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SPATIAL PRICE COMPETITION: A SEMIPARAMETRIC APPROACH

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We investigate the nature of price competition among firms that produce differentiated products and compete in markets that are limited in extent. We propose an instrumental variables series estimator for the matrix of cross price response coefficients, demonstrate that our estimator is consistent, and derive its asymptotic distribution. Our semiparametric approach allows us to discriminate among models of global competition, in which all products compete with all others, and local competition, in which products compete only with their neighbors. We apply our semiparametric estimator to data from U.S. wholesale gasoline markets and find that, in this market, competition is highly localized.

KEYWORDS: Price competition, differentiated products, spatial models, monopolistic competition, nonparametric estimation, series estimators, instrumental variables, wholesale gasoline.

1. INTRODUCTION

IN MANY INDUSTRIES, firms produce differentiated products and compete in markets that are limited in extent. Common features of those markets are that entry is easy and that firms supply similar but not identical goods (e.g., gasoline stations, movie theaters, and restaurants). In attempting to model such markets, one is naturally led to a variant of spatial or monopolistic competition. These game-theoretic models are typically based on the assumption that sales and profits depend on a vector of own and rival prices. The nature of this dependence, however, varies from model to model. Indeed, there is an entire spectrum of assumptions that ranges from extremely localized rivalry to symmetric competition.

For example, with one-dimensional spatial models, whether linear (Hotelling (1929)), circular (Salop (1979)), or vertical (Gabszewicz and Thisse (1979)), each firm competes directly only with its two neighbors, one on either side. In other words, conditional on neighbor prices, fluctuations in prices of more distant competitors have no effect on own sales.[2] In direct contrast to these spatial models

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[2] This assumes that there is no mill price undercutting (see Eaton and Lipsey (1976)).

where competition is local, models of monopolistic competition in the spirit of Chamberlin (1933) (e.g., Spence (1976) and Dixit and Stiglitz (1977)) are based on the notion that competition is not only global but also symmetric. Indeed, sales and profits depend only on the distribution of rival prices and not on the identities or locations of the firms that post those prices.[3]

In this paper, we develop an empirical technique that can be used to discriminate between local and global rivalry, where we use the words local and global in a general sense. Following Anderson, de Palma, and Thisse (1989), local competition denotes a situation in which firms compete directly only with their neighbors, but the dimension of the space need not be one, whereas global competition denotes a situation in which all products compete with all others, but competition need not be symmetric.

The answers to many questions concerning the efficiency of downstream markets depend crucially on the industry's location on the global/local spectrum. For example, it is well known that markets where competition is segmented are less competitive than those where competition is symmetric. In addition, markets with local competition tend to be characterized by brand proliferation and over entry, whereas when competition is global, there is no such tendency.[4] Given the importance of these questions from a policy point of view, it is desirable to have empirical methods that can locate particular industries on the local/global spectrum.[5]

We assume that price is the strategic variable and derive the firms' best reply or reaction functions. These functions form the basis of our empirical tests. If we were to impose considerable structure on the way in which firms interact, we could estimate the pricing rules by standard techniques. However, we do not choose this route. Instead, we use semiparametric methods to estimate the matrix of diversion ratios or reaction function slopes. In particular, we allow substitution patterns, and thus competitive responses, to depend in a possibly nonlinear fashion on a vector of distance measures that have been proposed in the literature. For example, we experiment with the (local) common-market-boundary measure that was used by Feenstra and Levinsohn (1995) and with the (global) Euclidean-distance measure that was used by Davis (1997). By allowing the pricing rules to be flexible, and by considering a number of frequently used distance measures, we are able to assess the nature of price competition in a given market.

[3] A number of models of differentiated products lie somewhere in between these two extremes. For example, with the characteristics approach to demand (Lancaster (1966), Baumol (1967), and Gorman (1980)) products compete along several dimensions. Moreover, as the number of dimensions increases, so does the number of neighbors.

[4] See Deneckere and Rothschild (1992) for a formal assessment of these issues in a model that encompasses both local and global competition. Vives (1999) claims that over entry is also the norm in models of global competition. However, we do not find his assumptions to be compelling.

[5] A number of researchers have calculated cross-price elasticities between all product pairs (e.g., Berry, Levinsohn, and Pakes (1995), Goldberg (1995), and Nevo (2000)). The models used, however, have generally been variants of a random-utility model in which all products compete with all others by assumption.

Although we estimate best reply functions (i.e., first-order conditions), our method could easily be used to estimate demand for differentiated products or a combination of demand and first order conditions.[6] The general problem is as follows. Suppose that there are n differentiated products where n is large. The data, however, consist of a single cross section or a short panel. In order to estimate the n^2 cross price elasticities, considerable structure must be imposed on the problem. The choice of structure to impose differentiates the various studies, and there are several general classes of models that deal with this problem.

Most empirical models of differentiated product markets in the industrial organization literature are cast in a discrete choice framework. In other words, consumers can purchase at most one brand of the differentiated product and are limited to one unit of that brand. Within this framework, there are two commonly used models: a global random utility model in which each product competes with every other, albeit with varying intensity, and a local spatial model in which most cross price elasticities are zero a priori.

A typical random utility model makes use of an individual utility function that is linear in product characteristics, product price, and an error term that is often assumed to have an extreme value distribution. Aggregation across consumers is accomplished by integrating with respect to that distribution, and it is well understood that the choice of distributional assumptions has strong implications for preferences. For example, if individual draws from the extreme value distribution are independent and identically distributed, one has a multinomial logit, which is symmetric. If, in contrast, consumer tastes are allowed to be correlated across products in a restricted fashion that involves a priori product groupings, one has a nested multinomial logit (NML). Finally, if the coefficients of the product characteristic variables are allowed to vary more generally, one has a random coefficients model that allows for very general patterns of substitution.[7]

Highly localized discrete choice models are much less common than random utility models. However, Bresnahan (1981, 1987) estimates a model of vertical differentiation with a single parameter that captures quality differences. With his model, products compete directly only with their two neighbors, one of higher and the other of lower quality. Feenstra and Levinsohn (1995), in contrast, allow for multiple dimensions of diversity and compute endogenous market boundaries in this larger space. They do this by assuming that the transport cost (or utility loss) function is quadratic in m-dimensional Euclidean space, where m is the number of characteristics. With both models, products that do not share a market boundary do not compete directly.

Whereas the discrete choice assumption seems more appropriate for some applications, variety in consumption seems more natural for others. For example, shoppers often purchase several brands of breakfast cereal in one shopping

[6] For an application that involves demand estimation, see Pinkse and Slade (2000).

[7] Examples of the use of a NML include Goldberg (1995), Verboven (1996), and Fershtman and Gandal (1998). Bresnahan, Stern, and Trajtenberg (1997) estimate a generalized extreme value model that is not hierarchical. Examples of random coefficients models include Berry, Levinsohn, and Pakes (1995), Davis (1998), Petrin (1998), and Nevo (2000). For a more comprehensive discussion of these models, see Berry (1994).

trip, and drinkers often consume variable amounts of several brands of beer in one evening on the town. With variety in consumption, consumers have convex indifference surfaces, and corner solutions are the exception, not the rule. Though common elsewhere, empirical models that embody a systematic taste for diversity are less common in the differentiated products literature. A recent study by Hausman, Leonard, and Zona (1994), however, is based on the notion that consumers can, and often do, purchase several varieties or brands. These authors consider a multi-stage budgeting problem, where individuals first decide how much of the product (beer) to consume, then decide which product types to purchase (e.g., premium, regular, or light), and finally select brands. The structure of their model, which involves a priori product groups, is thus similar to a NML. Substitution patterns within groups, however, are more flexible, but the number of brands that can be included in a group is more limited.

In this paper, we develop an alternative empirical approach to modeling competition among differentiated products that differs from a random utility model in several ways. First, it is applicable in situations in which consumers have a systematic taste for variety and thus might want to consume more than one product. Second, it nests local and global competition in a natural way.[8] Finally, it offers considerable flexibility in modeling substitution possibilities while, at the same time, being computationally less burdensome than a full fledged random coefficients model. Our approach also differs from a multistage budgeting model in that it does not rely on, but can encompass, a priori product groupings. In addition, it can be used when there is a very large number of products or brands in each group.

The organization of the paper is as follows. In the next section, we specify the theoretical model that forms the basis of our empirical tests. We begin with individual buyers and sellers, who can be heterogeneous in both observed and unobserved dimensions. Buyers are competitive firms whose profits depend on local demand and cost conditions, whereas sellers' are imperfectly competitive firms whose profits also depend on rival prices and on local market structure conditions. Our estimating equations are first order conditions that can be solved to obtain equilibria of the upstream product market game.

Section 3 deals with estimation. After a brief review of conventional estimation techniques, we present our semiparametric estimator. We propose a series estimator for the coefficients of rival prices, demonstrate that our estimator is consistent, and derive its asymptotic distribution. Our estimator makes use of a vector of measures of the distance between seller (product) pairs, where 'distance' locates sellers (products) in geographic (product characteristic) space. We assume that the strength of pairwise competition is determined by these measures. The functional form of this dependence, however, is determined by the data. Our estimator can handle endogenous prices and measurement error in a

[8] Berry (1994) demonstrates that, for example, the vertical model of Bresnahan (1987) is a special case of a random coefficients model. The different models, however, are not nested in an econometric sense.

straightforward fashion through the use of instrumental variables. We therefore also discuss the choice of instruments and suggest tests of their validity.

Section 4 describes the market and the data that are used in the application. We assess the nature of spatial price competition in U.S. wholesale gasoline markets. Petroleum products are shipped by pipeline or barge from refineries to terminals where they are sold to wholesalers at published prices. We use a cross section of prices, locations, and demand, cost, and market structure factors at terminals in the lower 48 states. Since we are interested in modeling price competition in geographic space, we experiment with several measures of geographic proximity for each terminal pair. These measures are dichotomous variables that indicate if firm j is i's nearest neighbor in some metric, if i and j share a market boundary, and if i and j share a boundary with a third competitor. A final measure, which is continuous, is the Euclidean distance between the terminals in which i and j are located. With all four measures, 'distance' can be determined exogenously as a function that only depends on kilometers or endogenously as a function that also depends on prices and transport costs.

In Section 5, we present our estimated pricing rules and our assessment of popular models of price competition. To anticipate results, we find that, in this market, competition is highly localized. Indeed a model in which each firm competes directly principally with its single nearest neighbor receives strongest support. Furthermore, conditional on being nearest neighbors, the distance between terminals is unimportant.

2. THE MODEL OF PRICE COMPETITION

In this section, we develop a model in which consumers purchase one or more of several variants of a differentiated product; a taste for diversity is thus a maintained hypothesis. Our framework nests local and global models of competition; the nature of competitive interactions is therefore to be assessed. Nesting is accomplished through the use of several notions of distance, or its inverse closeness.

Formally, suppose that there are n sellers of a differentiated product, with one seller for each variant.[9] Sellers as well as variants are indexed by $i = 1, \ldots, n$. The products, $q = (q_1, q_2, \ldots, q_n)^T$, sell at nominal prices $\tilde{p} = (\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n)^T$ that are parametric to the purchasers. Finally, each product or variant is associated with a characteristic, y_i. For notational simplicity, we assume that there is one characteristic per product. The generalization to multiple characteristics, however, is straightforward.

There are K buyers of q that are indexed by $k = 1, \ldots, K$. In our application, buyers are firms (wholesalers) who resell q (i.e., q is an input vector).[10] Each buyer is located at a point in geographic (or product characteristic) space and therefore has a unique profit function. We assume that the kth

[9] The generalization to multiproduct sellers is straightforward (see Pinkse and Slade (2000)).

[10] In other applications, buyers could be households or individuals (see Pinkse and Slade (2000)).

buyer or downstream firm can sell its output at a parametric nominal price, $\tilde{v}_k$, that also depends on its location.[11] Let buyer k's competitive profit function be $\tilde{\pi}_k(\tilde{v}_k, \tilde{p}, y)$, where $y = (y_1, y_2, \ldots, y_n)^T$. The aggregate profit function for the buying industry is then

$$\tilde{\Pi}(\tilde{v}, \tilde{p}, y) = \sum_k \tilde{\pi}_k(\tilde{v}_k, \tilde{p}, y), \tag{1}$$

where $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_K)^T$.

Up to this point, our analysis is completely general. In particular, Bliss (1975, pp. 68–69) shows that, in a competitive economy, *no* restrictions on individual demand functions are required for the existence of an aggregate profit function that generates the aggregate demands.[12] In other words, there is no loss of generality in treating a collection of price-taking firms as if they were a single price-taking maximizing unit. Moreover, given any aggregate profit function that satisfies standard regularity conditions, there exists an economy of firms whose profit-maximizing behavior generates the same aggregate demands.

As is common in the literature on derived demand, we approximate $\tilde{\Pi}$ with a flexible functional form, which is a second order approximation to an arbitrary profit function that places no restrictions on product substitution possibilities. There are many functional forms from which to choose. We use a normalized quadratic (Berndt, Fuss, and Waverman (1977) and McFadden (1978)), which is a quadratic function of prices that have been divided by an individual price or by an index of those prices.[13] We divide by an index of output prices, which we denote V, and define normalized prices, $p = V^{-1}\tilde{p}$ and $\bar{v} = V^{-1}\tilde{v}$. The approximation is then

$$\tilde{\Pi}(\tilde{v}, \tilde{p}, y) \approx V\left\{\tilde{\alpha}_1^T p + \tilde{\alpha}_2^T \bar{v} + \frac{V}{2}\left[p^T B^1 p + \bar{v}^T B^2 \bar{v} + p^T B^3 \bar{v}\right] + \frac{1}{2}\left[p^T B^4 y + \bar{v}^T B^5 y\right]\right\}. \tag{2}$$

Without loss of generality, one can choose units for downstream outputs such that V equals one. Furthermore, since $\bar{v}$ is constant in a cross section, it can be suppressed. The normalized quadratic profit function is then

$$\Pi(p, y) = a_0 + a^T p + \bar{a}^T y + \frac{1}{2}\left[p^T B^1 p + p^T B^4 y\right], \tag{3}$$

[11] $\tilde{v}_k$ can be a vector of prices without significantly altering the analysis.

[12] Individual or household utility functions, in contrast, can be combined into an aggregate utility function that is independent of the distribution of heterogeneity if and only if the indirect utility functions are in generalized Gorman polar form (see Gorman (1953, 1961) and Blackorby, Primont, and Russell (1978)).

[13] Diewert and Wales (1987) suggest using a price index rather than a single price. In our case, this is an important modification, since no single firm sells all products. It is also important that the normalizing price be exogenous.

where B^1 is an arbitrary symmetric positive semidefinite n by n matrix. Finally, by Hotelling's Lemma, downstream demand for product i is

$$q_i = \frac{\partial \widetilde{\Pi}}{\partial \tilde{p}_i} \cong \frac{\partial \Pi}{\partial p_i}\frac{\partial p_i}{\partial \tilde{p}_i} = \frac{\partial \Pi}{\partial p_i} = a_i + \sum_j \left[b_{ij}^1 p_j + b_{ij}^4 y_j\right] \tag{4}$$

where b_{ij}^k denotes the i, j element of B^k.

Turning to the upstream (imperfectly competitive) industry, we assume that each firm's marginal cost, C_i, is a linear function of a vector of cost factors, c_i, $C_i = \gamma^T c_i$. Given rival prices, the ith upstream seller chooses p_i to

$$\max_{p_i} (p_i - \gamma^T c_i)\left[a_i + \sum_j \left(b_{ij}^1 p_j + b_{ij}^4 y_j\right)\right] - F_i, \tag{5}$$

where F_i is a fixed cost. The first order condition for this maximization can be solved to yield seller i's reaction or best reply function,

$$p_i = R_i(p_{-i}) = \frac{1}{-2b_{ii}^1}\left(a_i - b_{ii}^1 \gamma^T c_i + \sum_{j \neq i} b_{ij}^1 p_j + \sum_j b_{ij}^4 y_j\right) \quad (i = 1, \ldots, n), \tag{6}$$

where $p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n)$.

