary to results obtained in the literature, we show that contracts cannot generally be used by a strategic party to commit. This difference between our result and the ones found in the literature arises from the modelling of the renegotiation process. When the renegotiation process is given a dynamic dimension, we show that *ex post* distortions are greatly reduced and hence cannot be used for commitment purposes.

More generally, our result may be interpreted as highlighting the different implications of allowing for a dynamic renegotiation process, as opposed to only a static process, before any actions are actually carried out.

## Appendix

■ Proofs of Propositions 1 and 2, Corollary 1, and Lemma 1, and Lemma 2 and its proof follow.

*Proof of Proposition 1.* We construct strategies and beliefs that support the outcome  $\{q_L^s, w_L^s, q_H^s, w_H^s, q_2^s\}$  as an equilibrium outcome of the game. Define  $V^s = \max_i V(q_i^s, w_i^s)$ . Define also the wage  $V^{-1}(q_1, V^s)$  implicitly by  $V(q_1, V^{-1}(q_1, V^s)) = V^s$ .

*Firm 1.* The manager initially offers  $\Omega^s(q_1)$  such that  $w_i^s = \Omega^s(q_i^s)$  and  $\Omega^s(q_1) = V^{-1}(q_1, V^s)$  for all  $q_1 \neq q_i^s$ .

<sup>10</sup> This fact is not exploited by Maskin and Tirole (1992), but referring to it gives a clearer intuition of the result than would a discussion of switches in the support of beliefs.

<sup>11</sup> The simple fact of selecting an outcome within a menu is enough to create an incentive to renegotiate. Beaudry and Poitevin (1993) examine the implications of allowing for renegotiation after status quo outcomes have been determined but before any specific actions are taken. For the parameterization at hand, we find a result comparable to that presented here.

In the renegotiation stage, regardless of his type, the manager reoffers  $\Omega^i(q_i)$  if it was the initial contract agreed upon. If the initial agreement is  $\Omega^i(q_i)$  different from  $\Omega^i(q_i)$ , the two types of managers offer a contract inducing an outcome equal to

$$\arg \left\{ \begin{array}{l} \max_{\substack{q_L, w_L, \\ q_H, w_H}} \mu_0 U(q_H, w_H, q'_2, H) + (1 - \mu_0) U(q_L, w_L, q'_2, L) \\ \text{subject to} \\ \text{(i) } V(q_H, w_H) \geq V(q'_H, w'_H) \\ \text{(ii) } V(q_L, w_L) \geq V(q'_L, w'_L) \\ \text{(iii) } U(q_t, w_t, q'_2, t') \geq U(q_t, w_t, q'_2, t') \text{ for } t, t' = L, H \end{array} \right\},$$

where  $q'_2$  is firm 2's best response to  $\Omega^i(q_i)$ . At the production stage, the manager orders production at its most preferred production level depending on the contract.

*Employee.* The employee accepts all initial contract offers that yield an expected value of at least  $V(0)$ , that is, any contract  $\Omega^i(q_i)$  such that  $\mu_0 V(q'_H, w'_H) + (1 - \mu_0) V(q'_L, w'_L) \geq V(0)$  where  $\{q'_i, w'_i\}$  is type  $i$ 's induced outcome under the contract  $\Omega^i(q_i)$ .

In the renegotiation stage, the employee accepts all contract offers inducing an outcome  $\{q_L, w_L, q_H, w_H\}$  that satisfies

- (i)  $V(q_H, w_H) \geq V(q'_H, w'_H)$
- (ii)  $V(q_L, w_L) \geq V(q'_L, w'_L)$
- (iii)  $U(q_t, w_t, q'_2, t) \geq U(q_t, w_t, q'_2, t')$  for  $t, t' = L, H$ ,

where  $\{q'_L, w'_L, q'_H, w'_H\}$  is the outcome under the initial agreement. This strategy is supported by the following beliefs. After the initial offer, the employee keeps his prior beliefs,  $\mu_0$ , because the manager cannot signal what he does not know. In the renegotiation stage, the employee keeps his prior beliefs if the renegotiation offer satisfies conditions (i)–(iii). If  $V(q_H, w_H) < V(q'_H, w'_H)$ , he believes that type  $H$  made that offer. In all other cases, he believes the offer was made by type  $L$ .

