Changing One’s Mind when the Facts Change: Incentives of Experts and the Design of Reporting Protocols

WEI LI
University of California, Riverside

First version received June 2004; final version accepted December 2006 (Eds.)

Experts often collect and report information over time. What reporting protocol elicits the most information? Here, a principal receives reports sequentially from an agent with privately known ability, who observes two signals about the state of the world. The signals differ in initial quality and, unlike previous work, differ in quality improvement. The paper finds that “mind changes” (inconsistent reports) can signal talent if a smart agent improves faster. Also, sequential reports dominate when the principal’s decision is very sensitive to information; a single report dominates if the mediocre agent’s signals improve faster or the agent is likely mediocre.

When the facts change, I change my mind. What do you do, sir? — John M. Keynes

1. INTRODUCTION

In most economic models of communication, information is collected once and transmitted in a single piece from a sender to a receiver (Crawford and Sobel, 1982; Aghion and Tirole, 1997; Morris, 2001). In many realistic settings, however, a sender receives multiple pieces of information over time and is asked to convey his opinion multiple times as more information comes in. Formal sequential reports are frequently observed in congressional committees, accounting (Dye and Verrecchia, 1995), capital budgeting (Arya et al., 1997), and the financial market (Penno, 1985). Informally, consultants, doctors, and other professionals are often asked to convey their early opinions before giving a final assessment.

In many of these environments, information received later may contain less noise than that received earlier and is thus of higher quality. For instance, a December survey of consumer demand for hybrid cars next year may be more accurate than a similar one taken in July. Moreover, the sender’s ability to observe the underlying true state of the world may improve as he becomes more familiar with the task at hand. In either case, the sender typically cares about how all his reports reflect on his ability. This paper investigates how an agent of privately known ability reacts strategically to improvement in the quality of his information under a sequential reports system. It also applies these insights to the optimal choice of reporting protocols. Namely, it identifies conditions under which the principal should require a single report after the agent has received all the information, and conditions under which she should ask for sequential reports instead.

1. In reply to accusation of inconsistency: The Economist, 1999-12-18, p.47.
2. Throughout this paper, the receiver of information who then makes decisions (the principal) is female and the sender of information (the agent) is male.
In this model, an agent delivers an initial report and a final report about the state of the world based on two private signals of increasing quality. After each signal, the agent sends a report to the principal, who makes a decision after the final report. Next, the true state becomes publicly observable. The same game is repeated in the second stage. The agent is one of two types: smart (type $H$) or mediocre (type $L$). Only the agent knows his type. A smart agent and a mediocre one differ not only in the level of signal quality but also in the slope of signal quality improvement. A smart agent learns about the true state of the world with higher initial accuracy than a mediocre one, but his signal quality improvement may be higher or lower than that of a mediocre agent. In the second stage, the agent is paid the expected value of his information, which depends on how smart he is perceived to be. As a result, the agent has an incentive to boost his reputation.

The main insight emerging from this model is that mind changes, or inconsistent reports, may signal high ability in equilibrium. This can happen if a smart agent’s signals improve faster than those of a mediocre one. Since a smart agent is more likely to receive and report an accurate first signal, a mediocre agent might want to “defend” his early report even when he receives conflicting signals. Thus, similar to Prendergast and Stole (1996), an agent may stick to a position likely to be wrong because changing his mind makes him appear incapable of finding the true state of the world earlier. However, unlike previous work (Scharfstein and Stein, 1990; Prendergast and Stole, 1996), a mediocre agent in this model is more likely to give consistent reports even though such consistency per se may indicate low ability in equilibrium.