Unfortunately, it is not possible to estimate the parameters of (6) from a single cross section or short panel of n firms. It is therefore necessary to place some structure on the parameters, which we do as follows.

Equation (6) shows that the intercepts of the best reply functions depend on the demand and cost factors, y and c. We drop the assumption that y_i is one dimensional, and, since our application is spatial, we partition y into national, regional, and local variables. To illustrate, a national factor might be the growth rate of GNP, a regional factor might be the deviation of regional from overall growth, and a local factor might be city population and/or per capita income. Furthermore, we assume that, whereas all product demands depend on national factors, the demand for q_i depends only on regional and local factors that are associated with the region and locality to which i belongs. We also partition the marginal cost variables into national, regional, and local factors (for example, crude oil price, regional shipping cost, and local wages, respectively).

The slopes of the best reply functions, $-b_{ij}^1/(2b_{ii}^1)$, are proportional to the diversion ratios that surface in the antitrust literature.[14] The diversion ratio from product i to j is the fraction of the lost customers of i that would switch to j if the price of i were to rise. It seems natural to assume that this fraction depends on the proximity of the two products in geographic (or characteristic) space. We therefore assume that the ratio depends on a vector, d_{ij}, of measures of the distance between the two products in some set of metrics.[15] To illustrate, if the

[14] See, e.g., Shapiro (1996). The factor of proportionality is 1/2.

[15] Even though the metrics measure only pairwise distance, they can depend on all prices and locations, as will be seen below.

products were brands of beer, the measures of distance (or its inverse, closeness) might be alcohol content proximity, market share proximity, and dummy variables that indicate whether brands belong to the same product group (e.g., premium, regular, or light). In our application, the elements of d_{ij} are measures of proximity in geographic space, which we discuss below.

These assumptions on the parameters of the aggregate profit function are clearly restrictive. Nevertheless, this simplification involves no conflict with the assumption that decisions are made separately, independently, and optimally by heterogeneous decision makers.

Let $X = (x_{ih}), h = 1, \ldots, H$, be a matrix of observed demand and cost variables. If, in addition, there are unobserved variables, u, the system of equations (6) can be written as

$$p = R(p) = A + X\beta + Gp + u, \tag{7}$$

where, in the parametric part of (7), A is a vector of intercepts that we treat as random effects and β is a vector of parameters that must be estimated. The matrix $G = (g_{ij})$ has zero diagonal elements, $g_{ii} = 0$, and off diagonal elements $g_{ij} = g(d_{ij}), i \neq j$, where $g(.)$ is a function that must be estimated. As we are interested in placing as little structure as possible on patterns of substitution and competitive interactions, we estimate $g(.)$ nonparametrically.[16]

Finally, the random variable u, which captures the influence of unobserved demand and cost variables, can be heteroskedastic and spatially correlated. We assume, however, that the unobserved characteristics, u, are mean independent of the observed characteristics, X, $E[u_i|X] = 0$. This strong conditional independence assumption is identical to the one that is made by most researchers in the area (see, e.g., Berry, Levinsohn, and Pakes (1995, p. 854)). Relative to other applications, however, the problems that are associated with the assumption are somewhat lessened in our context. Indeed, unlike product characteristics that tend to come in packages, observed and omitted geographic variables are less apt to be correlated. To illustrate, whereas larger cars tend to weigh more and have larger storage capacities and bigger engines, it is not as clear that larger cities have higher unemployment rates or more developed freeway systems. This means that x_i is less likely to be correlated with u_i in our context. Nevertheless, the conditional independence assumption is very strong, and when it is violated our estimator is inconsistent. For example, as in other contexts, it is possible that x_j belongs directly in the ith demand equation, in which case x_j will be correlated with u_i and thus will not be a valid instrument (see subsection 3.3).[17] Ultimately, the reasonableness of the conditional independence assumption for a particular application is an empirical issue, and the restrictions that are associated with that assumption should be tested on a case by case basis.

[16] The fact that g is a common function does not imply that buyers are similar, since g can be the sum of K buyer-specific functions, g_k.

[17] The fact that u_i can be correlated with u_j, in contrast, does not present a problem.

In specifying our model, we have implicitly assumed that all sellers are playing the same game. Furthermore, as is common in the literature, that game is assumed to be Bertrand.[18] If the game were to vary by region, our estimation strategy would not work well.[19] However, these problems are ubiquitous. For example, it is also possible that different segments of the automobile market are engaged in different games and that those games might not be Bertrand. Unfortunately, in the absence of good cost data, it is difficult to verify our equilibrium assumption or to assess whether it varies by region.[20]

3. ESTIMATION

3.1. *Estimation by Conventional Methods*

If one is willing to impose considerable structure on equation (7), it is possible to estimate this equation by conventional methods. In particular, one must parameterize the matrix G and either parameterize or ignore heterogeneity in the unobserved demand, cost, and market structure variables.

For example, if one assumes that $u_i \sim \text{i.i.d.}N(0, \sigma^2)$ and that G consists of an exogenously specified weighting matrix $\mathcal{W}$ that is scaled by a single unknown parameter ψ, $G = \psi\mathcal{W}$, the likelihood function is[21]

$$l = -\frac{N}{2}\ln(2\pi\sigma^2) + \ln|I_n - \psi\mathcal{W}| - \frac{1}{2\sigma^2}(p - \psi\mathcal{W}p - X\beta)^T(p - \psi\mathcal{W}p - X\beta), \tag{8}$$

where I_n is the identity matrix of size n. Equation (8) can be maximized to yield efficient estimates of β, ψ, and σ^2. Moreover, the estimation can be simplified by using the fact that

$$|I_n - \psi\mathcal{W}| = \prod_j (1 - \psi\epsilon_j), \tag{9}$$

where ϵ_j is the jth eigenvalue of $\mathcal{W}$.

This is the approach that is taken by Case (1991), who assumes that $\mathcal{W}_{ij} = 1$ if i and j share a market boundary and zero otherwise, and by Pinkse and Slade (1998), who experiment with several weighting matrices. Our objective, however, is to estimate G without imposing structure on the problem a priori. We therefore choose a semiparametric estimator.

[18] One can estimate a dynamic version of the model in which lagged rival prices appear on the right-hand side of the equation. When we do this we obtain very similar results.

[19] This is an argument in favor of not using time-series variation, since the game could also change over time.

[20] Pinkse and Slade (2000) use cost data to verify the equilibrium solution concept in a model in which demand functions are estimated using similar techniques.

[21] For a general discussion of maximum likelihood estimation in the presence of spatially lagged dependent variables and/or spatial residual correlation, see Anselin (1988).

3.2. *Estimation by Semiparametric Methods*

In this subsection, we propose a semiparametric series estimator for equation (7). Since prices appear on the right-hand side of this equation, we take an instrumental variables approach. We prove that our estimator is consistent, demonstrate that $\hat{\beta}$ and $\hat{g}$ are asymptotically normal, and show how to obtain their standard errors.

Our semiparametric estimator makes use of a vector, d_{ij}, of measures of distance between regions or outlets i and j in some metric.[22] For example, this vector could include the Euclidean distance between the geographic locations of the two outlets, a zero/one variable that indicates whether j is i's nearest neighbor, and a zero/one variable that indicates whether the two outlets are in the same broad region of the country. For other applications, measures of proximity in taste space, such as differences in speeds and storage capacities of computers, might be more relevant.

The price set at outlet i depends partly upon the prices set at other outlets and partly upon demand and cost factors, as noted earlier. Formally,[23]

$$p_i = \sum_{j \neq i} g(d_{ij}) p_j + \beta^T x_i + u_i \qquad (i = 1, \ldots, n). \tag{10}$$

The function g in (10) shows how the distance measures, d_{ij}, influence the strength of competition between products i and j. In the current subsection, we are interested in devising methods that will tell us something about the structure of g and will also give us consistent estimates of β, without specifying a functional form for g. It will still be up to the practitioner, however, to select the measures that are included in d_{ij}.

Since the discrete measures in d_{ij} can take finitely many values, one can without loss of generality assume that d_{ij} contains one compound discrete measure d_{ij}^D taking D^* different values,[24] $1, \ldots, D^*$, and a vector of continuous distance measures d_{ij}^C. For convenience of notation, in what follows we assume that there is only one continuous distance measure. The modifications that are required to deal with several continuous measures, however, are straightforward. We thus have

$$g(d) = \sum_{t=1}^{D^*} I(d^D = t) g_t(d^C), \tag{11}$$

where I is the indicator function that equals one when its argument is true and zero otherwise, and $g_t, t = 1, \ldots, D^*$, are continuous functions. Furthermore, each of the g_t's can be written as

$$g_t(d^C) = \sum_{l=1}^{\infty} \alpha_{tl} e_{tl}(d^C), \tag{12}$$

[22] We also experimented with models in which market structure variables enter g.

[23] The intercepts, A, have been dropped, since they can be included in the regressor matrix X.

[24] For example, if there are m^* dichotomous indicators, D^* will equal 2^{m*}.

where the α_{tl}'s are unknown coefficients, and the e_{tl}'s for each t form a basis of the function space to which g_t belongs.[25] For example, when g_t has compact support, common choices for series expansions are ones consisting of Fourier or polynomial functions.

Setting $e_1(d) = I(d^D = 1)e_{11}(d^C), \ldots, e_{D^*}(d) = I(d^D = D^*)e_{D^*1}(d^C), e_{D^*+1}(d) = I(d^D = 1)e_{12}(d^C), \ldots$ and letting $\alpha_l, l = 1, 2, \ldots,$ denote the corresponding coefficients, we have

$$g(d) = \sum_{l=1}^{\infty} \alpha_l e_l(d). \tag{13}$$

Accordingly,

$$\begin{aligned} p_i &= \sum_{l=1}^{\infty} \alpha_l \sum_{j \neq i} e_l(d_{ij}) p_j + \beta^T x_i + u_i, \\ &= \sum_{l=1}^{L_n} \alpha_l \sum_{j \neq i} e_l(d_{ij}) p_j + \beta^T x_i + v_i, \end{aligned} \tag{14}$$

where $v_i = u_i + r_i$ with $r_i = \sum_{l=L_n+1}^{\infty} \alpha_l \sum_{j \neq i} e_l(d_{ij}) p_j$, and L_n denotes the number of expansion terms to be estimated. Let $\alpha = [\alpha_1, \ldots, \alpha_{L_n}]^T$. In vector notation, (14) is then

$$p = Z\alpha + X\beta + v, \tag{15}$$

where Z is a matrix whose (l, i) element is $\sum_{j \neq i} e_l(d_{ij}) p_j$.

There are three concerns that must be addressed. First, Z contains current rival prices and is thus not independent of u, let alone of v. Second, the number of columns in Z, L_n, increases with the sample size. Finally, v is not an ordinary error term but contains neglected expansion terms, r, in addition to the ordinary error, u.

We deal with endogeneity by taking an instrumental variables approach. As the number of endogenous right-hand side variables increases with the sample size, so must the number of instruments. Since the variables that are to be instrumented are of the form $\sum_{j \neq i} e_l(d_{ij}) p_j, l = 1, \ldots, L_n$, it is intuitive to choose instruments of the form $\sum_{j \neq i} e_l(d_{ij}) x_{jh}$, where x_{jh} is a regressor for observation j. If x_{jh} explains much of the variation in p_j, then one would expect $\sum_{j \neq i} e_l(d_{ij}) x_{jh}$ to explain much of the variation in $\sum_{j \neq i} e_l(d_{ij}) p_j$. When only one exogenous regressor is used to construct instruments in this manner, the number of instruments is automatically the same as the number of endogenous right-hand side variables, namely L_n. Each additional exogenous regressor provides an additional L_n instruments.

The number of instruments, b_n, must be no less than the number of endogenous right-hand side variables L_n plus the number of exogenous regressors H.

[25] This is akin to the fact that any vector can be written as a linear combination of basis vectors of the Euclidean space to which the vector belongs.

Let B be the n by b_n matrix of instruments. In general, we use the notation P_Y to denote the orthogonal projection matrix onto the columns of Y, and M_Y to denote $I - P_Y$, where I is the identity matrix. Premultiply both sides of (15) by P_B to obtain

$$P_B p = P_B Z\alpha + P_B X\beta + P_B v = P_B W\theta + P_B v. \tag{16}$$

In this equation, $W = [Z|X]$, where $[Z|X]$ denotes the concatenation of the matrices Z and X, and $\theta = [\alpha^T, \beta^T]^T$. We can then estimate θ by the instrumental variables (IV) estimator

$$\hat{\theta} = (W^T P_B W)^{-1} W^T P_B p, \tag{17}$$

and hence

$$\hat{g}(d) = \sum_{l=1}^{L_n} \hat{\alpha}_l e_l(d). \tag{18}$$

The form of $\hat{\theta}$ is identical to that of the traditional parametric IV estimator, albeit that now the number of columns of the W and B matrices increases with the sample size. Without the matrix P_B in the definition of $\hat{\theta}$, consistency and asymptotic normality follow from Andrews (1991). To our knowledge, all results that appear below, in contrast, are new.

It is now possible to state the first theorem. Let $\lambda_{\min}$ and $\lambda_{\max}$ be functions whose images are respectively the smallest and largest eigenvalues of their arguments, and let # be a function whose image is the number of elements in its argument.

THEOREM 1 (Consistency): *If*:

(i) $E[p] = (I - G)^{-1} X\beta$, *there exist scalars* c_G *and* N *such that* $0 \le c_G < 1$, *and for all* $n > N$, *the eigenvalues of* G *are between* $-c_G$ *and* c_G,

(ii) $\limsup_{n\to\infty} \lambda_{\max}(\Omega) < \infty$, *where* $\Omega = V[u]$,

(iii) X *contains at least one 'locally measured' exogenous variable with nonzero coefficient*, (*i.e.*, *a variable that varies by individual location*), *the regressors are uniformly bounded, and* $X^T X/n$ *converges to a positive definite limit matrix*,

(iv) *for any fixed bounded set* D, $\lim_{n\to\infty} \#\{(i, j) : d_{ij} \in D\}/n < \infty$,

(v) *for some* $N > 0$ *and some sequence* $\{\zeta_n\}$, $\inf_{n>N} \lambda_{\min}(W^T P_B W/\zeta_n) \ge 1$ *a.s.*,

(vi) $\lim_{n\to\infty} \sup_{1\le i\le n,\, 1\le l<\infty} \sum_{j\ne i} |e_l(d_{ij})| < \infty$,

(vii) *for some fixed* $\lambda > 1$, $\sup_{1\le l<\infty} |\alpha_l l^{\lambda}| < \infty$,

(viii) $L_n \to \infty$, $nL_n^{2-2\lambda}\zeta_n^{-1} \to 0$, *and* $b_n\zeta_n^{-1} \to 0$, *as* $n \to \infty$;

then (a) $\hat{g}(d) - g(d) = o_p(1)$ *at almost all* d *and* (b) $\hat{\beta} - \beta = o_p(1)$.[26]

[26] $U_n = O(z_n)$ means that the limit as n approaches infinity of U_n/z_n is a finite constant. When O is replaced by o, this limit is zero. When O (o) has a subscript p, it refers to a probability limit.

The conditions of Theorem 1 not only ensure consistency of the estimators of g and β, but also implicitly guarantee that g and β are identified. A separate proof of identification would be longer than is merited.

We first summarize the practical implications of the conditions of Theorem 1 for our application, after which we discuss each condition in turn. The detailed discussion is more technical and can be skipped without loss of continuity.