*Firm 2.* Firm 2 produces  $q'_2$  as defined in the statement of the proposition.

It is easy to see that these strategies and beliefs constitute a PBE. Given the acceptance decisions of the employee at the initial and renegotiation stages, and given firm 2's output, firm 1 can do no better than offer  $\Omega^i(q_i)$ , which solves its maximization problem over all accepted and possibly renegotiated contracts. If this contract becomes the agreement, firm 1 cannot improve on its utility by renegotiating and therefore reoffers the same contract regardless of its type. If the initial agreement is different from  $\Omega^i(q_i)$ , firm 1 offers its most preferred contract over the set of accepted renegotiations. Given the manager's strategy and his own beliefs, the employee cannot gain by rejecting the initial offer. In the renegotiation stage, the employee accepts only incentive-compatible Pareto-improving renegotiations. This strategy is sequentially rational given his beliefs. Along the equilibrium path, the employee's beliefs satisfy Bayes' rule. Finally, firm 2 maximizes its expected profit given firm 1's output. *Q.E.D.*

*Proof of Corollary 1.* This is the direct consequence of the fact that the maximization problem in Proposition 1 implies  $q_H^* > q_H^i$ . The proof is therefore omitted. *Q.E.D.*

*Proof of Lemma 1.* Because the proof is different for each type, we divide the proof into two stages.

(a) We first show that type  $H$  cannot significantly overproduce. Suppose that the game ends with an outstanding contract denoted  $\hat{\Omega}(q_i)$ , which induces the outcome  $\{\hat{q}_H, \hat{w}_H, \hat{q}_L\}$  for type  $H$ . Suppose that  $\hat{q}_H = kQ/n > q_H^i$ , that is, type  $H$  overproduces in equilibrium and  $\hat{q}_H$  is obtained by the end of subperiod  $k$ .

We first examine the case in which the contract  $\hat{\Omega}(q_i)$  is signed before subperiod  $k$ , that is, the contract  $\hat{\Omega}(q_i)$  is the outstanding contract at the beginning of subperiod  $k$ . If the number of subperiods is large enough, there exists an outcome  $\{\hat{q}_H - Q/n, \hat{w}_H\}$  such that  $\hat{q}_H - Q/n > q_H^i$ , and such that this outcome is preferred relative to the equilibrium outcome by both type  $H$  and the employee regardless of the employee's beliefs. Suppose that in subperiod  $k$  the manager states that production will continue under the existing contract and proposes a renegotiated contract  $\tilde{\Omega}(q_i)$ , which specifies  $\tilde{w}_H = \hat{w}_H - Q/n$  and a very high wage for all other

levels of production. If that contract is rejected by the employee, then production continues until subperiod  $k + 1$ . In subperiod  $k + 1$ , because the manager overproduces at  $\hat{q}_H$ , there is no offer that can make the manager and the employee better off regardless of the employee's beliefs. Hence, there is no subgame in which a renegotiation would be accepted that the manager would be willing to offer. The manager would then stop production at the beginning of subperiod  $k + 1$ . On the other hand, if the employee accepts  $\bar{\Omega}(q_1)$ , he cannot be made worse off than at  $\{\hat{q}_H - Q/n, \bar{w}_H\}$  because he can always reject all future renegotiations. Therefore, the employee should accept the contract offer  $\bar{\Omega}(q_1)$ . This induces the manager to actually offer it, and therefore upsets the equilibrium outcome  $\{\hat{q}_H, \bar{w}_H\}$ .