The reason for this paradoxical result is that both consistency and accuracy affect the agent’s future wage, which is shown to be a convex function of his perceived ability. The wage is convex because the principal’s optimal decision in the second stage may depend crucially on the reports’ accuracy. For example, if a wrong decision leads to huge losses, an agent perceived to be smart provides information of disproportionately higher value and accordingly receives a disproportionally higher wage than a mediocre one. Specifically, being consistently right leads to the highest possible second-stage wage, whereas being consistently wrong leads to the lowest. Thus, repeating one’s early report is a risky “gamble” to receive the highest wage. Since type $L$’s signals improve slower than type $H$’s, he is more willing to take on this gamble at the potential cost of appearing consistently wrong. Furthermore, the more convex is the wage function, the more likely type $L$ lies against his final signal to appear consistent. Type $H$, however, has more confidence in his information improvement and is thus willing to change his mind when his signals differ.\(^3\)

Mind changes, therefore, are valued more as a sign of ability if the principal’s second-stage decision depends so strongly on the reports’ accuracy that the agent’s wage function is highly convex. In this case, before the true state is observed in the first stage, the agent who changes his mind is more likely to be smart than the one who is consistent. This consideration matters in areas such as major economic reforms, wars, and high stake financial manoeuvres because the principal can improve her first-stage decision using this sequencing information. If the principal’s optimal decision is not very sensitive to the reports’ accuracy, however, consistency is more valued in equilibrium.

Moreover, this model shows that if the mediocre agent improves faster than the smart agent, both types of agents lie against their better, more informative second signal. In fact, after receiving conflicting signals, type $H$ becomes less confident in his second signal than type $L$. Hence, he is more willing to repeat his initial report to appear consistent, which drives type $L$ to imitate. In equilibrium, the second report becomes useless, and the principal does not benefit at all from the agent’s improved information.

\(^3\) Some experimental and sociological evidence for increasing commitment to a wrong project is consistent with this model’s prediction (Staw, 1981, 1992). Wicklund and Braun (1987) show that people more confident in their ability seem to be less committed to their early positions than the less confident ones.
The model so far demonstrates that, due to reputational concerns, it may not be in the agent’s interest to report each signal truthfully after it is received. The principal, however, is only concerned with the reports’ accuracy and may want to choose a reporting protocol to encourage truthful reporting. One natural question is whether the principal should require sequential reports at all, given its possible inefficiency. It would seem that requiring a single report (or reports) after the agent has received all signals is optimal because it eliminates any incentive to appear consistent.

As an answer to this question, the model shows that the optimal reporting protocol depends on how strongly the principal’s decision relies on the reports’ accuracy. The advantage of requiring a final report is that the agent is judged on his accuracy alone, which leads him to report his best estimate of the state (his final signal) truthfully. The disadvantage is that the agent ignores his informative initial signal. Therefore, if the principal’s optimal decision depends primarily on which state of the world is more likely, a final report is optimal: the agent has no incentive to appear consistent by distorting his second signal. Moreover, a final report is also preferable when the mediocre agent’s signals improve faster. The sequential reports system, however, is optimal when a very precise estimate of the true state is crucial to the principal’s optimal decision. The two reports (even though the second one may be distorted) offer the principal finer information and may lead to a better decision than one, truthful final report.

It is important to emphasize that this result hinges on the timing, not the number, of the reports. This paper shows that requiring sequential reports cannot be replicated by requiring two reports at the end. Under the sequential reports system, an agent always reports his initial signal truthfully, even though he may distort his final report to appear consistent. If the principal requires both reports at the end, then the agent may simply repeat his final signal, which is the agent’s best estimate of the state, in order to appear both consistent and accurate. As a result, his first signal is lost in equilibrium, just like when one final report is required.

Previous research has shown that in a multi-agent setting, economic agents may want to be consistent with some early movers or to conform to existing consensus because they want to increase the market’s perception of their ability (Scharfstein and Stein, 1990). In the current model, both reports are associated with the agent, and distortions occur even when only his own information is used.4

Ottaviani and Sorensen (2006a,b) analyse a static reputational cheap-talk game with very general distributions of the state and the agent’s (expert’s) type. They find that full revelation or truth telling is generically impossible in this setting. Either no informative equilibrium exists or the experts can only communicate part of their information, for example, “high” or “low” despite a rich signal and message space. The current model adopts simple distributions of the agent’s type and the state to focus on the dynamic aspect of the agent’s incentive problem, that is, the interaction between the agent’s initial and final report given his reputational concerns.