The conditions of Theorem 1 embody three main assumptions. The first is that dependence between random variables at different locations (terminals in our application) does not decrease too slowly as the 'distance' between them increases. More precisely, when more terminals are located between terminals i and j, the dependence between the random variables at i and j should lessen.[27] Fully symmetric competition is therefore explicitly excluded.

Secondly, the price response functions (g_t) should be reasonably smooth functions of the continuous distance measures (d^C or Euclidean distance in our application) for each value of t. Extreme fluctuations appear unlikely in our context; in fact, even the much stronger assumption of monotonicity of the price response functions would not be unreasonable there.

The final assumption is that there be at least one local instrument for the price at each terminal. This means that not only must there be at least one exogenous regressor, but also at least one of those regressors must vary locally (i.e., it must be possible for it to take different values at different terminals).

Note that we do not make any assumptions concerning homogeneity across terminals. Indeed, we allow for more than heteroskedasticity, since aspects of the conditional distribution in addition to the second moments can depend on regressor values. We now proceed with a more detailed discussion of individual conditions.

Condition (i) excludes the possibility of spatial unit roots, which can occur, for example, when firms match rival price changes. Although of interest, allowing for spatial unit roots (or indeed even stronger forms of spatial dependence) leads to a hopeless statistical quagmire.

Condition (ii) says that our errors can be heteroskedastic and correlated with variance covariance matrix Ω. The maximum eigenvalue restriction on Ω is satisfied when the error variances are uniformly bounded and the autocovariances are summable. The autocovariances are summable if $\{u_i\}$ satisfies what is called a mixing condition and the locations of our observations are suitably spread out. In other words, correlation must decay suitably fast with distance.

The restriction of one 'locally measured' exogenous variable in condition (iii) is needed for identification purposes.[28] If all variables vary only regionally, the coefficient on the price of a neighboring observation is generally not identified

[27] The conditions of Theorem 1 are in fact even weaker than this in respects irrelevant to our application.

[28] On the issue of identification, our model is not subject to the reflection problem (Manski (1993)), since the values of our regressor variables vary with location but are not solely determined by, and are unlikely to depend in a linear fashion on, geographic location however measured.

separately from the exogenous variables of the observation itself, since the observations share the same set of exogenous regressors. The boundedness condition on the regressors can be relaxed at the expense of stronger conditions elsewhere. Finally, the third part of condition (iii) is standard, except that the exogenous variables are assumed to be deterministic. One can allow for random exogenous variables with some additional notation.

In conjunction with condition (i), condition (iii) ensures that we can at least identify the coefficients of the exogenous variables and the first term in each expansion (the constant). Identification of the remaining expansion term coefficients is achieved by sufficient variation in the distances, as assumed in condition (iv). Condition (iv) implies that, in general, additional observations cause the area studied to grow rather than the density of observations to increase.

Without condition (v), our right-hand side variables would be collinear. Since the number of regressors in Z (and hence W) increases with the sample size, the dimension of $W^T P_B W/n$ also increases. Hence the number of eigenvalues of $W^T P_B W/n$ increases, and there is no guarantee that they will be bounded away from zero uniformly in n and across eigenvalues. Condition (v), which is a standard, albeit not very primitive, assumption in the series estimation literature, deals with this problem. The sequence $\{\zeta_n\}$ is technical in that it has no practical relevance. We are only concerned with the existence of such a sequence. If $W^T P_B W$ is invertible, such a sequence can always be found.[29] Condition (viii), however, imposes additional restrictions on $\{\zeta_n\}$.[30]

Condition (vi) restricts the locations of the observations as well as the functions e_l. When the support of a particular e_l is finite, there are only finitely many neighboring observations for which $e_l(d_{ij})$ is nonzero, and condition (vi) is satisfied automatically. When in the limit there are infinitely many j's for which $e_l(d_{ij}) \neq 0$, condition (vi) requires that $e_l(d)$ decline suitably fast as $d \to \infty$ and that the observations be sufficiently spread out. In other words, we do not want an infinite number of observations in a limited area.[31]

Condition (vii), which is a "smoothness condition," is also standard in the series estimation literature. It is almost equivalent to the condition $\sum_{l=1}^{\infty} \alpha_l l^{\lambda-1-\lambda_\varepsilon} < \infty$ where λ_ε is an arbitrarily small positive number. When a polynomial expansion is used as in our application, this condition can be expressed in terms of the Sobolev norm, i.e. $\|g\|_{St} = \sup_d \sum_{i=1}^{t} |g^{(i)}(d)|$, where $g^{(i)}$ is the ith derivative of g. With polynomial expansions, an initial transformation is carried out to the $[0, 1]$ interval before powers are taken. Condition (vii) is therefore similar to $\|g\|_{St} < \infty$, where t is the smallest integer no less than $\lambda - 1 - \lambda_\varepsilon$. Hence, $\lambda > 1$ means that g must have bounded first derivatives, $\lambda > 4$ means that its fourth derivatives must be bounded, and so forth. Hence the greater the number λ, the more derivatives are bounded, and hence the smoother is this function.[32]

[29] If $W^T P_B W$ is singular, the instruments were poorly chosen.

[30] One can think of $\{\zeta_n\}$ as a sequence that increases almost as fast as n.

[31] More formally, we do not want any fixed and bounded set to contain more than an asymptotically negligible fraction of the observations.

[32] There are different notions of smoothness, and this is but one of them.

Finally, condition (viii) restricts the rate at which L_n and b_n go to infinity. This condition says nothing about the optimal rate, let alone the optimal choice. From the proof, however, it can be deduced that the optimal rate of increase for L_n and b_n is $n^{1/(2\lambda-1)}$, assuming that L_n and b_n increase at the same rate.[33]

Theorem 1 establishes the consistency of $\hat{\beta}$ and $\hat{g}$. For the purpose of inference, it is also useful to establish the limiting distribution of $\hat{\beta}$, which is achieved in Theorem 2. In Theorem 3 we establish the limiting distribution of $\hat{g}$ at fixed values of d.

$\Omega = V[u]$ can be written as $\sum_{i=1}^{n} \lambda_i^* \eta_i \eta_i^T$, where the η_i are orthonormal eigenvectors and the λ_i^* are the corresponding eigenvalues. Using this notation, we have the following:

THEOREM 2 (Asymptotic Normality of $\hat{\beta}$): *If in addition to the conditions of Theorem* 1:

(ix) $\max_i n^{-1}\lambda_i^* \|X^T P_B M_{P_B E[Z]} P_B \eta_i\|^2 \to 0$ *as* $n \to \infty$,

(x) *the elements of the matrix of instruments* B *are uniformly bounded, and for* N *and* ζ_n *of condition* (v), $\inf_{n>N} \lambda_{\min}(B^T B/\zeta_n) \geq 1$, $\inf_{n>N} \lambda_{\min}(Z^T P_B Z/\zeta_n) \geq 1$, $\inf_{n>N} \lambda_{\min}(\overline{Z}^T P_B \overline{Z}/\zeta_n) \geq 1$, *where* $\overline{Z} = E[Z]$,

(xi) $nb_n^2 L_n \zeta_n^{-2} \to 0$, *as* $n \to \infty$;

then $(X^T P_B M_{P_B Z} P_B \Omega P_B M_{P_B Z} P_B X)^{-1/2} X^T P_B M_{P_B Z} P_B X(\hat{\beta} - \beta) \xrightarrow{\mathcal{L}} N(0, I)$.

The conditions of Theorem 2 and Theorems 3 and 4 below are technical and their discussion can be skipped by readers who are only interested in the application. The main practical implication is that the conditions on the rate at which dependence decreases with increasing distance are more restrictive than in the conditions of Theorem 1.

Condition (ix) is a technical condition that is used to exclude the possibility that a single observation has an asymptotically nonnegligible effect. It is generally satisfied when the off diagonal elements of Ω decline sufficiently fast. When Ω is diagonal, it is implied by condition (ii).

With longer proofs, the boundedness of the instruments in condition (x) could be avoided.[34] The second and third eigenvalue conditions are similar to those in condition (v). The condition on the eigenvalues of $B^T B$ could be tied to a sequence other than $\{\zeta_n\}$; tying it to $\{\zeta_n\}$, however, reduces the number of sequences used.

Condition (xi), like condition (viii), is uninformative about the optimal choice of L_n and b_n, and there are situations in which no sequences $\{L_n\}$ and $\{b_n\}$ that

[33] No additional efficiency can be gained if they do not. This implies that $\hat{g}(d) - g(d) = O_p(n^{(1-\lambda)/(2\lambda-1)})$ at almost all d and that $\hat{\beta} - \beta = O_p(n^{(1-\lambda)/(2\lambda-1)})$. The greater the value of λ, therefore, and hence the smoother g, the greater the asymptotic efficiency of the estimator, were the value of λ known. Indeed, for $\lambda = \infty$ there are only finitely many nonzero coefficients and hence a rate of $n^{-1/2}$ could be achieved if this fact were known a priori. This is in line with the standard results for a fixed number of regressors.

[34] Given the choice of instruments for our application, the boundedness condition is not overly restrictive.

satisfy the conditions can be found. However, one might let L_n and b_n increase at a rate of $n^{1/5}$, in which case the condition would require that ζ_n go to infinity at a rate exceeding $n^{4/5}$.

THEOREM 3 (Asymptotic Normality of $\hat{g}$): *If in addition to the Conditions of Theorems* 1 *and* 2:

(xii) $\inf_{n>N} \lambda_{\min}(W^T P_B \Omega P_B W/\zeta_n) \geq 1$,

(xiii) $nb_n^{2/3} L_n^{1/3} \zeta_n^{-4/3} \to 0$ *and* $nL_n^{(2/3)(1-\lambda)} \zeta_n^{-2/3} \to 0$, *as* $n \to \infty$;

then $\widehat{\Omega}_g^{-1/2}\{\hat{g}(d) - g(d)\} \xrightarrow{\mathcal{L}} N(0,1)$, *where* $\widehat{\Omega}_g = \|\Omega^{1/2} P_B W (W^T P_B W)^{-1} \Phi^T e\|^2$, *with* $e = [e_1(d), \ldots, e_{L_n}(d)]^T$ *and* $\Phi \in \mathbb{R}^{L_n \times (L_n + H)}$ *is the matrix*

$$\Phi = [0 \quad I]. \tag{19}$$

The conditions of Theorem 3 are of the same form as those used earlier. Condition (xii) is equivalent to condition (v) when Ω is proportional to the identity matrix, i.e. when the errors are independent and identically distributed. Condition (xiii) is stronger than condition (xi) but can still be satisfied. Indeed, when L_n and b_n increase at a rate of $n^{1/5}$, ζ_n should increase at a rate exceeding $n^{9/10}$ to satisfy the first half of condition (xiii); if L_n and b_n increase at a slower rate, so can ζ_n. Having L_n and b_n increase slowly, however, requires a greater degree of smoothness on g in the form of λ taking greater values to satisfy the second half of condition (xiii); for $L_n = n^{1/5}$, $\lambda = 4$ is sufficient; for $L_n = n^{1/10}$, $\lambda > 6$ is needed.

Theorems 2 and 3 determine the matrices that are used to rescale $\hat{\beta} - \beta$ and $\hat{g}(d) - g(d)$ to get limiting standard normal distributions. However, Theorems 2 and 3 are silent as to estimation. In Theorem 4 we derive a consistent estimator of matrices of the form $\text{plim}_{n\to\infty} n^{-1} J^T \Omega J$, where J is a matrix of uniformly bounded variables such as regressors. In the standard linear model with regressors J, the variance matrix is $(J^T J)^{-1} J^T \Omega J (J^T J)^{-1}$. Since the structure of Ω can be very complicated, and we do not wish to impose any structure on the covariances (including homogeneity), estimation of Ω by itself is infeasible.

Instead, we use a simple generalization of White's (1980) heteroskedasticity-consistent covariance matrix estimator, or indeed of the Newey–West (1987) covariance matrix estimator.[35] When Ω is a diagonal matrix, White (1980) suggests estimating

$$\Psi = \text{plim}_{n\to\infty} n^{-1} J^T \Omega J \tag{20}$$

by $\widehat{\Psi} = n^{-1} J^T \widehat{\Omega} J$, where $\widehat{\Omega}$ is a diagonal matrix with diagonal elements $\hat{u}_i^2$.

When the u_i's are correlated, White's approach does not suffice. If the errors are stationary one can use the Newey–West estimator. In our case, the errors are possibly nonstationary,[36] so instead we use a matrix $\widehat{\Omega}$ with

$$\widehat{\Omega}_{ij} = \iota_{ij} \hat{u}_i \hat{u}_j, \tag{21}$$

[35] Another relevant reference is Andrews and Monahan (1995).

[36] Stationarity is used here to mean that the joint distribution can depend on locations, not just on the distance between locations, and not to denote a unit root.

where the ι_{ij} are weights that, for fixed i and j, converge to 1 as n increases.[37] Although, in general, $\widehat{\Omega}_{ij}$ does not converge to Ω_{ij}, $\widehat{\Psi}$ can be shown to converge to Ψ for suitably chosen ι_{ij}.[38] We suggest that ι_{ij} be set to 1 for the values of i and j for which Ω_{ij} is likely to be the greatest. Moreover, ϕ_n should approach infinity with n, where[39]

$$\phi_n = \max_{j\le n} \sum_i \iota_{ij} + \max_{i\le n} \sum_j \iota_{ij}. \tag{22}$$

Note that although we do not know the value of any of the Ω_{ij}'s, it is not difficult to make an educated guess about which correlations are likely to be large, which is all that is required. For example, for each i, one can rank the observations according to one of the distance measures in d_{ij}. Moreover, if the weights are chosen such that $\iota_{ij} = \iota_{ji}$, $\widehat{\Psi}$ will be symmetric. If a distance measure, K_{ij}, is used for ranking purposes, and if this measure is not symmetric, one can rank according to $(K_{ij} + K_{ji})/2$.

We are now in a position to state the final theorem. Let $\Omega_{i(j)}$ denote the element of Ω corresponding to $\iota_{i(j)}$, where $\iota_{i(j)}$ is the jth order statistic of $\iota_{i1}, \ldots, \iota_{in}$.

THEOREM 4 (Covariance Matrix Estimation): *If in addition to the conditions of Theorem* 2:

(xiv) $\sup_i E[u_i^4] < \infty$ *and for some* $\widetilde{C} > 0$ *and some* $\chi > 0$,

$$\sup_{n>N} \sup_{i,j} \|\Omega_{i(j)}\| j^{1+\chi} < \widetilde{C},$$

(xv) *for any* i, j, k, l *and any random variables* $\mu_i, \mu_j, \mu_k, \mu_l$ *with* $\sup_i E|\mu_i|^4 < \infty$ *that can be written as* $\mu_i = \mu(X_i, Z_i, B_i, p_i)$,

$$|\operatorname{cov}[\mu_i\mu_k, \mu_j\mu_l]| \le \sqrt{\max\{E[\mu_i^2\mu_k^2]E[\mu_j^2\mu_l^2], E|\mu_1\mu_2\mu_3\mu_4|\}}\{\delta_{ij} + \delta_{il} + \delta_{jk} + \delta_{kl}\},$$

where the 'mixing numbers' $\{\delta_{ij}\}$ *satisfy* $\sup_i \sum_{j=1}^n |\delta_{ij} + \delta_{ji}| = O(1)$ *and the indices correspond to the indices of the observations*,

(xvi) *the vector of residuals* $\hat{u}$ *satisfies* $\|\hat{u} - u\|^2 = O_p(\Lambda_n)$, *and* $\phi_n \to \infty$, $\phi_n^2 n^{-1} \to 0$, $\Lambda_n \phi_n^{1/2} n^{-1/2} \to 0$, *as* $n \to \infty$,

(xvii) *the weights* ι_{ik} *are positive and bounded,* $\iota_{i(k)} = 0$ *for all* $k > \phi_n$, *and* $|\iota_{i(k)} - 1| \le \overline{C} k \phi_n^{-1}$ *for all* $k \le \phi_n$;

then $\widehat{\Psi} - \Psi = o_p(1)$.