Suppose now that the contract  $\hat{\Omega}(q_1)$  is signed at the beginning of subperiod  $k$  or  $k + 1$ . In both cases, for the contract  $\hat{\Omega}(q_1)$  to be offered and accepted, it must be the case that the previous contract  $\bar{\Omega}(q_1)$  induces an outcome  $\bar{q}_H > \hat{q}_H$  for type  $H$ . Otherwise, it would not be individually rational for one of the parties to agree to  $\hat{\Omega}(q_1)$ . Therefore, the contract  $\bar{\Omega}(q_1)$  induces an outcome  $\{\bar{q}_H, \bar{w}_H\}$  for type  $H$  who stops production in subperiod  $k' > k + 1$  because  $\bar{q}_H > \hat{q}_H$ . By the previous argument we know that in subperiod  $k' - 1$  the manager can decide to continue production, and then offer a contract that improves on  $\bar{\Omega}(q_1)$  for both parties regardless of beliefs. This offer could be such that it yields  $V(\bar{q}_H, \bar{w}_H) + \nu$  where  $\nu > 0$  is arbitrarily small. The employee would then accept the offer regardless of his beliefs. By backward induction, there exists a contract that the manager can offer in subperiod  $k' - 2$  that improves on the manager's utility as well as on  $V(\bar{q}_H, \bar{w}_H) + \nu$ . If this offer is rejected, the employee knows that he will accept a worse offer in subperiod  $k' - 1$ . He therefore accepts it. The same argument applies to subperiod  $k' - 3$  and so forth until the manager can make an offer that will be accepted and that he prefers to  $\{\hat{q}_H, \bar{w}_H\}$ , thus upsetting the equilibrium.

In conclusion, for any  $\epsilon > 0$  such that  $\hat{q}_H > q_H^d + \epsilon$ , define  $n_H^d(\epsilon)$  such that  $\hat{q}_H - Q/n_H^d(\epsilon) > q_H^d$ . In this case the proof shows that the equilibrium can be upset by an appropriate offer. Hence,  $q_H^d$  cannot be significantly larger than  $q_H^c$ , that is, type  $H$  cannot significantly overproduce.

(b) We now show that type  $L$  cannot significantly overproduce. Suppose that the game ends with the outstanding contract being  $\hat{\Omega}(q_1)$  and inducing the outcome  $\{\hat{q}_L, \hat{w}_L, \hat{q}_2\}$  for type  $L$ . Suppose that  $\hat{q}_L = kQ/n > q_L^c$ , that is, type  $L$  overproduces in equilibrium and the production level  $\hat{q}_L$  is obtained by the end of subperiod  $k$ . Note that  $\hat{q}_L < q_H^c$  because type  $H$  cannot overproduce, and  $\hat{q}_L < \hat{q}_H$  by standard incentive-compatibility constraints. Hence type  $H$  would be underproducing at  $\hat{q}_L$  relative to his *ex post* efficient level.

We first examine the case in which the contract  $\hat{\Omega}(q_1)$  is signed before subperiod  $k$ , that is, the contract  $\hat{\Omega}(q_1)$  is the outstanding contract at the beginning of subperiod  $k$ . If the number of subperiods is large enough, there exists an outcome  $\{\hat{q}_L - Q/n, \bar{w}_L\}$  such that  $\hat{q}_L - Q/n > q_L^c$ , and that would make both type  $L$  and the employee better off relative to the equilibrium payoff if the employee believes the manager is of type  $L$ . Let  $\bar{w}_H$  be defined so that the following inequality is satisfied:

$$U(\hat{q}_L - Q/n, \bar{w}_L, \hat{q}_2, H) < U(q_H^c, \bar{w}_H, \hat{q}_2, H) < U(\hat{q}_L, \hat{w}_L, \hat{q}_2, H).$$

The first inequality states that type  $H$  prefers an *ex post* efficient outcome earning the wage  $\bar{w}_H$  rather than type  $L$ 's induced outcome. The second inequality ensures that type  $H$  prefers the outcome induced by the contract  $\hat{\Omega}(q_1)$  to the outcome  $\{q_H^c, \bar{w}_H\}$ . Suppose that in subperiod  $k$  the manager states that production will continue under the existing contract, and offers an alternative contract  $\bar{\Omega}(q_1)$  that specifies