More closely related to this paper, Prendergast and Stole (1996) consider a reputational concerns model in which a manager with privately known ability receives noisy signals about the true profitability of his investments over time, and the more capable manager receives signals with higher precision. In their model, the market infers each manager’s signal precision from the period-to-period change in his investment choices. Initially, large changes away from the prior indicate high-quality information and therefore high precision, and each manager exaggerates. Eventually, changes in investment indicate (many) past errors, and everyone becomes too conservative. Therefore, exaggeration is beneficial only because the agent has no reputational stake in the prior, and “admitting” that a previous investment choice was bad always hurts reputation. In

---

4. In reputational herding models, such as Scharfstein and Stein (1990), an agent wants to conform to the early mover because smart agents tend to receive the same signal and to agree with each other. All else being equal, a later mover may avoid a contrarian position because it reduces the market’s perception of his talent.
contrast, the present paper shows that with improvement in signal quality, admitting a previous mistake can enhance one’s reputation in equilibrium.

The paper proceeds as follows: Section 2 sets up the model. Section 3 characterizes the equilibria of the sequential reporting game. Section 4 identifies the optimal reporting protocol in different environments, and Section 5 discusses several key assumptions and extensions. Section 6 concludes. All proofs are in the appendix.

2. THE TWO-SIGNAL MODEL

A principal needs to make a decision based on two sequential reports from an agent. Although the model is clearly more general, this paper will couch it in a concrete story: the owner of a company needs to make an investment decision \( a \in \{0, 1\} \) after reviewing a consultant’s initial report \( (m_0) \) and final report \( (m_1) \) on a project’s profitability. The profitability depends on the true state of world \( s \), which is \textit{ex ante} good or bad \( (s \in \{g, b\}) \) with equal probability. It is easiest to equate state with profitability: no investment \( (a = 0) \) yields zero, while investment \( (a = 1) \) brings net profit \( g > 0 \) and \( b < 0 \) when the state is \( g \) and \( b \), respectively. Moreover, the owner does not invest without further information, that is, \( g + b \leq 0 \).

Before setting up the sequential reporting game formally, it may be useful to provide some real-world examples of the situations this model describes. First, in an application to stock markets, an analyst receives multiple pieces of information about a company over time and releases multiple stock recommendations. Eventually, the company’s true profitability becomes known, and the investors can evaluate the analyst’s ability. Second, in an application to the political arena, a politician initiates a war according to his private information. Then, he receives new information and needs to decide whether to continue the war or to change course. Later, the truth becomes observable and the voters can discipline the politician through elections.

2.1. Environment and information

The agent works in two stages \( N = A, B \). In each stage, the true state of the world \( s \) is, independently, either good \( (g) \) or bad \( (b) \). Events within stage \( A \) proceed as follows:

- At \( t = 0 \): the agent receives a fixed wage \( w^A \);
- At \( t = 0.5 \): the agent receives his first signal \( i_0 \in \{g, b\} \) and then sends an initial report \( m_0 \in \{g, b\} \) as to which state his initial signal indicates;
- At \( t = 1 \): the agent receives his second signal \( i_1 \in \{g, b\} \) and then sends a final report \( m_1 \in \{g, b\} \) as to which state his second signal indicates;
- At \( t = 2 \): the principal makes an investment decision \( a \in \{0, 1\} \) based on the reports; \( ^6 \)
- At \( t = 3 \): the true state of world becomes observable (but not verifiable) to all.

Stage \( B \) is identical to stage \( A \) : the agent receives a fixed wage \( w^B \) and delivers two sequential reports. Then, the principal makes an investment decision and the game ends. Timing of this game is illustrated in Figure 1.