[37] We have ignored the implicit dependence on n in our notation here.

[38] It would be a mistake to set $\iota_{ij} = 1$ for all i and j. Indeed, when J is the matrix of regressors in a standard linear regression model that is estimated by ordinary least squares, $\widehat{\Psi}$ is $n^{-1} J^T \hat{u}\hat{u}^T J = 0$, since $J^T \hat{u} = 0$ is the first-order condition of the least squares estimator.

[39] Alternatively, we could let the ι_{ij}'s decrease suitably fast with the perceived value of Ω_{ij}. The advantage of the latter scheme is that for suitably chosen ι_{ij}, $\widehat{\Omega}_{ij}$ and hence $\widehat{\Psi}$ can be guaranteed to be positive semidefinite. Indeed, one could use weights similar to those in Newey–West (1987), albeit accounting for the nonstationarity, $\iota_{i(j)} = 1 - j/(\phi_n + 1)$, where $\iota_{i(j)}$ is the jth order statistic of $\iota_{i1}, \ldots, \iota_{in}$.

Except for the assumed existence of fourth moments, condition (xiv) is not much stronger than condition (ii).[40]

Condition (xv) is rather technical. With more structure on the process, a simpler condition would suffice. For example, with stationary time series or spatial data, condition (xv) is implied by strong mixing (see, e.g., Ibragimov and Linnik (1970, Lemma 1, p. 306)). In practical terms, the rate at which dependence decreases with distance is further constrained.

In a standard linear regression model, Λ_n in condition (xvi) is $O(1)$. We allow Λ_n to increase with the sample size, as is appropriate for non- and semiparametric regression models.[41]

Finally, condition (xvii) restricts the choice of the weights $\{\iota_{ij}\}$. It permits weights that decline in a similar way to those in Newey and West (1987) as well as zero/one weights. Unfortunately, it does not provide guidance concerning the most efficient choice of weights. Indeed, in the absence of stationarity, the problem of determining the optimal choice is probably intractable.

3.3. *The Instruments*

Rival prices appear on the right-hand side of our estimating equation, and, since we are dealing with imperfect competition, all prices are at least potentially jointly determined and thus endogenous. Appropriate instruments for these regressors must therefore be found. Moreover, we need variables that vary by location. Any of the locally measured included exogenous variables is a candidate for an instrument. In addition, locally measured exogenous variables from rival markets can be used.

While this is a common choice,[42] the use of rival variables is somewhat different here. To illustrate, consider a random utility discrete choice demand equation. The endogenous variable in that equation is own price, p_i. With our estimating equation, the endogenous variables are a subset of rival prices, p_j with $j \neq i$. We therefore use the variables x_j as instruments for p_j, whereas most other researchers use x_j as instruments for p_i. In spite of this difference, the validity of either practice depends on the validity of the conditional independence assumption, $E[u_i|X] = 0$. It is therefore clear that this assumption should be tested.

[40] Assuming the existence of fourth moments is not always reasonable (e.g. in financial time series) but it does not seem unreasonable in our application.

[41] For example, with our semiparametric model, $\hat{u} - u = p - W(W^T P_B W)^{-1} W^T P_B p - u = -W(W^T P_B W)^{-1} W^T P_B u$, and hence

$$\|\hat{u} - u\|^2 \leq \|P_{P_B W} u\|^2 \lambda_{\max}(W^T W) \lambda_{\max}((W^T P_B W)^{-1}) = O_p(L_n n \zeta_n^{-1})$$

by condition (v). Condition (xvi) then says that $n^{1/2} L_n \phi_n^{-1/2} \zeta_n^{-1} \to 0$, as $n \to \infty$, which is satisfied, for example, if one chooses $L_n \sim n^{1/5}$ and $\phi_n \sim n^{1/4}$, as long as $\zeta_n \to \infty$ at a rate faster than $n^{23/40}$. Note that these rates do not violate condition (xi) when ζ_n increases at a rate faster than $n^{4/5}$. For $\lambda_{\max}(W^T W) = O_p(n)$ some additional restrictions would need to be imposed on the dependent variable. The example merely illustrates condition (xvi).

[42] See, for example, Berry, Levinsohn, and Pakes (1995), who use rival product characteristics as instruments.

In most applications, there are more moment conditions than there are unknown parameters. The excess moment conditions can be employed to test the validity of the set of moment conditions by means of an overidentification test. In particular, exogeneity of the instruments can be assessed. As in the generalized method of moments (GMM) literature, it is possible to use the $\mathcal{J}$ statistic for this purpose, where $\mathcal{J}$ is proportional to the minimized value of the GMM objective function, $(p - W\hat{\theta})^T B(B^T \widehat{\Omega} B)^{-1} B^T (p - W\hat{\theta})$, and the notation $\widehat{\Omega}$ indicates that our first stage estimate of the matrix in parentheses is used. The $\mathcal{J}$ statistic is distributed χ^2 with degrees of freedom equal to the number of instruments minus the number of regression coefficients.

4. WHOLESALE GASOLINE MARKETS

4.1. *The Market*

Most large American cities have nearby terminals where refined petroleum products are sold to wholesalers. These products are first shipped from a refinery to a terminal by pipeline or barge and then trucked by wholesalers to retail service stations.[43]

Sellers, or refiners who produce gasoline, can be grouped into two broad classes, majors and independents. Major brand gasoline bears the trademark of one of the large integrated oil companies (e.g., Exxon or Shell). Independents, in contrast, tend to be smaller and less fully integrated. Sellers post product prices at each terminal site. Posted prices fluctuate frequently, and the spread between the highest and lowest price at a given terminal can be as much as fifteen cents per gallon. Major sellers charge higher prices and secure their customers through brand loyalty, long term contractual arrangements, or company affiliation. Independent sellers, in contrast, charge lower prices and sell a higher proportion of unbranded gasoline.[44] A typical terminal is supplied by approximately twelve companies.

Buyers, or wholesalers who purchase and truck gasoline, can also be grouped into two classes. Some are independent "jobbers." Jobbers own and operate some stations; they also supply stations that they do not operate. The other group of wholesalers is affiliated with a refiner. These affiliated wholesalers also purchase gasoline that they truck to their own stations or sell to independent marketers. Affiliated wholesalers buy only from their parent companies, whereas jobbers are not tied to particular sellers.

We assume that the upstream market is imperfectly competitive. To enter this market, one must acquire a refinery, which involves a substantial investment. The downstream market, in contrast, is assumed to be perfectly competitive. Any

[43] A number of researchers have studied this market including Slade (1986), Spiller and Huang (1986), and Borenstein and Shepard (1996).

[44] The distinction between major and independent is not the same as the distinction between branded and unbranded. Indeed, both major and independent refiners sell both branded and unbranded products.

individual who acquires a truck that can transport petroleum products can enter the downstream market, and there are many small independent wholesalers as a consequence.

There are also two types of gasoline sold at each terminal—branded and unbranded. Branded products are normally sold to service stations that bear the name of the refiner, whereas unbranded gasoline is produced by a branded refiner but sold as a homogeneous product that cannot be resold under the brand name. Buyers of unbranded gasoline from wholesalers are normally small independent marketers.

Arbitrage occurs when buyers select a terminal site that offers the lowest delivered price (rack price plus transport, information, and other costs).[45] When a buyer is willing to truck gasoline long distances in order to minimize costs, he will almost certainly purchase from a low price seller at the distant terminal. However, nonprice factors also influence a buyer's choice, and many of those factors are not observed by the econometrician. These include traffic density, road quality, and bottlenecks such as bridges and tunnels that must be negotiated en route.

Arbitrage can also occur on the seller side of the market. For example, shipments that were originally destined for one city can be diverted to another where the price is expected to be higher. Market equilibration from the seller side, however, usually takes more time, since it involves revising pipeline schedules that are normally set well in advance.

We expect that most of the pricing arbitrage that occurs across terminals results from the actions of independent jobbers who purchase unbranded products, since independent jobbers are in a position to take advantage of the lower unbranded price. For this reason, in our empirical work, we analyze the low, unbranded rack price at each terminal. Purchasing patterns in that market are not complicated by dynamic issues such as brand loyalty and switching costs. Furthermore, unbranded prices are not discounted and are therefore true transaction prices.

The branded and unbranded markets are fairly well segregated. Indeed, branded sellers must sell branded products to their regular customers, and branded buyers must purchase the products of their affiliated refiners. For this reason, we assess inter rather than intraterminal price competition. Our analysis, however, is conditioned on several local market structure variables, including the average of the prices posted by the branded sellers. In order to motivate an estimating equation that includes branded price, Appendix B develops a formal model in which there are both branded and unbranded sellers at each terminal.

A given wholesaler can supply terminals in more than one metropolitan area and will therefore often purchase gasoline from more than one terminal. Moreover, as relative prices vary, the fraction of a wholesaler's purchases that are supplied by a given terminal will also vary. For example, if a buyer patronizes two terminals, A and B, and if the price at A rises relative to the price at B, the wholesaler will purchase gasoline from A only for those stations that are very

[45] A rack price is a price at which wholesalers purchase petroleum products for resale. Rack products (and prices) can be branded or unbranded.

close to A. A larger fraction of stations, however, will be supplied by gasoline purchased from B.

Regular unleaded gasoline is physically an almost completely homogeneous commodity. Nevertheless we consider the product to be differentiated. Indeed, there are a number of factors that cause individual sellers to face downward sloping demand. These include geographic location, brand loyalty, and the presence or absence of gasoline additives. With unbranded gasoline, however, only the former is of primary importance.

Finally, the industry divides the country into five regions or petroleum allocation districts (PADs). PADs one through five are in the east, midwest, south, northwest, and southwest respectively. Oil companies often treat PADs or groups of PADs as relevant markets. In particular, there is very little pipeline shipment of product between regions located to the east of the Rocky mountains (PADs 1–3) and those located to the west (4 and 5), and companies that market in both areas usually operate the two regions independently. Buyers, however, who transport by truck, not pipeline, can arbitrage across PADs.

4.2. *Data and Preliminary Data Analysis*

The data set is a cross section of 305 terminals in the lower 48 states. Although there are 312 terminals in the U.S., observations were eliminated if there were no price data for the week of interest (i.e., if there were no unbranded sellers) or if the site was located in Hawaii or Alaska.

Terminal prices are published weekly by the Oil Price Information Service (OPIS), a private data collection agency. Our prices are for the third week in October of 1993. They are unbranded rack prices to resellers, f.o.b. terminal. This means that the buyer bears the transport cost, as in our model. Both regular and premium unleaded prices are available. We focus on the regular unleaded price because the volume of sales in that category is greater. Prices are measured in cents per gallon and are denoted *PRICE*.

Explanatory variables include the observed demand, cost, and market structure factors. Data for those variables are as follows: Gasoline spot prices are included to capture overall economic conditions in the oil industry. Spot markets, where only unbranded gasoline is sold, are larger in geographic extent and more competitive than terminal markets. Spot markets are located in New York, the Gulf Coast, the Midwest, Los Angeles, San Francisco, and the Northwest. Spot prices are on average lower than terminal prices, but are not a practical alternative for wholesalers, due to the distances and transactions costs involved. The spot price of gasoline for terminal i is the price that prevailed in the spot market that was closest to i. The spot price, like the terminal price, is for the third week in October of 1993. This variable, which is published by OPIS and is measured in cents per gallon, is denoted *SPOT*.

Changes in gasoline inventories are a measure of supply/demand imbalance. The percentage change in stocks is 100 times the stock in the third week in October minus the stock in the second week divided by the stock in the second week.

Inventory data are available from OPIS for each PAD. Moreover, OPIS divides PAD one (the East Coast) into the Northeast and Southeast coasts. Percentage changes in inventories are denoted *STOCK*.

City population and household income are local demand variables. Population, which is for the year 1989, is published by the Bureau of the Census.[46] It is measured in millions of persons and is denoted *POP*. Average household income, which also comes from census data, is also for 1989. This variable is measured in thousands of dollars per year and is denoted *INC*.

City wage rates (*WAGE*) are a measure of local labor costs. Wage rate data are average annual pay in 1992 for workers covered by unemployment insurance. Wage rates for metropolitan areas of the U.S. are published by the Bureau of Labor Statistics. For locations not named in metropolitan areas, the wage rate in the closest city was used.[47]

Certain regions of the country require that gasoline burned in the region contain methyl terciary butyl ether (MTBE). This gasoline additive enhances oxygen in the fuel and is associated with cleaner burning. It also increases production costs. A dummy variable, *MTBE*, was constructed to equal one if a terminal was located in a region where MTBE was required.[48] These data are also published by OPIS.

We use the number of competing sellers at a terminal, *NCOMP*, to capture variations in local market structure. In addition, we construct an average branded price for each terminal, *BRPRICE*, as well as the number of branded sellers at that terminal, *NBRAND*. These variables are used to control for within-terminal competition.

Dummy variables that distinguish the five petroleum allocation districts were created. These variables, which are denoted PAD_i, $i = 1, \ldots, 5$, are fixed effects in the regression equations. In particular, they capture broad regional differences, such as differences in regional transport costs.[49] The inclusion of these fixed effects controls for the unobserved regional demand and cost factors.[50]

Table I lists summary statistics for each of the variables with the exception of the PAD dummies. It shows that branded prices are approximately two and one half cents per gallon higher and spot prices are two cents per gallon lower than unbranded terminal prices. It also shows that variation in most of the explanatory variables is sizable, especially city population. Since *POP* has a thin right tail, we use the natural logarithm of *POP*, *LPOP*, in the estimation.

[46] 1989 is the year of the closest census.

[47] Geographic locations were determined with the help of a number of publications that are listed in the references.

[48] There are only nine MTBE terminals in the sample. It is not clear whether they should be included in the analysis, or whether they form a separate market. The flavor of our results, however, is completely insensitive to this choice.

[49] For example, shipment by water, which is possible in coastal regions, is cheaper than shipment by pipeline.

[50] Aggregate (national) demand and cost factors are included in the constant term.

TABLE I
SUMMARY STATISTICS

	Unbranded Rack Price (*PRICE*) ¢/gallon	Spot Price (*SPOT*) ¢/gallon	Changes in Stocks (*STOCK*) % change	Population (*POP*) 10^6 people	Household Income (*INC*) 10^3 $/year	Annual Wage (*WAGE*) 10^3 $/year	Number of Competitors (*NCOMP*) Firms selling unleaded at own terminal	Average Branded Price (*BRPRICE*) ¢/gallon	MTBE (*MTBE*) Dummy Variable
Mean	57.53	55.45	0.26	0.41	30.47	23.62	12.35	60.16	0.03
S.D.	6.08	4.81	2.03	1.14	5.61	3.43	6.30	5.88	0.17
Minimum	49.25	48.05	−2.42	0.0003	17.83	16.58	1.00	52.15	0.00
Maximum	73.00	59.75	4.27	11.40	59.99	38.80	30.00	74.49	1.00

4.3. *The Metrics*

To implement the estimation, we must specify the elements of the distance vector d. Each measure is an $n \times n$ matrix with typical element i, j. We experimented with four notions of closeness or distance: terminals that are nearest neighbors, that share a market boundary, that share a market boundary with a third competitor, and the Euclidean distance between terminals. Moreover, each of these measures can be exogenously or endogenously determined.

The elements of the first matrix, denoted *NNX* where X stands for exogenous, are dummy variables that equal one if outlet j is i's nearest neighbor and zero otherwise, where i's nearest neighbor is located in the terminal that is the shortest Euclidean distance from i.[51] With the second nearest neighbor matrix, *NNP*, the nearest neighbor is determined endogenously by prices and transport costs as well as by kilometers, and the letter P is used to indicate that distance is price determined. Specifically, let EU_{ij} be the Euclidean distance between locations i and j. Then j is i's endogenous nearest neighbor if $DP_{ij} = p_j + \tau \times EU_{ij}$ is smaller for j than for any other terminal, where τ denotes the (linear) transport cost.[52] In other words, outlet j is i's endogenous nearest neighbor if j has the lowest delivered price at i's location.