$$\bar{w}_L = \hat{\Omega}(\hat{q}_L - Q/n), \bar{w}_H = \bar{\Omega}(q_H^c)$$

and a very high wage for all other levels of production. If that contract is rejected by the employee, then production continues until subperiod  $k + 1$ . In subperiod  $k + 1$ , because type  $L$  overproduces at  $\hat{q}_L$ , there is no offer that can make type  $L$  and the employee better off, hence there is no subgame in which a renegotiation would be accepted that type  $L$  would be willing to offer. Type  $L$  then stops production at the beginning of subperiod  $k + 1$ . As for type  $H$ , there is no subgame that induces an outcome worse than  $\{\hat{q}_H, \bar{w}_H\}$  for the employee or the manager because either can always enforce the contract  $\hat{\Omega}(q_1)$ . On the other hand, suppose the employee accepts  $\bar{\Omega}(q_1)$ . If the manager is of type  $L$ , he stops production in subperiod  $k$ . In that case the employee would earn  $V(\hat{q}_L - Q/n, \bar{w}_L) > V(\hat{q}_L, \hat{w}_L)$ . Therefore, the employee should accept the contract offer  $\bar{\Omega}(q_1)$  if he believes the manager is of type  $L$ . If the manager is of type  $H$ , the contract would lead to the outcome  $\{q_H^c, \bar{w}_H\}$ . Therefore, if the employee believes that the manager is of type  $H$ , he should also accept the contract because no future renegotiation can give the employee higher profits than  $V(q_H^c, \bar{w}_H)$ , which would be offered by type  $H$ . Hence, regardless of beliefs, the employee should always accept the contract  $\bar{\Omega}(q_1)$ . This induces type  $L$  to actually offer the contract  $\bar{\Omega}(q_1)$ , and thus upsets the equilibrium outcome  $\{\hat{q}_L, \hat{w}_L\}$ .

Suppose now that the contract  $\hat{\Omega}(q_1)$  is signed at the beginning of subperiod  $k$  or  $k + 1$ . In both cases, for the contract  $\hat{\Omega}(q_1)$  to be offered and accepted, it must be the case that the previous contract  $\bar{\Omega}(q_1)$  induced an outcome  $\bar{q}_L > \hat{q}_L$  for type  $L$ . Otherwise it would not be individually rational for one of the parties to agree to  $\hat{\Omega}(q_1)$ . Furthermore, type  $H$  must underproduce at  $\bar{q}_L$ , otherwise by the argument of part (a) of this proof, both types could make offers that would be accepted and that would improve on their respective equilibrium

utility. The contract  $\bar{\Omega}(q_1)$  induces an outcome  $\{\bar{q}_L, \bar{w}_L\}$  for type  $L$  who stops production in period  $k' > k$ , because  $\bar{q}_L > \hat{q}_L$ . By the argument above, we know that in subperiod  $k' - 1$  the manager can decide to continue production and then offer a contract that improves on  $\bar{\Omega}(q_1)$  for both parties regardless of beliefs. This offer could be such that it yields  $V(\bar{q}_L, \bar{w}_L) + \nu$  where  $\nu > 0$  is arbitrarily small for type  $L$ , and specifies a high enough wage for all other output levels (as in the previous case). The employee would then accept the contract regardless of his beliefs. By backward induction, there exists a contract that the manager can offer in subperiod  $k' - 2$  that improves on the manager's utility as well as on that of the employee, regardless of his beliefs. If this offer is rejected, the employee knows that he will accept a worse offer in subperiod  $k' - 1$ . He therefore accepts it. The same argument applies to subperiod  $k' - 3$  and so forth until the manager can make an offer that improves on  $\{\hat{q}_L, \hat{w}_L\}$  and that would be accepted by the employee, thus upsetting the equilibrium.

In conclusion, for any  $\epsilon > 0$  such that  $\hat{q}_L > q_L^c + \epsilon$ , define  $n_L^d(\epsilon)$  such that  $\hat{q}_L - Q/n_L^d(\epsilon) > q_L^c$ . In this case the proof shows that the equilibrium can be upset by an appropriate offer. Hence,  $q_L^d$  cannot be significantly larger than  $q_L^c$ , that is, type  $L$  cannot significantly overproduce.

(c) In both parts of the proof we have specified a minimal number of subperiods for which an equilibrium with overproduction by one type could be broken. By taking  $n^d(\epsilon) = \max_i n_i^d(\epsilon)$  we satisfy the statement of the proposition. *Q.E.D.*

The following lemma will be used in the proof of Proposition 2.