5. Some models such as Scharfstein and Stein (1990) employ a reduced form second stage in which the agent’s wage is his posterior probability of being smart. Modelling two stages fully, however, makes it possible to study explicitly the shape of the agent’s wage function in the second stage, which influences the agent’s truth-telling incentives in the first stage. Lemma 1 characterizes this wage function.

6. The assumption that the principal only decides after receiving both reports is not crucial to the results of the model, as long as she can observe the second report. In the examples above, the investors or voters may take some action based on the early report, but incentives similar to the current model’s would arise if the principal can still use both reports and the (later) observed state to evaluate the agent.
The agent is smart (type $H$) with probability $\eta$ and mediocre (type $L$) with probability $1 - \eta$. While the distribution of the state and that of the agent’s type are common knowledge, only the agent knows his type $\theta$. The agent receives two private signals $i_0$ and $i_1$, which are independent conditional on the state. Let $p_0 \equiv \Pr(i_0 = s \mid H, s)$ and $p_1 \equiv \Pr(i_1 = s \mid H, s)$ denote the qualities of type $H$’s signals. Assume for simplicity that type $L$’s initial signal is uninformative, and let $r \equiv \Pr(i_1 = s \mid L, s)$ denote the quality of type $L$’s second signal. Type $H$’s signals are assumed to be more accurate than the corresponding ones of type $L$, that is, $p_0 > \frac{1}{2}$ and $p_1 \geq r$. Moreover, for both types of agents, the second signal is assumed to be more accurate than the first:

$$p_1 > p_0, \quad r > \frac{1}{2}.$$ 

Note also that with symmetric distribution of the state, the initial signal $i_0$ itself is not informative about ability, that is, $\Pr(i_0 = g \mid H) = \Pr(i_0 = g \mid L) = \frac{1}{2}$. This allows the analysis to focus on the dynamic incentive problems of the agent, who is not tempted to lie in his initial report.  

### 2.2. Pay-offs

The principal and the agent are risk neutral, but the principal cannot transfer the ownership of the project to the agent (e.g., due to credit constraints). Let $m^N$ be the history of reports in stage $N$ and $\Pi^N$ be the stage $N$ profit. Also, let $\hat{\eta} \equiv \Pr(H \mid m^A, s)$ be the principal’s posterior estimate of the agent being type $H$ given his first-stage reports as well as the observed state. The expected profit of the principal in stage $N$ is

$$\Pi^A = \sum_{m^A} [\Pr(g, m^A)g + \Pr(b, m^A)b]a(m^A, \eta), \quad \Pi^B = \sum_{m^B} [\Pr(g, m^B)g + \Pr(b, m^B)b]a(m^B, \hat{\eta}).$$

The principal chooses action $a$ after each report sequence $m^N$ to maximize $E \Pi = \Pi^A - w^A + E[\Pi^B - w^B \mid m^A, s]$, her total profit net of wage payments.

The true state $s$ and the reports are assumed to be observable but not verifiable; therefore, the agent cannot be paid conditional on the accuracy of reports, nor can contracts be written on the reports. Also, the principal is assumed to operate in a perfectly competitive environment. Thus, the agent is only motivated by his second-stage wage, which is simply the expected value of his information given the principal’s posterior belief about his type. 

---

7. As long as type $H$’s initial signal is more accurate than type $L$’s, the main results hold with slight modifications.

8. Strict improvement in the agent’s signal quality, rather than better information from multiple signals alone, is necessary for him to report his second signal truthfully in equilibrium. Without it, the agent has little confidence in his second signal and is likely to repeat his initial report instead. See (1.3) of Proposition 1 for details.

9. Allowing asymmetric state distribution introduces potential lying in the initial report in addition to the dynamic incentive problems. For example, if state $g$ is much more likely than state $b$, type $H$ is more likely to report that the state is $g$ because his initial signal is more accurate. This gives type $L$ an incentive to report $s = g$ with some probability even when his first signal is $b$. This effect surfaces in Prendergast (1993) and Prat (2005).