The elements of the first common boundary matrix, denoted *CBX*, are dummy variables that equal one if i and j share an exogenous market boundary but are not nearest neighbors, and zero otherwise. To determine exogenous market boundaries, the continental U.S. was partitioned into nonoverlapping, all-inclusive regions. This partition was constructed so that i's market contains all customers who are at least as close (in Euclidean distance) to i as to any other terminal. The boundary between markets i and j thus consists of customers who are equidistant from the two and are not closer to any other terminal. In other words, i's market contains the set of buyers who would purchase from i if transport costs were determined solely by geographic distance and all customers based

[51] Note that i need not be j's nearest neighbor. In other words, this measure is not symmetric.

[52] We use a transport cost parameter, τ, of 0.05 cents/gallon/kilometer, a number that was provided by Shell Oil Company.

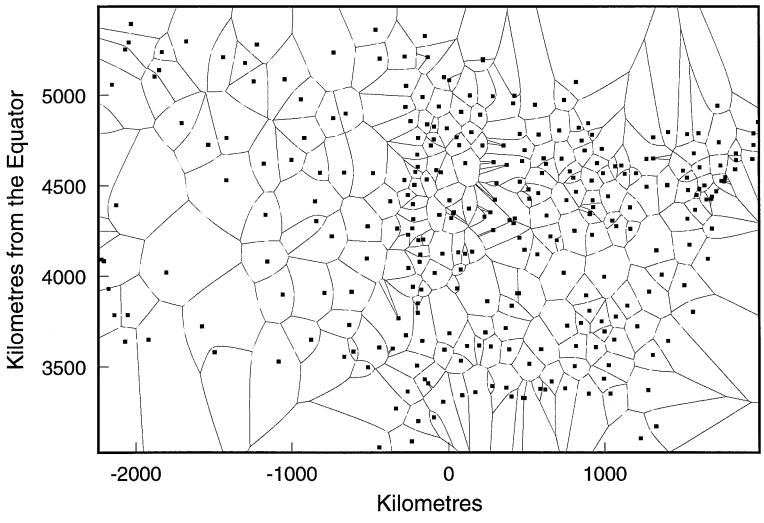

FIGURE 1.—Endogenous market areas.

their purchases on transport costs alone. We use $NCBX_i$ to denote the number of firm i's exogenous common boundary neighbors.

The second common boundary measure, with associated matrix *CBP*, is similar to the first except that market boundaries are endogenously determined. To illustrate, consider two terminals, i and j, that share a common boundary in the sense of *CBX*. The boundary between the two is determined by the perpendicular bisector of the line that joins the two locations. With *CBP*, the boundary between the two is also determined by a curve that is orthogonal to the line joining the two locations. However, instead of being a bisector of this line, its position is determined by the relationships $p_i + \tau \times EU_{1ij} = p_j + \tau \times EU_{2ij}$ and $EU_{1ij} + EU_{2ij} = EU_{ij}$. In other words, the boundary between markets i and j consists of customers for whom the delivered prices of the two sellers are the same, and i's market area consists of those customers for whom i's delivered price is less than or equal to the delivered prices of all other sellers. These market areas are depicted in Figure 1. We let $NCBP_i$ denote the number of i's endogenous common boundary neighbors. On average, sellers have six common boundary neighbors. Since the determination of common boundary markets is more complex, we give details in Appendix C.[53]

Second order common boundary measures allow for indirect competition. The first second order matrix, *CBX2*, consists of dummy variables that equal one if i and j do not share a market boundary in the sense of *CBX*, but each shares a boundary with a third seller, k. We let $NCBX2_i$ denote the number of firm

[53] This measure of closeness is very similar to the one used by Feenstra and Levinsohn (1995). The principal difference is that our transport costs are linear whereas theirs are quadratic.

i's exogenous second order common boundary competitors. The second second order matrix, *CBP2*, is defined in a similar manner except that markets are based on delivered prices instead of Euclidean distances. We let $NCBP2_i$ denote the number of *i*'s endogenous second order common boundary competitors. On average, sellers have twenty second order common boundary neighbors.

We also constructed continuous, global measures of closeness. Elements of the first matrix, *EDX*, are functions of the Euclidean distance between locations *i* and *j*, whereas elements of the second matrix, *EDP*, are functions of delivered prices. The function that we use is $1/(0.01 \times XX_{ij} + 1)$, where $XX_{ij} = EU_{ij}$ for *EDX*, and $XX_{ij} = EU_{ij} + (p_j - p_i)/\tau$ for *EDP*.[54] These measures are thus smooth, decreasing, convex functions of exogenous and endogenous distance, respectively.

Finally, we need measures of rival prices that correspond to each weighting matrix. These are constructed by interacting the distance measures with prices. To illustrate, the variable *NNPPRICE* was constructed by premultiplying the vector *PRICE* by the matrix *NNP*. This means that the weighted average of rival prices is simply one times the price of the endogenously determined nearest neighbor. Other rival price measures, e.g., *CBPPRICE*, *CBP2PRICE*, and *EDPPRICE*, were constructed in a similar fashion. Each is a weighted average of rival prices where the weights or strengths of rivalry are determined by the distance measures.

4.4. *Construction of Instruments*

Rival price regressors can be endogenous for two reasons: they are weighted averages of prices, and both prices and weights can be endogenous. As noted earlier, we use the characteristics of rival markets, x_j with $j \neq i$, as instruments for the prices in those markets, p_j. The groups of markets that interact, however, is an issue that must be determined by the data. For this reason, for each continuous terminal-specific exogenous variable (i.e., population, income, wage, and the number of competitors at the terminal), we create an instrument by premultiplying this variable by a weighting matrix.[55] To illustrate, when a specification includes the endogenous nearest neighbor price, *NNPPRICE*, additional instruments are created using products of the exogenous nearest neighbor weighting matrix and the vectors of exogenous variables, $NNX \times LPOP, NNX \times INC$, etc.[56] This means that exogenous variables from nearest neighbor terminals are used as instruments. Averages of common boundary location variables are used when the equation contains *CBPPRICE*, and so forth, whereas all four sets of instruments are used when all four rival price measures are included in a specification.

[54] $EU_{ij} + (p_j - p_i)/\tau$ is the (asymmetric) endogenous Euclidean distance between terminals *i* and *j*. Indeed, when p_j is equal to p_i, it is the exogenous Euclidean distance. However, when prices differ, this distance is adjusted by an amount that refects both the price difference (positive or negative) and the transport cost.

[55] We assume that the number of competitors at the terminal is predetermined, since it is fixed when the pricing decisions are made.

[56] We multiply by *NNX* rather than *NNP* because *NNP* is endogenous.

5. EMPIRICAL RESULTS

5.1. *Parametric Estimates*

Prior to examining the semiparametric estimates, it is useful to look at ordinary least squares (OLS) and parametric instrumental variables (IV) estimates of the pricing equation (10). Due to the presence of spatially lagged dependent variables, OLS (IV) estimates of the coefficients of this equation are (can be) inconsistent.[57] Nevertheless, they are suggestive.

Table II contains OLS estimates of price response functions that include each endogenous rival price measure separately, as well as specifications with average branded price at the same terminal and with all rival price measures simultaneously. The dependent variable in each specification is the unbranded rack price, *PRICE*. The variable *NNPPRICE* appears in the first specification, *CBPPRICE* appears in the second, and so forth. The table shows that the coefficients of the rival price variables decline as one moves from local to global measures of competition (i.e., from specifications 1 to 4). However, this is not the best way to determine the relative importance of the various measures of rivalry, since the units of measurement differ. It is preferable to assess this issue in a less direct manner. First, notice that the influence of the spot price, as measured by either the magnitude or the significance of its coefficient, grows as one moves from local to more global notions of competition. This means that, when rivalry is measured inappropriately, the spot price becomes a proxy for the omitted more appropriate rival price measure. Second, notice that both R^2 and the significance of the rival price coefficients decline as one moves from specifications 1 to 4. Individually, each of these facts implies that local measures of competition outperform global measures. Taken together, the evidence in favor of local competition is overwhelming.

One can use the same methods to assess the influence of the average branded price at the same terminal (specification 5). Indeed, intraterminal competition between branded and unbranded gasoline appears to be less intense than interterminal competition among sellers of unbranded gasoline at nearest neighbor and common boundary outlets. However, intraterminal competition is stronger than competition at second order common boundary locations.

When all rival price measures are included in a single equation (specification 6), the significance of the coefficients of the interterminal rival price variables declines as one moves from local to global measures. Furthermore, in this specification, the influence of the spot price is less strong than in any of the others.

Table II reveals another empirical regularity—as the number of competitors rises, prices fall. Moreover, this conclusion does not depend on whether the number of competitors at the same terminal (*NCOMP* and *NBRAND*) or at neighboring terminals (*NCBP* and *NCBP2*) are used as market structure measures.

[57] Parametric IV estimators are inconsistent if the functional form that is imposed on g is incorrect. Our IV estimators are therefore consistent only when we have selected the correct distance measure and, when this measure is discrete, the magnitude of the price response does not depend on Euclidean distance.

TABLE II
OLS PRICE RESPONSE FUNCTIONS
Endogenous Market Boundaries

Rival-Price Variable	Nearest-Neighbor 1	Common-Boundary 2	Second-Order Common-Boundary 3	Euclidean Distance 4	Branded Price at Same Terminal 5	All Rival Prices 6
NNPPRICE	0.793					0.394
	(0.038)					(0.042)
CBPPRICE		0.146				0.041
		(0.008)				(0.010)
CB2PRICE			0.026			0.006
			(0.0025)			(0.004)
EDPPRICE				0.002		0.0004
				(0.0014)		(0.0006)
BRPRICE					0.489	0.375
					(0.032)	(0.034)
# of competitors	−0.084	−0.016	−0.035	−0.089		0.223
	(0.027)	(0.032)	(0.039)	(0.042)		(0.032)
# of CB competitors		−8.888				−2.520
		(0.475)				(0.813)
# of 2nd order competitors			−1.226			−0.312
			(0.111)			(0.260)
# of branded competitors					−0.636	−0.557
					(0.047)	(0.054)
Spot price	0.309	0.402	0.736	0.900	0.604	0.234
	(0.067)	(0.071)	(0.083)	(0.097)	(0.075)	(0.047)
	$R^2 = 0.84$	$R^2 = 0.82$	$R^2 = 0.73$	$R^2 = 0.61$	$R^2 = 0.78$	$R^2 = 0.93$

Notes: Supply and demand variables and PAD fixed effects not shown. Standard errors in parentheses.

The only exception is the positive coefficient of *NCOMP* in the specification that contains all market structure measures.

Next consider the IV regressions. Table III, which contains the IV estimates, shows that the significance of the rival price coefficients declines when these variables are instrumented. The qualitative nature of the results, however, does not change. This table also shows that, when the four distance weighted rival prices are considered simultaneously, only the coefficient of the nearest neighbor rival price is significant at conventional levels. Conditional on being nearest neighbors, therefore, no other notion of closeness matters.

A number of tests of specification were made. First, we assessed the explanatory power of the exogenous variables (i.e., if the instruments can explain the endogenous variables). For the null hypothesis of no explanatory power, p values were all 0.00. Next, we assessed the exogeneity of the instruments. The last row in Table III shows tests of overidentification as measured by the $\mathcal{J}$ statistic. With the exception of the specification that includes only the continuous inverse distance measure of rivalry, *EDPPRICE*, the moment conditions are satisfied.

TABLE III
IV PRICE RESPONSE FUNCTIONS
Endogenous Market Boundaries

Rival-Price Variable	Nearest-Neighbor 1	Common-Boundary 2	Second-Order Common-Boundary 3	Euclidean Distance 4	Branded Price at Same Terminal 5	All Rival Prices 6
NNPPRICE	0.921					0.593
	(0.181)					(0.145)
CBPPRICE		0.173				−0.071
		(0.017)				(0.089)
CBP2PRICE			0.071			0.031
			(0.009)			(0.028)
EDPPRICE				0.003		0.002
				(0.002)		(0.0012)
BRPRICE					0.575	0.483
					(0.155)	(0.115)
# of competitors	−0.082	−0.075	−0.010	−0.085		0.269
	(0.027)	(0.375)	(0.050)	(0.043)		(0.084)
# of CB competitors		−10.435				−5.299
		(0.957)				(6.624)
# of 2nd order competitors			−3.088			−1.703
			(0.359)			(1.548)
# of branded competitors					−0.752	−0.721
					(0.215)	(0.176)
Spot price	0.212	0.309	0.454	0.896	0.547	0.243
	(0.151)	(0.088)	(0.130)	(0.096)	(0.127)	(0.087)
	$R^2 = 0.84$	$R^2 = 0.82$	$R^2 = 0.61$	$R^2 = 0.43$	$R^2 = 0.78$	$R^2 = 0.89$
	$J\ Stat = 2.2$	$J\ Stat = 0.5$	$J\ Stat = 0.02$	$J\ Stat = 10.0$	$J\ Stat = 0.1$	$J\ Stat = 1.2$
	$df = 3$	$df = 2$	$df = 2$	$df = 3$	$df = 3$	$df = 8$

Notes: Supply and demand variables and PAD fixed effects not shown. Standard errors in parentheses. The J statistic is a test of overidentification. *df* denotes the degrees of freedom for the J statistic.

Rejection in the one case is perhaps due to the fact that the equation is misspecified. Since the $\mathcal{J}$ statistic is a joint test of model specification and instrument validity, rejection does not necessarily imply that the instruments are invalid. For a final assessment of instrument validity, we tested whether the characteristics of rival markets (i.e., the instruments) should enter the best reply equation directly.[58] For the null hypothesis that they do not belong in that equation, the p values were all over 0.15. The instruments therefore appear to be valid.

[58] To illustrate, if the rival price measure was the nearest neighbor price, we included the nearest neighbor characteristics in the estimating equation and used second order, nearest neighbor characteristics as instruments for nearest neighbor price.

We also estimated specifications that contain the exogenous distance measures. Those results are extremely similar to the specifications that are shown in Tables II and III.[59]

5.2. *Semiparametric Estimates*

We have carried out our semiparametric estimation procedure for a number of different specifications. With all specifications, we use polynomials with five expansion terms per discrete distance measure for the functions e_l.

As with the parametric IV regressions, an adjustment must be made to the way that the instruments are constructed when endogenous distance weights are used. We solve the endogenous distance problem by using instruments of the form $\sum_{j\neq i} e_l(EDX_{ij})x_{jh}$ instead of $\sum_{j\neq i} e_l(EDP_{ij})x_{jh}$, where h denotes the regressor that is used to construct the instrument, and EDX_{ij} (EDP_{ij}) are the exogenous (endogenous) Euclidean distance measures.

The asymptotic covariance matrix was estimated using the results of Theorem 4. The main concern is the choice of the weights ι_{ij}. The weights were chosen such that $\iota_{ij} = 1$ if j is among i's four nearest neighbors and i is among j's four nearest neighbors, $\iota_{ij} = 0.5$ if j is among i's four nearest neighbors or i is among j's four nearest neighbors (but not both), and $\iota_{ij} = 0$ otherwise. In these calculations, Euclidean distance was used as a measure of closeness.[60]

The results are summarized in Figure 2. This figure corresponds to specifications that include endogenous nearest neighbor, common boundary, and second order common boundary prices separately and nearest neighbor and common boundary prices together. Euclidean distances enter all four specifications.[61] The continuous lines in the figure are estimated responses to a one cent price increase by the nearest neighbor or by a single common boundary or second order common boundary neighbor, respectively, at various endogenous Euclidean distances from one's terminal. The dashed lines are 95% asymptotic pointwise confidence bands.