**Lemma 2.** Consider an outstanding contract  $\hat{\Omega}(q_1)$ . Suppose  $j - 1$  periods have passed and define as  $\hat{\Omega}_j(q_1)$  the truncation of  $\hat{\Omega}(q_1)$  for all subperiods  $k \geq j$ . Finally define  $\hat{V}_j = \min_{j-1 \leq k \leq Q} V(kQ/n, \hat{\Omega}_j(kQ/n))$  with  $\hat{k}_j$  being the largest  $k$  solving this problem. Then the employee cannot earn significantly more than  $\hat{V}_j$  on either type's induced outcome.

*Proof of Lemma 2.* (a) Suppose first that type  $t$  overproduces at  $\hat{k}_j$ . A backward-induction argument similar to the argument made in the proof of Lemma 1 shows that in subperiod  $\hat{k}_j$ , type  $t$  can make a contractual offer  $\hat{\Omega}$  that promises the employee more than  $\hat{V}_j$ , regardless of his beliefs, with the induced outcome arbitrarily close to  $\hat{V}_j$ . Then, in the preceding subperiod, there exists an offer that improves on  $\hat{\Omega}$  that will also be accepted by the employee. This argument applies until subperiod  $j$ , when the manager can make an offer with the induced outcome arbitrarily close to  $\hat{V}_j$  and that will be accepted by the employee. This implies that the employee cannot earn significantly more than  $\hat{V}_j$ .

(b) Suppose now that type  $t$  underproduces at  $\hat{k}_j$ . Then, in subperiod  $\hat{k}_j$ , the type- $t$  manager can offer a contract  $\hat{\Omega}$  inducing an outcome slightly better than  $\hat{V}_j$  for the employee with production expected to stop in the next subperiod. Simultaneously, the manager offers to stop with the outstanding contract  $\hat{\Omega}$  if the renegotiation is refused. The employee then accepts the renegotiation knowing that he can always guarantee himself the payoff induced by  $\hat{\Omega}$ . In the next subperiod, the manager can repeat this strategy until he produces at his *ex post* efficient level, yielding a payoff just slightly higher than  $\hat{V}_j$  for the employee. This implies that the employee cannot earn significantly more than  $\hat{V}_j$ . *Q.E.D.*

*Proof of Proposition 2.* The statement of the proposition follows immediately from Lemma 1. No type can significantly overproduce, and therefore the best the manager can do is to produce a number of units not significantly larger than his *ex post* efficient level. This is expected by firm 2, which produces at  $q_2^d$ , its Cournot production level. Hence, in equilibrium firm 1 cannot earn significantly more than  $U^c$ . Suppose the payoff  $U^c + \epsilon$  is obtained by optimally producing  $q_1^d + \epsilon'(\epsilon)$ . By taking  $n^d(\epsilon) = \max_i n_i^d(\epsilon'(\epsilon))$ , as in the proof of Lemma 1, then the manager cannot produce more than  $q_1^d + \epsilon'(\epsilon)$ , which implies that his payoff will be  $U^d \leq U^c + \epsilon$ . Furthermore, his payoff cannot be less than  $U^c$  because the manager can always offer a contract that induces the *ex post* efficient production levels as outcomes. Therefore, if an equilibrium exists, and if renegotiation can occur often enough, the manager cannot earn significantly more than the payoff that he would earn if the contract was not observable. To complete the proof we need to show that an equilibrium does in fact exist. We now make a slight abuse of notation and provide equilibrium strategies and beliefs for the limit case when  $\epsilon'(\epsilon) = 0$ . It is clear that the proof extends to the case with  $\epsilon'(\epsilon) > 0$ .

First define the following outcome:  $\mathbb{O}_j(\hat{\Omega}_j) = \{q_{ij}^d = k_{ij}^d Q/n, w_i^d(\hat{\Omega}_j)\}$ , where

$$\{q_{ij}^d = k_{ij}^d Q/n, w_i^d(\hat{\Omega}_j)\} = \arg \left\{ \max_{w, t} U(kQ/n, w, q_2^d, t) \text{ subject to } V(kQ/n, w) \geq \hat{V}_j \right\}.$$