10. Hence, all rents accrue to the agent if he is perceived as talented, and the principal is only concerned about her first-stage profit in choosing a reporting protocol. If the expert market is not perfectly competitive, the principal may gain (partially) from her updated knowledge of the agent’s talent in stage $B$. For example, she may put more valuable and important projects in stage $B$, after learning about the agent’s ability in stage $A$. 

© 2007 The Review of Economic Studies Limited
Assume that the agent reports truthfully in stage $B$ (later shown to be part of an equilibrium). Let $a^*(m^B, \hat{\eta})$ denote the principal’s optimal action given reports $m^B$ and the posterior estimate of the agent’s talent $\hat{\eta}$. Then, the agent’s wage is $w(\hat{\eta}) = \Pi^B(m^B, \hat{\eta}) |_{a=a^*}$. Moreover, the following is true.

**Lemma 1.** (1) $w(\hat{\eta})$ is a convex, non-decreasing, and piecewise-linear function of $\hat{\eta}$, the posterior probability that the agent is smart; (2) $w(\hat{\eta})$ is affine and strictly increasing if the principal’s optimal action $a^*(m^B, \hat{\eta})$ depends only on the agent’s reports $m^B$.

Similar to that shown in Blackwell (1953), the value function of the agent’s information is convex in the principal’s beliefs about the agent’s type. This is because the principal can make better (and potentially different) decisions given two different posterior distributions of the agent’s type than when she has to decide given a convex combination of these distributions. Intuitively, imagine that the agent’s type is known in the second stage. Then, for any report sequence $m^B$, the principal can choose the most profitable action given the agent’s type and does no worse than if she has to choose an action knowing only the agent’s type distribution.

**Example 1.** A simple convex pay-off function. Suppose that state $s = b$ is sufficiently bad that the principal is only willing to invest if she strongly believes that $s = g$, then

\[
w(\hat{\eta}) = \begin{cases} 0, & \text{if } \hat{\eta} < \eta_1, \\ \tau_1(\hat{\eta} - \eta_1), & \text{if } \hat{\eta} \in [\eta_1, \eta_2), \\ \tau_2(\hat{\eta} - \eta_2) + \tau_1(\eta_2 - \eta_1), & \text{if } \hat{\eta} \in [\eta_2, 1], \end{cases}
\]

where $\tau_1 < \tau_2$. The agent is “fired” after stage $A$ if his perceived ability is below cut off value $\eta_1$. Otherwise, he is retained and his wage depends on where $\hat{\eta}$ falls: he gets either a good wage or a star wage. Intuitively, even the best news from a (likely) mediocre agent is not enough to change the principal’s decision from no investment (the default) to investment. Good news from an agent likely to be smart, however, may induce her to invest and earn a higher expected profit.

The exact shape of the pay-off function depends on the difference of signal quality between types as well as the project-specific values $g, b$. One special case is when the principal’s optimal action depends only on the reports regardless of $\hat{\eta}$. For instance, if $rg + (1-r)b \geq 0$ and $(1-p_0)p_1g + p_0(1-p_1)b \geq 0$, report sequences $(g, g)$ or $(b, g)$ from any agent yield a non-negative expected profit, thus the principal always invests. The wage function in this case is simply $w(\hat{\eta}) = \frac{1}{2} [rg + (1-r)b + (g-b)(p_1-r)\hat{\eta}]$.

Albeit simple, this lemma shows that the reduced-form approach used in many reputational concerns models is a special case. In those models, the agent maximizes the posterior probability that he is smart because his future wage is linear in such probability (Scharfstein and Stein, 1990; Prendergast and Stole, 1996). Such an approach implicitly assumes that the principal’s future decision problem is not very sensitive to the agent’s forecasting accuracy. One economic implication, to be explored partially below, is that the implicit incentive itself may be highly convex in professions where key information is provided by experts concerned about their reputation. This

11. The agent’s wage is the value of his information over what the principal would obtain by default, which is zero because her optimal action without further information is no investment.