A striking feature of the graphs is that the nearest neighbor's price has a strong effect on own price, whereas common boundary neighbor prices, whether first or second order, have much less of an impact. Indeed, the function $g_0(d) = 0$ is entirely within the common boundary confidence bands. One is forced to conclude that competition is extremely local.

Another striking feature is that all graphs are rather flat, suggesting that, for example, when two terminals are nearest neighbors, the Euclidean distance

[59] An appendix with the results from the exogenous distance estimations can be obtained from the authors upon request.

[60] Unlike the OLS and IV regressions, Euclidean distance enters all of the semiparametric regressions.

[61] We also estimated an equation with only the (endogenous) Euclidean distance measure. The results are in line with what one would expect on the basis of Figure 2: an initial steep decline to about 50 kilometers, after which the price response function hugs the horizontal axis. Furthermore, after about 25 kilometers, the response is not significantly different from zero.

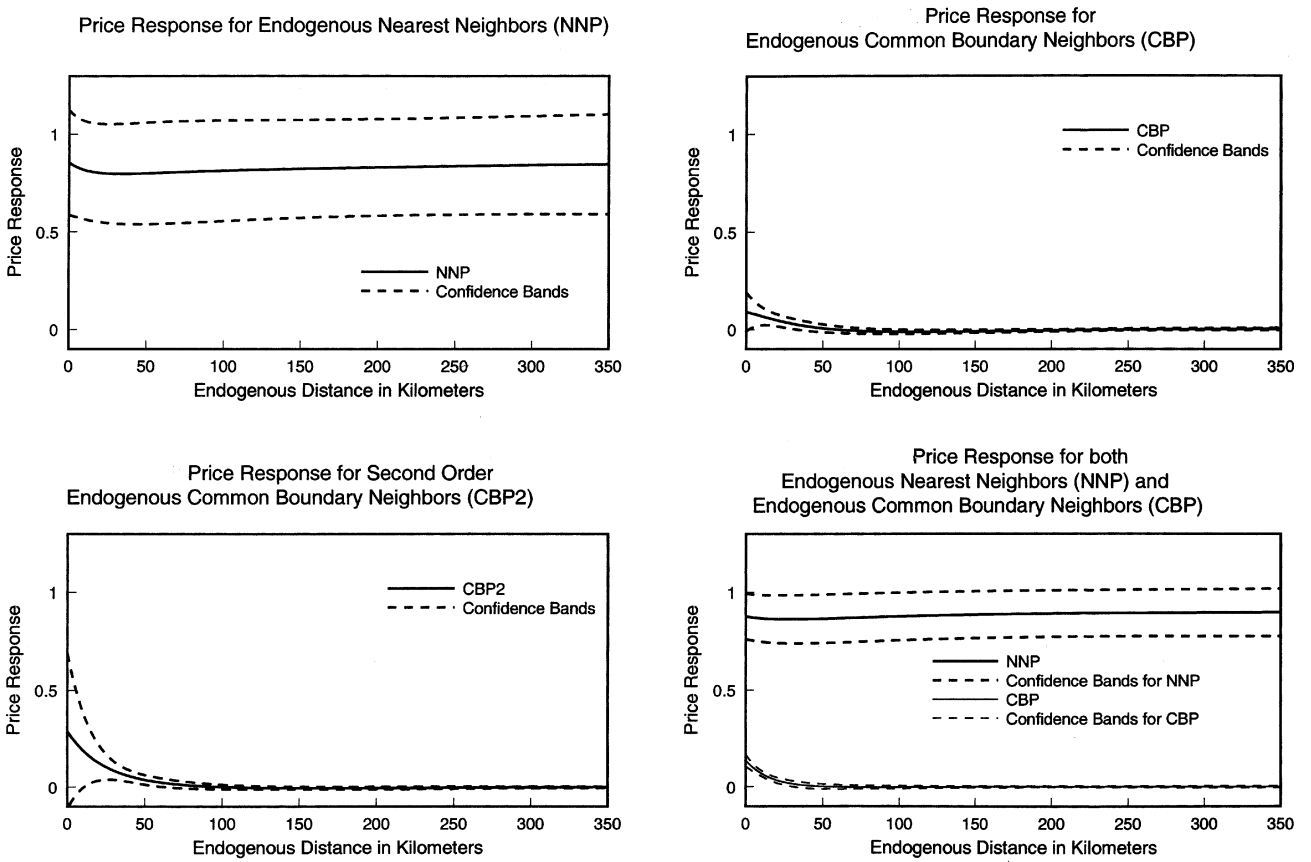

FIGURE 2.—Price responses for endogenous nearest neighbors, common boundary neighbors, and second order common boundary neighbors.

between those terminals is not an important determinant of rivalry. In other words, conditional on being the nearest neighbor, the effect of a price change at a nearby terminal is almost exactly the same as the effect of a change at a terminal that is more than 200 kilometers away. This finding is perhaps due to the fact that competition occurs at the margin, and small price changes can cause buyers that are located on boundaries to switch suppliers, regardless of the distance between terminals.

Table IV contains OLS, IV, and semiparametric IV estimates of the coefficients of all of the exogenous variables. Only the specification with nearest neighbor price is shown. The table reveals that unbranded rack prices tend to be higher when the number of competitors at the same terminal is smaller, when spot prices are higher, stocks are falling, population is lower, income is higher, wages are lower, and the additive MTBE is required. All of these results are as expected except for the effect of wages. Not all, however, are significant. The coefficients of the petroleum allocation district or PAD dummy variables, differ by specification. The difference, however, is approximately constant, implying that only the constant term varies. Finally, note that both specifications satisfy the overidentification restrictions as measured by the $\mathcal{J}$ statistic.

If the magnitude of the price response is indeed not affected by Euclidean distance, then both the parametric and the semiparametric IV estimators are consistent. The similarity in the regression coefficients in the second and third columns of Table IV is therefore not coincidental. The finding that, in our application, the parametric IV estimator appears to be consistent does not detract from the usefulness of our semiparametric estimation method. First, without having computed the semiparametric IV estimates we would not have known that the magnitude of the price response is insensitive to Euclidean distance, once we account for nearest neighbor effects. Furthermore, our estimation method is applicable in a much wider context than the one considered here.

5.3. *Further Analysis and Potential Applications*

All factors considered thus far lead to the conclusion that direct competition among terminals is extremely local and that therefore markets are small. However, we have not considered domino effects, where a price change at terminal i triggers a change at terminal j, which in turn triggers a change at terminal k, and so forth. For example, with the Hotelling model, firms compete directly only with their nearest neighbors on either side. Nevertheless, all firms compete indirectly through domino effects.

We assess this issue in two ways. First, in Figure 3, nearest neighbor terminals are connected by straight lines. This construction allows for both direct and indirect competition through spatial domino effects. Nevertheless, markets are very small.[62] Indeed, if markets are determined solely by nearest neighbor

[62] With the Hotelling model, in contrast, if neighbors were joined to one another, eventually all locations would be connected.

TABLE IV

OLS, IV, AND SEMIPARAMETRIC ESTIMATES
Price Response Functions with Endogenous Market Boundaries

	OLS	IV	Semiparametric IV
Nearest neighbor price	0.793	0.921	(See Figure 2)
	(0.038)	(0.181)	
# of competitors	−0.084	−0.082	−0.095
	(0.027)	(0.027)	(0.023)
Spot price	0.309	0.212	0.235
	(0.067)	(0.151)	(0.107)
% Δ stocks	−0.078	−0.045	−0.040
	(0.101)	(0.112)	(0.067)
log(population)	−0.128	−0.105	−0.177
	(0.079)	(0.087)	(0.048)
Average income	0.092	0.085	0.063
	(0.030)	(0.032)	(0.031)
Wage	−0.119	−0.082	−0.076
	(0.052)	(0.074)	(0.059)
MTBE	2.674	2.815	3.163
(gasoline additive)	(1.008)	(1.046)	(1.861)
PAD_1	−2.640	−5.301	1.444
	(3.536)	(5.145)	(4.183)
PAD_2	−3.851	−6.209	0.415
	(3.991)	(5.210)	(4.151)
PAD_3	−2.392	−4.829	1.283
	(3.486)	(4.893)	(4.277)
PAD_4	−2.085	−6.001	1.070
	(3.985)	(6.762)	(5.350)
PAD_5	−2.391	−5.479	1.164
	(3.646)	(5.655)	(5.821)
	$R^2 = 0.84$	$R^2 = 0.84$	
		$J\ Stat = 2.2$	$J\ Stat = 8.4$
		$df = 3$	$df = 10$

Notes: Standard errors in parentheses. The J statistic is a test of overidentification. df denotes the degrees of freedom for the J statistic.

relationships, as we find, each set of connected terminals constitutes a separate geographic market for wholesale gasoline.

Second, it is possible that our finding of extremely local competition depends on the fact that our model is static. This would be the case, for example, if more distant firms responded to price changes with a lag that was longer than a week, in which case we would have a temporal domino effect. To assess this issue, we experimented with a specification in which rival prices are lagged one month. When we ran regressions with lagged prices; however, our conclusions remained unchanged.

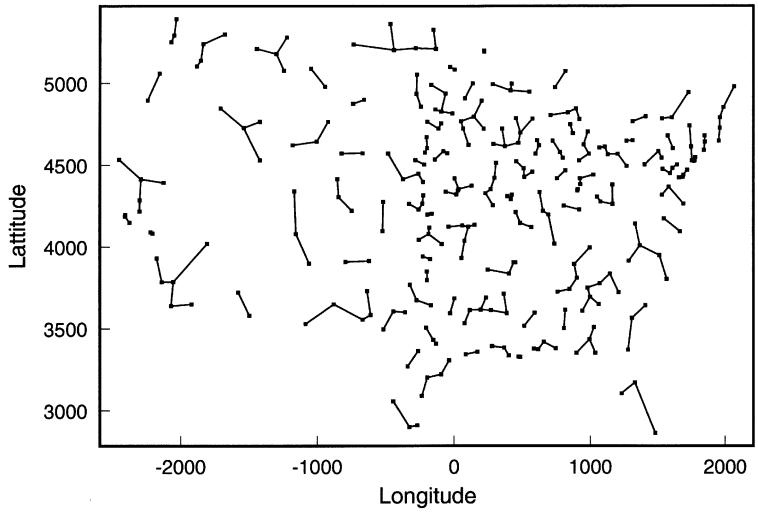

FIGURE 3.—Markets.

We can relate our findings to the empirical market definition literature. For example, Slade (1986), Spiller and Huang (1986), and Scheffman and Spiller (1987) assess the U.S. wholesale gasoline market and address the question of whether two or more terminals are in the same geographic market for antitrust purposes. The answers in those studies depend on the size and significance of price correlations or cross price elasticities of demand. However, the authors assess competition between pairs of terminals in isolation. In other words, if there are six regions of interest, each of the fifteen possible pairs must be assessed separately. The method of this paper can be used to define markets in a more integrated framework. Moreover, the techniques used in the earlier studies are less well suited to applications in which products are both physically and spatially differentiated and to situations where firms produce multiple products.[63]

There are many possible ways to use our empirical findings. We illustrate by considering the price effects of a terminal closing. Clearly openings could be assessed as well. Our model predicts that if, for example, the terminal in Seattle were to close, wholesale prices in Tacoma and Anacortes would increase by more than three cents per gallon. This finding is due to the fact that the three terminals constitute a separate wholesale market. If competition were symmetric, in contrast, the effect of this or, more generally, any closing would be negligible.

[63] Many recent studies bypass the process of market definition altogether and use differentiated product models to evaluate the effects of a merger directly (see Werden and Froeb (1994), Hausman, Leonard, and Zona (1994), Nevo (2000), and Pinkse and Slade (2000)). Nevertheless, market definition is still an integral part of antitrust analysis.

6. SUMMARY AND CONCLUSIONS

The notion of being neighbors or in close proximity surfaces in many economic problems. For example, these concepts can refer to sellers located in geographic space, or they can be applied to product types or industrial sectors. Although intuitively we know what these words mean, in practice we do not have good methods of measuring their strength. In this paper, we propose a method of estimating proximity that places minimal structure on the problem. Specifically, our procedure requires that the econometrician select a number of possible measures of closeness but allow the data to determine the importance of those measures and how each interacts with the others in a possibly nonlinear fashion.

We apply our estimation technique to the problem of determining the nature of spatial price competition in wholesale gasoline markets and find that, in this market, competition is highly localized. Indeed, we experiment with measures of distance that include being nearest neighbors, having markets with common boundaries, and being located a certain Euclidean distance apart, and find that only the first is a strong determinant of the strength of interterminal rivalry. In particular, direct rivalry decays abruptly with distance, not in a more gradual manner, as would be the case if the market were global. Our results, however, are stronger than mere rejection of global competition; we find that wholesale price competition is even more local than in a typical Hotelling model, where firms compete directly with all competitors with whom they share a market boundary. We use semiparametric methods to guide this analysis, and we think that the relationships among the distance measures are illustrated much more effectively and parsimoniously in Figure 2 than by the results of a specification search using conventional estimation techniques.

The market for crude oil is clearly worldwide. We conclude, however, that refined product markets are much smaller, a finding that could be used in a number of policy applications. We have illustrated, using a quantitative example, the impact of closing a terminal site. In addition, the qualitative nature of our results could be used in evaluating mergers among refiners.[64] Indeed, when integrated refiners merge, one must consider the impacts on markets at all stages in the production and sale of gasoline, and the wholesale market is probably the smallest in that sequence. This means that competition authorities should scrutinize the wholesale market most closely when evaluating such mergers.

There are a number of other empirical research areas where our proposed estimation technique could be fruitfully applied. We suggest one here, but clearly there are others. The problem that we choose to emphasize is the determination of the nature and strength of R&D spillovers across firms or industrial sectors. Intuitively one expects that spillovers will occur among industries that are in some sense close. It is not obvious, however, how one should measure closeness.

[64] Before assessing mergers between refiners quantitatively, one would need to develop a model of intraterminal price competition in more detail. For an application of the method developed here to a quantitative evaluation of mergers and divestitures, see Pinkse and Slade (2000).

Indeed, different researchers propose different measures, such as patent citations across sectors or input/output flows among sectors. With our technique, the measures that have been proposed by others would be the exogenously specified measures of closeness, and the data would determine their relevance.

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APPENDIX A: PROOFS

Let $P_C = W(W^T P_B W)^{-1} W^T P_B$.