Note that  $q_{ij}^d$  is independent of  $\hat{\Omega}_j$  because it equals either type  $t$ 's *ex post* efficient production level (which is itself independent of the wage level), or it equals  $(j - 1)Q/n$ . Construct the contract  $\bar{\Omega}_j(q_1, \hat{\Omega})$  that induces the outcomes  $\mathbb{O}_j$  by  $\bar{\Omega}_j(q_1, \hat{\Omega}) = V^{-1}(q_1, \hat{V}_j) \forall q_1 \geq (j - 1)Q/n$ , where  $V^{-1}(q_1, \bar{V})$  is defined implicitly by  $V(q_1, V^{-1}(q_1, \bar{V})) = \bar{V}$ . The contract  $\bar{\Omega}_j(\cdot, \hat{\Omega})$  represents the renegotiated contract that will be offered by the manager in subperiod  $j$  when the outstanding contract is  $\hat{\Omega}$ . Finally define the contract  $\Omega_j(q_1)$  as subperiod  $j$ 's offer.



We now construct strategies and beliefs that support an equilibrium outcome which yields the highest expected payoff to firm 1's manager. Define the following contract:

$$\Omega^d(q_1) = V^{-1}(q_1, V(0)) \quad \forall 0 \leq q_1 \leq Q.$$

$$\Omega_i = \begin{cases} \Omega_0 = \Omega^d(q_1) \\ \text{For all } j = 1, \dots, n, \\ \Omega_{ij} = \begin{cases} \{\bar{\Omega}_j(q_1, \hat{\Omega}), 0\} & \text{if } (j-1)Q/n < k_{ij}^d Q/n \\ \{\bar{\Omega}_j(q_1, \hat{\Omega}), 1\} & \text{otherwise} \end{cases} \\ z_{ij} = \begin{cases} 1 & \text{if } (j-1)Q/n < k_{ij}^d Q/n \\ 0 & \text{otherwise.} \end{cases} \end{cases}$$

$$\sigma = \begin{cases} \sigma_0 = \begin{cases} 1 & \text{if } \mu_0 V(\mathbb{C}_{H1}(\Omega_0)) + (1 - \mu_0) V(\mathbb{C}_{L1}(\Omega_0)) \geq V(0) \\ 0 & \text{otherwise} \end{cases} \\ \text{For all } j = 1, \dots, n, \\ \sigma_j = \begin{cases} 1 & \text{if } \mu_0 V(\mathbb{C}_{Hj}(\Omega_j)) + (1 - \mu_0) V(\mathbb{C}_{Lj}(\Omega_j)) \\ & \geq \mu_0 V(\mathbb{C}_{Hj}(\hat{\Omega})) + (1 - \mu_0) V(\mathbb{C}_{Lj}(\hat{\Omega})) \\ 0 & \text{otherwise.} \end{cases} \end{cases}$$

$$\mu = \begin{cases} \mu_0 \text{ for } j = 0 \\ \text{For all } j = 1, \dots, n, \\ \mu_j = \begin{cases} 1 & \text{if } \Omega_j(q_1) = \bar{\Omega}_j(q_1, \hat{\Omega}) \text{ and } (j-1)Q/n > q_{Lj}^d \\ \mu_0 & \text{otherwise.} \end{cases} \end{cases}$$

$$\lambda = q_2^d = \operatorname{argmax}_q \mu_0 \pi^d(q_{H1}^d, q) + (1 - \mu_0) \pi^d(q_{L1}^d, q).$$

These strategies and beliefs do constitute a PBE. Along the equilibrium path, the manager offers the contract  $\Omega^d(q_1)$ , which is accepted. He then repeats his offer until  $q_n^d$  is reached, at which point he elects to stop production. All repeated offers are trivially accepted. Firm 2 produces at its Cournot level  $q_2^d$ . Beliefs satisfy Bayes' rule along the equilibrium path. The initial offer is believed to come from either type. As long as the initial offer is reoffered and type  $L$ 's output has not been reached, beliefs equal the priors. When type  $L$ 's outcome is passed, the employee believes he is dealing with a type  $H$  as long as the manager repeats his initial offer.

Off the equilibrium path, the manager always offers his preferred contract among the set that will be accepted. The employee accepts all contracts that do not decrease his payoff anticipating the future renegotiations. *Q.E.D.*

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