12. Both $\tau_1$ and $\tau_2$ are determined using the model’s parameters: $\tau_1 = \frac{1}{2} \left( (p_0p_1 - \frac{1}{2})g + (1 - p_0)(1 - p_1) - \frac{1 - p_1}{p_0} \right)$ and $\tau_2 = \frac{1}{2} (g - b)(p_1 - r)$. Moreover, this wage function is true if $rg + (1-r)b < 0$ and $(1-p_0)p_1g + p_0(1-p_1)b > 0$. The first inequality means that report sequence $(b, g)$ from type $L$ is not good enough news about the state to warrant investment, while the second one means that the same sequence from type $H$ is.
convex incentive encourages even risk-neutral experts to take on more risk in sending reports or giving advice. Moreover, the higher is the premium on the accuracy of an expert’s advice, the more convex this implicit incentive becomes.

2.3. Equilibrium
The analysis adopts the concept of perfect Bayesian equilibrium. In the first stage, the agent’s strategy is a function that maps his type, his signals, as well as the history of reports (if any) to new report(s): \( m_0 : \Theta \times I_0 \to \Delta(g, b) \) and \( m_1 : \Theta \times I_0 \times M_0 \times I_1 \to \Delta(g, b) \). His strategy in the second stage is similarly defined. The principal chooses an action given the reports and updates her beliefs in a Bayesian way. The equilibrium consists of a triple \( (m^*, a^*, \hat{\eta}) \) such that \( m^* \) maximizes the agent’s expected wage in the second stage given his signals and the principal’s inference. The principal chooses \( a^* \) to maximize her net profit, and \( \hat{\eta} \) is her posterior belief given the agent’s strategy.

Two well-known equilibrium multiplicity problems exist in cheap-talk games. First, there always exist “babbling” equilibria in which all messages are taken to be meaningless and ignored by the receiver.\(^{13}\) The following analysis restricts attention to characterizing informative equilibria and to identifying when they exist.\(^{14}\) Second, there exists another, unimportant type of multiple equilibria: because the meaning of messages in cheap-talk games is endogenously determined in equilibrium, any permutation of messages across meanings yields another equilibrium. This paper deals with this problem by assuming that both the principal and the agent use and understand the literal meaning of the reports; whether they think the reports are credible depends on the equilibrium strategies.\(^{15}\)

3. EQUILIBRIUM INFORMATION REVELATION
This section categorizes the equilibrium strategies of the agent, focusing on how information revelation depends on both the initial difference and the improvement in the agent’s signal quality.

To begin with, there always exists an equilibrium in the second stage in which the agent reports both signals truthfully. The reason is that the agent’s wage \( w(\hat{\eta}) \) does not depend on his second-stage performance, and he has no further reputational concerns at the end of his career. Since there is no conflict of interest between the principal and the agent, it is assumed that this truth-telling equilibrium is always played in stage \( B \).

In the first stage, assume that both types of agents report signal \( i_0 \) truthfully (shown later to be part of the equilibrium).\(^{16}\) Without loss of generality, the agent’s pure continuation strategy after receiving \( i_1 \) is to report true \( i_1 \) or to repeat \( m_0 = i_0 \).\(^{17}\) Observe that it cannot be an equilibrium for type \( H \) to always report \( i_1 \) truthfully and for type \( L \) to always repeat his first

---

\(^{13}\) Farrell (1993) argue that the babbling equilibria are frequently implausible in games with some common interest. Blume, Kim and Sobel (1993) show that they are unstable in the long run from an evolutionary viewpoint.

\(^{14}\) Because the principal requires multiple reports in this model, there may exist a particular type of informative equilibrium in which one report is useful but no information can be transmitted in the other report due to the agent’s incentive problems.

\(^{15}\) Myerson (1989) and Farrell (1993) show that this type of multiplicity disappears in a rich language such as English because both the sender and the receiver may use the literal meaning of a message without believing its content. See also footnote 17 for an example.