PROOF OF THEOREM 1: First (a). Note that $\int(\hat{g}(d) - g(d))^2 = \|\hat{\alpha} - \alpha\|^2 + \sum_{l=L_n+1}^{\infty} \alpha_l^2$. The second term on the right-hand side is $O(\sum_{l=L_n+1}^{\infty} l^{-2\lambda})$ by condition (vii), which is $O(L_n^{1-2\lambda}) = o(1)$ by conditions (vii) and (viii). For (a) it remains to be shown that $\|\hat{\alpha} - \alpha\|^2 = o_p(1)$ and for (b) that $\|\hat{\beta} - \beta\|^2 = o_p(1)$. We show that $\|\hat{\alpha} - \alpha\|^2 + \|\hat{\beta} - \beta\|^2 = \|\hat{\theta} - \theta\|^2 = o_p(1)$. Now $\|\hat{\theta} - \theta\|^2 = \|(W^T W)^{-1/2} P_C(u + r)\|^2$. Note that $\lambda_{\max}(W^T W)^{-1} = O_p(\zeta_n^{-1})$ by condition (v). Since $P_C = P_C P_B$ we consider $\|P_C P_B u\|^2$ and $\|P_C r\|^2$. Note that $\|P_C P_B u\|^2 \leq \|P_B u\|^2$ and that $E\|P_B u\|^2 = \operatorname{tr}(P_B E[uu^T]) = \operatorname{tr}(P_B \Omega P_B) \leq \lambda_{\max}(W) \operatorname{tr}(P_B) = O_p(b_n)$ where the second equality follows from condition (ii) and the last equality from condition (ii) and the fact that P_B is an orthogonal projection matrix. Finally, $\|P_C r\|^2 \leq \|r\|^2$ and

$$E\|r\|^2 = \sum_{i=1}^{n} E\left\{ \sum_{l=L_n+1}^{\infty} \alpha_l \sum_{j \neq i} e_l(d_{ij}) p_j \right\}^2 \leq n\left(\sum_{l=L_n+1}^{\infty} |\alpha_l| \right)^2 \left(\sup_{i,l} \sum_{j \neq i} |e_l(d_{ij})| \right)^2 \sup_i E p_i^2$$

$$\leq Cn\left(\sum_{l=L_n+1}^{\infty} l^{-\lambda} \right)^2 = O_p(nL_n^{2-2\lambda})$$

for some $C > 0$ where the first inequality follows from the Schwarz inequality and the second from conditions (i), (vi), (iii), and (vii). Finally, $nL_n^{2-2\lambda}\zeta_n^{-1} + b_n\zeta_n^{-1} \to 0$ as $n \to \infty$ by condition (viii). *Q.E.D.*

LEMMA 1: $\limsup_{n\to\infty} \lambda_{\max}(V[p]) < \infty$.

PROOF: Consider

$$\lambda_{\max}(V[p]) = \lambda_{\max}\{(I - G)^{-1} \Omega (I - G^T)^{-1}\} \leq \lambda_{\max}\{(I - G)^{-1}(I - G^T)^{-1}\} \lambda_{\max}(\Omega).$$

Finally, $\limsup_{n\to\infty} \lambda_{\max}\{(I - G)^{-1}(I - G^T)^{-1}\} < \infty$ by condition (i) and $\limsup_{n\to\infty} \lambda_{\max}(\Omega) < \infty$ by condition (ii). *Q.E.D.*

LEMMA 2: *Let* $\widetilde{Z} = Z - E[Z]$. *Then* $\sup_{\|\omega\|=1} \|P_B \widetilde{Z} \omega\|^2 = O_p(nb_n L_n \zeta_n^{-1})$.

PROOF: Note that $Z = (I \otimes p^T)A$ where $A \in \mathbb{R}^{n^2 \times L}$ has $(n(i-1)+j, l)$ element $e_l(d_{ij})$. Consider

$$E\left[\sup_{\|\omega\|=1} \|P_B \widetilde{Z}\omega\|^2\right] \le \operatorname{tr} E\left[\widetilde{Z}^T P_B \widetilde{Z}\right] = \sum_{i,j=1}^{n} \operatorname{tr}\left(Q_i^T E\left[\tilde{p}\tilde{p}^T\right] Q_j B_i^T (B^T B)^{-1} B_j\right),$$

where $Q_i \in \mathbb{R}^{n \times L}$ has (j, l) element $e_l(d_{ij})$ for $j \neq i$ and 0 for $j = i$, and $\tilde{p} = p - E[p]$. Denote $\Omega^p = E[\tilde{p}\tilde{p}^T] = V[p]$. Note that

$$\begin{aligned}\sum_{i,j=1}^{n} \operatorname{tr}\left(Q_i^T E\left[\tilde{p}\tilde{p}^T\right] Q_j B_i^T (B^T B)^{-1} B_j\right) &= \sum_{l=1}^{L_n} \sum_{i=1}^{n} B_i^T (B^T B)^{-1/2} Q_{il}^T \Omega^p \sum_{j=1}^{n} Q_{jl} (B^T B)^{-1/2} B_j \\ &\le \lambda_{\max}(\Omega^p) \sum_{l=1}^{L_n} \sum_{i,j=1}^{n} B_i^T (B^T B)^{-1} B_j Q_{il}^T Q_{jl},\end{aligned}$$

where Q_{il} is the lth column of Q_i and $\Omega^p = V[p]$. By Lemma 1, $\lambda_{\max}(\Omega^p) = O(1)$. Further, note that $|B_i^T (B^T B)^{-1} B_j| \le B_i^T (B^T B)^{-1} B_i + B_j^T (B^T B)^{-1} B_j$. But $\sup_i |B_i^T (B^T B)^{-1} B_i| \le \sup_i \|B_i\|^2 \times \lambda_{\max}(B^T B)^{-1} = O(b_n \zeta_n^{-1})$ by condition (x). Hence we still need to show that $\sum_{l=1}^{L_n} \sum_{i,j=1}^{n} |Q_{il}^T Q_{jl}| = O(nL_n)$. The left-hand side is $\sum_{l=1}^{L_n} \sum_{i,j,t=1}^{n} |e_l(d_{it}) e_l(d_{jt})| \le nL_n \sup_{l,t} (\sum_{i=1}^{n} |e_l(d_{it})|)^2 = O(nL_n)$, by condition (vi). *Q.E.D.*

LEMMA 3: *Let $\mathscr{B} = P_B Z$, $\mathscr{A} = P_B \overline{Z}$, $\mathscr{C} = \mathscr{A} - \mathscr{B}$. Then for any two vectors ω, ν,*

$$\begin{aligned}\left|\omega^T (M_{\mathscr{A}} - M_{\mathscr{B}}) \nu\right| &\le \lambda_{\max}\left((\mathscr{C}^T \mathscr{C})^{1/2}\right)\left\{\lambda_{\max}\left((\mathscr{A}^T \mathscr{A})^{-1/2}\right) + \lambda_{\max}\left((\mathscr{B}^T \mathscr{B})^{-1/2}\right)\right\} \\ &\quad \times \{\|P_{\mathscr{B}}\omega\|(\|P_{\mathscr{C}}\nu\| + 2\|P_{\mathscr{A}}\nu\|) + \|P_{\mathscr{C}}\omega\| \, \|P_{\mathscr{A}}\nu\|\} \\ &= O_p\left(n^{1/2} b_n^{1/2} L_n^{1/2} \zeta_n^{-1}\right)\left\{\|P_{\mathscr{B}}\omega\|(\|P_{\mathscr{C}}\nu\| + 2\|P_{\mathscr{A}}\nu\|) + \|P_{\mathscr{C}}\omega\| \, \|P_{\mathscr{A}}\nu\|\right\}.\end{aligned}$$

PROOF: Define within the context of this lemma $S_Y = Y(Y^T Y)^{-1}$ and $\mathscr{D}_Y = Y(Y^T Y)^{-1/2}$, for $Y = \mathscr{A}, \mathscr{B}, \mathscr{C}$. Note that

$$M_{\mathscr{A}} - M_{\mathscr{B}} = P_{\mathscr{B}} - P_{\mathscr{A}} = S_{\mathscr{B}} \mathscr{C}^T + P_{\mathscr{B}} \mathscr{C} S_{\mathscr{A}}^T + S_{\mathscr{B}} \mathscr{C}^T P_{\mathscr{A}}^T + \mathscr{C} S_{\mathscr{A}}^T = \widetilde{Q}_1 + \widetilde{Q}_2 + \widetilde{Q}_3 + \widetilde{Q}_4$$

for implicitly defined $\widetilde{Q}_1, \widetilde{Q}_2, \widetilde{Q}_3, \widetilde{Q}_4$. Note that

$$\begin{aligned}|\omega^T \widetilde{Q}_1 \nu| &= \left|\omega^T D_{\mathscr{B}} (\mathscr{B}^T \mathscr{B})^{-1/2} (\mathscr{C}^T \mathscr{C})^{1/2} D_{\mathscr{C}}^T \nu\right| \\ &\le \lambda_{\max}\left((\mathscr{B}^T \mathscr{B})^{-1/2}\right) \lambda_{\max}\left((\mathscr{C}^T \mathscr{C})^{1/2}\right) \|P_{\mathscr{B}}\omega\| \, \|P_{\mathscr{C}}\nu\|, \\ \widetilde{Q}_2 &\le \lambda_{\max}\left((\mathscr{A}^T \mathscr{A})^{-1/2}\right) \lambda_{\max}\left((\mathscr{C}^T \mathscr{C})^{1/2}\right) \|P_{\mathscr{B}}\omega\| \, \|P_{\mathscr{A}}\nu\|, \\ \widetilde{Q}_3 &\le \lambda_{\max}\left((\mathscr{B}^T \mathscr{B})^{-1/2}\right) \lambda_{\max}\left((\mathscr{C}^T \mathscr{C})^{1/2}\right) \|P_{\mathscr{B}}\omega\| \, \|P_{\mathscr{A}}\nu\|, \\ \widetilde{Q}_4 &\le \lambda_{\max}\left((\mathscr{A}^T \mathscr{A})^{-1/2}\right) \lambda_{\max}\left((\mathscr{C}^T \mathscr{C})^{1/2}\right) \|P_{\mathscr{C}}\omega\| \, \|P_{\mathscr{A}}\nu\|.\end{aligned}$$

The final equality in the lemma statement follows from Lemma 2 and condition (x). *Q.E.D.*

LEMMA 4: $\|X^T (M_{\mathscr{A}} - M_{\mathscr{B}}) u\| = O_p(nb_n L_n^{1/2} \zeta_n^{-1}) = o_p(n^{1/2})$.

PROOF: Without loss of generality we prove that $|X_{\cdot 1}^T (M_{\mathscr{A}} - M_{\mathscr{B}}) u| = o_p(n^{1/2})$, where $X_{\cdot 1}$ is the first column of X. In Lemma 3, take $\omega = X_{\cdot 1}$ and $\nu = u$. Observe that (i) $\|P_{\mathscr{B}}\omega\| = \|P_{\mathscr{B}} X_{\cdot 1}\| = O(n^{1/2})$,

(ii) $\|P_{\mathscr{C}}\omega\| = \|P_{\mathscr{C}}X_{\cdot 1}\| = O_p(n^{1/2})$, (iii) $\|P_{\mathscr{C}}\nu\| = \|P_{\mathscr{C}}u\| = \|P_{\mathscr{C}}P_B u\| \le \|P_B u\| = O_p(b_n^{1/2})$, (iv) $\|P_{\mathscr{A}}\nu\| = \|P_{\mathscr{A}}u\| = \|P_{\mathscr{A}}P_B u\| \le \|P_B u\| = O_p(b_n^{1/2})$, such that

$$\begin{aligned}\|X^T(M_{\mathscr{A}} - M_{\mathscr{B}})u\| &= O_p(n^{1/2}b_n^{1/2}L_n^{1/2}\zeta_n^{-1})O_p(n^{1/2}(b_n^{1/2}+b_n^{1/2}) + n^{1/2}b_n^{1/2}) \\ &= O_p(nb_nL_n^{1/2}\zeta_n^{-1}) = o_p(n^{1/2})\end{aligned}$$

since $n^{1/2}b_nL_n^{1/2}\zeta_n^{-1} \to 0$ by condition (xi). *Q.E.D.*

LEMMA 5: $X^T(M_{\mathscr{A}} - M_{\mathscr{B}})\Omega(M_{\mathscr{A}} - M_{\mathscr{B}})X = o_p(n)$.

PROOF: This can be proved along the lines of Lemma 4. *Q.E.D.*

PROOF OF THEOREM 2: We need to prove that $(X^TP_BM_{\mathscr{A}}P_B\Omega P_BM_{\mathscr{A}}P_BX)^{-1/2}X^TP_BM_{\mathscr{A}}P_Bu \xrightarrow{\mathscr{L}} N(0,I)$. Let $s = \Omega^{-1/2}u$ such that $\{s_i\}$ is i.i.d. From Lemmas 4 and 5 (noting that $P_B(M_{\mathscr{A}} - M_{\mathscr{B}}) = M_{\mathscr{A}} - M_{\mathscr{B}}$) it follows that instead it is sufficient to prove that

$$\begin{aligned}&\left(X^TP_BM_{\mathscr{B}}P_B\Omega P_BM_{\mathscr{B}}P_BX\right)^{-1/2}X^TP_BM_{\mathscr{B}}P_Bu \\ &\quad= \left(X^TP_BM_{\mathscr{B}}P_B\Omega P_BM_{\mathscr{B}}P_BX\right)^{-1/2}X^TP_BM_{\mathscr{B}}P_B\Omega^{1/2}s \xrightarrow{\mathscr{L}} N(0,I).\end{aligned}$$

Because of the independence of the s_i this is an immediate consequence of Eicker's (1963) central limit theorem using condition (ix).[65] *Q.E.D.*

PROOF OF THEOREM 3: First observe that

$$\begin{aligned}\widehat{\Omega}_g &\ge \lambda_{\min}\left((W^TP_BW)^{-1}W^TP_B\Omega P_BW(W^TP_BW)^{-1}\right)\|\Phi^Te\|^2 \\ &\ge \lambda_{\min}(W^TP_B\Omega P_BW)\lambda_{\min}((W^TP_BW)^{-2})\|\Phi^Te\|^2,\end{aligned}$$

which is of order no less than $\zeta_n n^{-2}\|\Phi^Te\|^2$. Hence, $\widehat{\Omega}_g^{-1/2} = O_p(\zeta_n^{-1/2}n\|\Phi^Te\|^{-1})$.

Note that

$$\hat{g}(d) - g(d) = e^T\Phi(W^TP_BW)^{-1}W^TP_B(p - W\alpha) - \sum_{l=L_n+1}^{\infty}\alpha_l e_l(d).$$

First,

$$\left|\sum_{l=L_n+1}^{\infty}\alpha_l e_l(d)\right| = O\left(\sum_{l=L_n+1}^{\infty}|\alpha_l|\right) = O\left(\sum_{l=L_n+1}^{\infty}l^{-\lambda}\right) = O\left(L_n^{1-\lambda}\right).$$

Hence $\widehat{\Omega}_g^{-1/2}\sum_{l=L_n+1}^{\infty}\alpha_l e_l(d) = O_p(\zeta_n^{-1/2}n\|\Phi^Te\|^{-1}L_n^{1-\lambda}) = O_p(\zeta_n^{-1/2}nL_n^{1-\lambda}) = o_p(1)$ by condition (xiii).

Now look at the first term,

$$\begin{aligned}e^T\Phi(W^TP_BW)^{-1}W^TP_B(p - W\alpha) &= e^T\Phi(W^TP_BW)^{-1}W^TP_Bu + e^T\Phi(W^TP_BW)^{-1}W^TP_Br \\ &= S_u + S_r.\end{aligned}$$

Note that

$$|S_r| \le \|\Phi^Te\|\lambda_{\max}((W^TP_BW)^{-1/2})\|r\| = \|\Phi^Te\|O_p(\zeta_n^{-1/2})O_p(n^{1/2}L_n^{1-\lambda}) = O_p(n^{1/2}\zeta_n^{-1/2}L_n^{1-\lambda}),$$

[65] Reference found in Andrews (1991).

where the rate for $\|r\|$ was established in the proof to Theorem 1. Thus, $\widehat{\Phi}_g^{-1/2} S_r = O_p(n^{3/2}\zeta_n^{-1} L_n^{1-\lambda}) = o_p(1)$ by condition (xiii).

Next, split up $S_u = (S_u - S_u^*) + S_u^*$, where S_u^* is S_u with the W's replaced with $\overline{W}$'s. We first look at $S_u - S_u^* = e^T \Phi(S_{\mathscr{A}} - S_{\mathscr{B}})u$, where $S_{\mathscr{A}}, S_{\mathscr{B}}, \mathscr{A}, \mathscr{B}$ are as in Lemma 3. But $S_{\mathscr{A}} - S_{\mathscr{B}} = (\mathscr{B}^T\mathscr{B})^{-1}\mathscr{C}^T + (\mathscr{B}^T\mathscr{B})^{-1}\mathscr{C}^T P_{\mathscr{A}} + S_{\mathscr{B}}\mathscr{C} S_{\mathscr{A}}^T$ and hence

$$\begin{aligned}
|e^T \Phi(S_{\mathscr{A}} - S_{\mathscr{B}})u| &\le \left\{\lambda_{\max}\left((\mathscr{B}^T\mathscr{B})^{-1/2}\right)\lambda_{\max}\left((\mathscr{A}^T\mathscr{A})^{-1/2}\right) + \lambda_{\max}\left((\mathscr{B}^T\mathscr{B})^{-1}\right)\right\} \\
&\quad \times \lambda_{\max}(\mathscr{C}^T\mathscr{C})^{1/2}\|\Phi^T e\|(\|P_{\mathscr{C}} u\| + \|P_{\mathscr{A}} u\|) \\
&= O_p(\zeta_n^{-1}) O_p\left(n^{1/2} b_n^{1/2} L_n^{1/2} \zeta_n^{-1/2}\right)\|\Phi^T e\| O_p\left(b_n^{1/2}\right) \\
&= O_p\left(n^{1/2} b_n L_n^{1/2} \zeta_n^{-3/2}\right)\|\Phi^T e\|
\end{aligned}$$

by Lemma 2, (the derivations in) Lemma 4, and condition (x). Hence $\widehat{\Omega}_g^{-1/2}(S_u - S_u^*) = O_p(n^{3/2} b_n L_n^{1/2} \zeta_n^{-2}) = o_p(1)$ by condition (xiii).

Along the same lines it can be shown that the difference between $\widehat{\Omega}_g$ and $\overline{\Omega}_g$, which is defined as $\widehat{\Omega}_g$ with the W's replaced with $\overline{W}$'s, is of smaller order than $\overline{\Omega}_g$.

Finally, let $s_L = \Omega^{1/2} P_B \overline{W}(\overline{W}^T P_B \overline{W})^{-1}\Phi^T m$ and $s = \Omega^{-1/2} u$. Then s is i.i.d. and hence by Eicker's (1963) result, $\overline{\Omega}_g^{-1/2} S_u^* = (s_L^T/\|s_L\|)s \xrightarrow{\mathscr{L}} N(0,1)$. *Q.E.D.*

LEMMA 6: $n^{-1}\sum_{i,k=1}^n (\iota_{ik} - 1)\Omega_{ik} = o(1)$.

PROOF: The left-hand side is for some $C > 0$ and sufficiently large n bounded in absolute value by

$$n^{-1}\sum_{i=1}^n \sum_{k=1}^{\phi_n} |\iota_{i(k)} - 1|\,|\Omega_{i(k)}| + n^{-1}\sum_{i=1}^n \sum_{k=\phi_n+1}^n |\iota_{i(k)} - 1|\,|\Omega_{i(k)}|.$$

Now,

$$n^{-1}\sum_{i=1}^n \sum_{k=1}^{\phi_n} |\iota_{i(k)} - 1|\,|\Omega_{i(k)}| \le \widetilde{C}\overline{C} n^{-1}\sum_{i=1}^n \sum_{k=1}^{\phi_n} k\phi_n^{-1} k^{-1-\chi} = \widetilde{C}\overline{C} n^{-1}\phi_n^{-1}\sum_{i=1}^n \sum_{k=1}^{\phi_n} k^{-\chi} = O(\phi_n^{-\chi})$$

by conditions (xiv) and (xvii). Further,

$$n^{-1}\sum_{i=1}^n \sum_{k=\phi_n+1}^n |\iota_{i(k)} - 1|\,|\Omega_{i(k)}| \le \widetilde{C} n^{-1}\sum_{i=1}^n \sum_{k=\phi_n+1}^n k^{1+\chi}$$

by conditions (xiv) and (xvii). Finally, $n^{-1}\sum_{i=1}^n \sum_{k=\phi_n+1}^n k^{1+\chi} = O(\phi_n^{-\chi}) = o(1)$. *Q.E.D.*

LEMMA 7: $n^{-1}\sum_{i,k=1}^n \iota_{ik}(u_i u_k - \Omega_{ik}) = o_p(1)$.

PROOF: The squared left-hand side expectation is $n^{-2}\sum_{i,j,k,l=1}^n \iota_{ik}\iota_{jl} E[(u_i u_k - \Omega_{ik})(u_j u_l - \Omega_{jl})]$ which is bounded by $Cn^{-2}\sum_{i,j,k,l=1}^n \iota_{ik}\iota_{jl}\{\delta_{ij} + \delta_{il} + \delta_{jk} + \delta_{jl}\}$ for some finite $C > 0$ by the uniform boundedness of $E[u_i^4]$ imposed in condition (xiv) and by condition (xv). Then

$$n^{-2}\sum_{i,j,k,l=1}^n \iota_{ik}\iota_{jl}\delta_{ij} \le n^{-2}\sum_{i,j=1}^n \delta_{ij} \sup_i \sum_{k=1}^n \iota_{ik} \sup_j \sum_{l=1}^n \iota_{jl} \le (\phi_n/n)^2 \sum_{i,j=1}^n \delta_{ij} = O(\phi_n^2 n^{-1}) = o(1)$$

by condition (xvii). *Q.E.D.*

LEMMA 8: $n^{-1}\sum_{i,k=1}^n \iota_{ik}(\hat{u}_i - u_i)(\hat{u}_k - u_k) = o_p(1)$.

PROOF: By the Schwarz inequality the left-hand side is bounded by $n^{-1}\{\sum_{i,k=1}^n \iota_{ik}^2\}^{1/2}\|\hat{u}-u\|^2 = O_p(n^{-1}n^{1/2}\phi_n^{1/2}\Lambda_n) = o_p(1)$ by condition (xvi). *Q.E.D.*

LEMMA 9: $n^{-1}\sum_{i,k=1}^n \iota_{ik}(\hat{u}_i - u_i)u_k = o_p(1)$.

PROOF: By the Schwarz inequality the left-hand side is bounded by

$$n^{-1}\|\hat{u}-u\|^{1/2}\left\{\sum_{i=1}^n\left(\sum_{k=1}^n \iota_{ik}u_k\right)^2\right\}^{1/2}.$$

Note that $\|\hat{u}-u\| = O_p(\Lambda_n)$ by condition (xvi). Finally,

$$\sum_{i=1}^n E\left[\left(\sum_{k=1}^n \iota_{ik}u_k\right)^2\right] = \sum_{i,k,l=1}^n \iota_{ik}\iota_{il}\Omega_{kl} \le \sup_{k,l}\sum_{i=1}^n |\iota_{ik}\iota_{il}| \sum_{k,l=1}^n \Omega_{kl} = O(\phi_n n)$$

by the summability of the covariances in condition (xiv) and by condition (xvii). Hence,

$$n^{-1}\|\hat{u}-u\|^{1/2}\left\{\sum_{i=1}^n\left(\sum_{k=1}^n \iota_{ik}u_k\right)^2\right\}^{1/2} = O_p\left(n^{-1}\Lambda_n^{1/2}n^{1/2}\phi_n^{1/2}\right) = O_p\left(n^{-1/2}\Lambda_n^{1/2}\phi_n^{1/2}\right) = o_p(1)$$

by condition (xvi). *Q.E.D.*

PROOF OF THEOREM 4: To simplify notation, we only consider the case in which J is a vector of ones. This can be done without loss of generality since the elements of J are bounded and deterministic. Thus we prove that $n^{-1}\sum_{i,k=1}^n\{\iota_{ik}\hat{u}_i\hat{u}_k - \Omega_{ik}\} = o_p(1)$. The left-hand side can be split up into (i) $n^{-1}\sum_{i,k=1}^n \iota_{ik}(\hat{u}_i\hat{u}_k - u_iu_k)$, (ii) $n^{-1}\sum_{i,k=1}^n \iota_{ik}(u_iu_k - \Omega_{ik})$, and (iii) $n^{-1}\sum_{i,k=1}^n(\iota_{ik}-1)\Omega_{ik}$. Expression (iii) is taken care of in Lemma 6 and (ii) in Lemma 7. (i) remains, which is

$$n^{-1}\sum_{i,k=1}^n \iota_{ik}(\hat{u}_i-u_i)(\hat{u}_k-u_k) + n^{-1}\sum_{i,k=1}^n \iota_{ik}u_k(\hat{u}_i-u_i) + n^{-1}\sum_{i,k=1}^n \iota_{ik}u_i(\hat{u}_k-u_k).$$

The first term is dealt with in Lemma 8 and the remaining two in Lemma 9. *Q.E.D.*

APPENDIX B: A MODEL WITH SEVERAL COMPETITORS AT ONE TERMINAL

Suppose that there are two sellers at terminal $i, i = 1, \ldots, n$, one branded and one unbranded. Let $p_B = (p_{B1}, \ldots, p_{Bn})^T$ be the vector of branded prices and $p_U = (p_{U1}, \ldots, p_{Un})^T$ be the vector of unbranded prices. We assume that the branded and unbranded markets are somewhat segregated, and that there is no direct interterminal competition between branded and unbranded sellers. However, at a given location, branded and unbranded products compete.

To motivate this assumption, using the data from our application, we obtain the following correlation coefficients: $\rho(p_U, p_B) = 0.426$, $\rho(p_U, p_{NNU}) = 0.940$, and $\rho(p_U, p_{NNB}) = 0.375$, where p_{NNU} (p_{NNB}) is the unbranded (branded) price at the nearest neighbor terminal. Clearly, the principal competition is between unbranded gasoline at nearest neighbor locations. The correlation coefficient between branded and unbranded prices at the same location is less than half of the correlation coefficient between unbranded prices at neighboring terminals, and the correlation between unbranded and branded prices at neighboring terminals is even smaller.

We therefore assume that a typical unbranded firm's profit, π_{Ui}, depends on the vector p_U, the scalar p_{Bi}, and the vector of local demand, cost, and market structure variables, x_i. Unbranded seller i chooses p_{Ui} to

$$\max \pi_{Ui}(p_U, p_{Bi}, x_i), \tag{23}$$

given p_{U-i} and p_{Bi}, where $p_{U-i} = (p_{U1}, \ldots, p_{Ui-1}, p_{Ui+1}, \ldots, p_{Un})^T$. The first order condition for this maximization is $f_i(p_U, p_{Bi}, x_i) = 0$, which can be solved for i's best response or reaction function,

$p_{Ui} = R_i(p_{U-i}, p_{Bi}, x_i)$. Given our assumptions on profits, this function, which is linear, can be written as

$$p_{Ui} = R_i(p_{U-i}, p_{Bi}, x_i) = \sum_{j \neq i} g_{ij} p_{Uj} + \beta^T X_i + \gamma p_{Bi} + u_i, \tag{24}$$

which is identical to equation (10) except that branded price at the same terminal has been added.

APPENDIX C: THE MARKET BOUNDARY ALGORITHMS

Below is a description of the algorithm that determines which neighbors share an endogenous boundary. Since endogenous boundaries are not piecewise linear, the algorithm is somewhat more complicated than that for exogenous boundaries, which are linear. Appendix C.1 contains a rough outline of the algorithm, whereas Appendix C.2 contains some technical detail on specific steps.

C.1. *Outline of the Algorithm*

The algorithm follows three steps.

(i) *Determine the Set of Corner Points*: Let b be an arbitrary location. Define $\tilde{d}_i(b) = p_i + \tau \mathrm{EU}_{b,i}$, where EU is the Euclidean distance between b and terminal i. Then $\mathscr{E}_{ij} = \{b : \tilde{d}_i(b) = \tilde{d}_j(b)\}$ is the set of locations at which jobbers are indifferent between buying from terminals i and j. If prices are equal or transport costs infinite, this is a straight line; otherwise it is a curve convex to the location of the higher priced terminal.

Define sets of *intersection points* $\mathscr{I}^*_{ijk}$ by $\mathscr{I}^*_{ijk} = \mathscr{E}_{ij} \cap \mathscr{E}_{ik} \cap \mathscr{E}_{jk}$. When prices are equal there generally is one intersection point, but when they are different there could be more. Finally, define sets of *corner points* $\mathscr{I}_{ijk}$ by $\mathscr{I}_{ijk} = \{b \in \mathscr{I}^*_{ijk} : \nexists i^* \neq i, j, k : \tilde{d}_{i^*}(b) < \tilde{d}_i(b)\}$. $\mathscr{I}_{ijk}$ is hence the set of intersection points for terminals i, j, k for which there is no terminal i^* closer (in terms of $\tilde{d}$) to the intersection point than terminals i, j, k are to the intersection point.

(ii) *Determine whether i and j Share an Endogenous Boundary*: When $\mathscr{I}_{ijk} \neq \varnothing$, then i shares an endogenous boundary with both j and k, and j and k also share an endogenous boundary. We have ignored boundaries that are located far outside the geographical area studied.

(iii) *Creating an Endogenous Common Boundary Graph*: One can graph the boundaries of the endogenous market areas. When this is done using the data of our application, the result is Figure 1.

C.2. *Some Technical Details*

Determining the Intersection Points:

Denote the observation longitudes and latitudes by $(\kappa_{xi}, \kappa_{yi})$, $i = 1, \ldots, n$, and the longitude and latitude of an intersection point by $(\varepsilon_x, \varepsilon_y)$. Then to determine the intersection points (if any) of observations 1, 2, and 3, the following set of nonlinear equations needs to be solved for ε_x, ε_y, and ς, where ς is the delivered price at the intersection point:

$$\begin{cases} (\varepsilon_x - \kappa_{x1})^2 + (\varepsilon_y - \kappa_{y1})^2 = (\varsigma - p_1)^2/\tau^2, \\ (\varepsilon_x - \kappa_{x2})^2 + (\varepsilon_y - \kappa_{y2})^2 = (\varsigma - p_2)^2/\tau^2, \\ (\varepsilon_x - \kappa_{x3})^2 + (\varepsilon_y - \kappa_{y3})^2 = (\varsigma - p_3)^2/\tau^2. \end{cases} \tag{25}$$

Subtracting the first from the second and third equalities, one obtains

$$\begin{cases} 2(\kappa_{x1} - \kappa_{x2})\varepsilon_x + 2(\kappa_{y1} - \kappa_{y2})\varepsilon_y + 2(p_2 - p_1)\tau^{-2}\varsigma = (p_2^2 - p_1^2)\tau^{-2} + \kappa_{x1}^2 + \kappa_{y1}^2 - \kappa_{x2}^2 - \kappa_{y2}^2, \\ 2(\kappa_{x1} - \kappa_{x3})\varepsilon_x + 2(\kappa_{y1} - \kappa_{y3})\varepsilon_y + 2(p_3 - p_1)\tau^{-2}\varsigma = (p_3^2 - p_1^2)\tau^{-2} + \kappa_{x1}^2 + \kappa_{y1}^2 - \kappa_{x3}^2 - \kappa_{y3}^2. \end{cases} \tag{26}$$

Any two of ε_x, ε_y, and ς can be expressed in terms of the third and substituted into any one of the earlier nonlinear equations.[66] One is then left with a single quadratic expression with zero, one, or two solutions (intersection points).

[66] One generally wants the largest number (in absolute value) in the denominator.

Mill Price Undercutting

We encountered the problem that, for the transportation cost used ($\tau = 0.05$ cents per gallon per kilometer), there were occasions of mill price undercutting. We have solved this by adjusting prices (for the purpose of determining common boundary neighbors only) until prices plus transport costs from all other terminal locations exceed the price at the terminal (the mill price). The price adjustments were chosen to minimize the number of price changes necessary.

A Miscellaneous Issue

It is possible for a terminal at the center of the area studied to have only two (endogenous) common boundary neighbors (and this phenomenon occurred in our application). This is due to the fact that the boundaries are not straight lines. This phenomenon manifests itself when price differences between neighboring terminals are large. The terminal with two common boundary neighbors then also has only two corner points, both with the same two neighbors.

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