\(^{16}\) There exists another type of equilibrium in which the initial report is completely uninformative: the agent babbles and then reports his second signal truthfully. This equilibrium is similar to that under a final report system (see Proposition 3). In the sequential reports system, existence of this equilibrium depends crucially on the principal’s belief upon hearing the uninformative initial report. See Section 5(A) for further discussions.

\(^{17}\) Restricting attention to the literal meaning of messages eliminates many uninteresting equilibria. For example, suppose there exists a full revelation equilibrium in which everyone reports the opposite of his signals, and the principal knows that, this equilibrium is equivalent to one in which everyone just reports the true signals.
report regardless of $i_1$, or vice versa. Suppose so, then type $H$ and type $L$ can be distinguished perfectly on the equilibrium path when $i_0 \neq i_1$, in which case type $L$ has a strong incentive to deviate. Hence, at most three possible continuation equilibria exist: a “full revelation equilibrium” in which both types of agents report their second signal truthfully, a “full pooling equilibrium” in which both types simply repeat their initial report, and finally, a “partial revelation equilibrium” in which the agent repeats his initial report with some probability.

3.1. Signal quality improvement and the agent’s equilibrium incentives

This section focuses on how the agent’s incentive to report his second, more informative signal truthfully depends on his type and signal quality. Since both types of agents receive better second signals, it is necessary to define a measure of signal quality improvement. A smart agent is considered to improve faster than a mediocre one if the following condition holds:

$$1 - r \geq \frac{p_0(1 - p_1)}{p_1(1 - p_0)},$$

while a mediocre agent improves faster if it does not hold. Inequality (1) compares the confidence of an agent in his second signal relative to the first when his signals disagree, which is important because the key decision an agent faces is what to report after receiving conflicting signals. In particular, the left hand side of inequality (1) measures the probability ratio that a type $L$ agent’s second signal is wrong vs. his second signal is right; the right hand side is the same ratio for type $H$. When this condition holds, type $H$ trusts his second signal more than type $L$. The opposite is true when it fails to hold.

Consider a benchmark case when both types of agent report truthfully. A comparison of posterior probabilities that the agent is smart given his reports and the observed true state suggests that both the accuracy and the sequencing of reports affect his posterior reputation:

$$\Pr(H \mid i_0 = i_1 = s) > \Pr(H \mid i_0 \neq s, i_1 = s) > \Pr(H \mid i_0 = s, i_1 \neq s) > \Pr(H \mid i_0 \neq s, i_1 \neq s).$$

Denote the above four posterior probabilities, respectively, as $CR$, $R$, $W$, and $CW$: $CR$ stands for consistently right, $R$ for a right change of mind, $W$ for a wrong change of mind, and lastly, $CW$ stands for consistently wrong. The above inequalities suggest that, for instance, given a correct final report, a change of mind is bad for the agent’s reputation because it means that he is wrong at the beginning. And being consistently wrong is even worse: it implies two faulty signals.

In the current model, however, the agent may not report truthfully due to reputational concerns. The following proposition characterizes the agent’s equilibrium behaviour with sequential reports.

**Proposition 1.** If $\eta < \overline{\eta} \in \left(\frac{1}{3}, 1\right)$, and

(1.1) if the smart agent improves faster: there exists a cut off value $p_{L0}^*$ such that if $p_0 \leq p_{L0}^*$, a unique full revelation equilibrium exists in which the agent reports both signals truthfully.\(^{19}\)

\(^{18}\) Exact expressions are in Appendix B, the analysis of Proposition 1.

\(^{19}\) The cut off value $\overline{\eta}$ is defined in Appendix B. The condition $\eta < \overline{\eta}$ guarantees the monotonicity of the agent’s mixing probability with respect to the key parameter $p_0$. Sometimes, for example, when $p_1 \approx 1$ or $r \approx \frac{1}{2}$, $\overline{\eta} \approx 1$, this restriction is trivial. In other cases, however, the wage function depends on specific parameters $(g, b)$ and the equilibrium cannot be characterized in general when $\eta$ is sufficiently high (see Section 5(B)).

\(^{20}\) The cut off values $p_{L0}^*$ and $p_{H0}^*$ are defined in Appendix B. As $p_0$ increases, a correct initial report becomes a better signal of ability. Which type of agent is more attracted to deviate from reporting his true second signal depends on his signal quality improvement. The cut off value $p_{L0}^*$ is the value above which type $L$ deviates from truthful reporting, and $p_{H0}^*$ is that for type $H$. 
If \( p_0 \geq p_0^L \), there exists a unique partial revelation equilibrium in which the agent reports his first signal truthfully (\( m_0 = i_0 \)). In the second report, type \( H \) always reports truthfully. Type \( L \) reports truthfully if \( i_0 = i_1 \), but repeats \( m_0 \) with probability \( \pi^* \in (0, 1) \) if \( i_1 \neq i_0 \). Moreover, type \( L \)'s lying probability \( \pi^* \) increases with \( p_0 \).

(1.2) If the mediocre agent improves faster: there exists a cut off value \( p_0^H \) such that if \( p_0 \leq p_0^H \), a unique full revelation equilibrium exists. If \( p_0 \geq p_0^H \), a full pooling equilibrium exists in which the agent reports \( m_0 = m_1 = i_0 \). Moreover, the second report is uninformative in any equilibrium.

(1.3) If the agent’s signal quality does not improve \( (p_0 = p_1, r = \frac{1}{2}) \), a full pooling equilibrium exists in which the agent reports \( m_0 = m_1 = i_0 \). Moreover, \( m_1 \) is uninformative in any equilibrium.

First, Proposition 1 shows that the agent reports both signals truthfully if \( p_0 \) is sufficiently low. If both type \( H \) and type \( L \) receive fairly uninformative first signals, changing one’s mind is not a bad signal of ability while giving a wrong final report is. In this case, accuracy of the final report becomes the key indicator of ability and the agent reports his second, more informative signal truthfully. If \( p_0 \) becomes sufficiently high, however, such truthful reporting is impossible: a correct first report is increasingly more likely to reflect high ability. Consider an extreme example where type \( H \)'s first signal is almost perfect \( (p_0 \approx 1) \). Then, a mediocre agent repeats his first report with probability one regardless of his second signal. Any mind change shows that he is type \( L \) for sure, while repeating his first report makes him appear smart sometimes.

Second, if the smart agent improves faster, Proposition 1 shows that the mediocre agent lies more as the smart agent’s initial signal becomes more informative. To see this, note that after receiving conflicting signals, the more an agent believes in his later (and better) signal, the less attractive repeating his first report becomes. The smart agent reports his second signal truthfully because he is more confident in its accuracy. He knows that lying is likely to lead to consistently wrong reports, yielding the lowest reputational pay-off. The mediocre agent, however, has less confidence in his second signal and thus is more attracted by the possibility of appearing consistently right. In equilibrium, inefficiency due to the mediocre agent’s lying can be quite high: the principal’s information may deteriorate significantly even if the probability that the agent is smart is negligible. This occurs when a mediocre agent repeats his first uninformative report with a high probability despite a high-quality second signal. Consider the following example.

Example 2. One good apple may ruin the barrel. Suppose that \( \eta = 0.001, p_1 = 1, r = 0.9 \), and \( \omega(\hat{\eta}) = \hat{\eta} \). That is, the agent is extremely likely to be mediocre, but the mediocre agent's second signal is very accurate. In equilibrium, however, type \( L \) repeats his first report with probability \( \pi^*(p_0) = 9.982p_0 + 0.078p_0^2 - 9 \). Clearly, \( \pi^* \) increases in \( p_0 \). When \( p_0 = 0.95, \pi^* = 0.5 \). A type \( L \) agent lies against his highly informative signal \( i_1 \) and reports his useless signal \( i_0 \) half of the time, even though the prior probability he is smart is only one out of a thousand.

Third, more subtly, Proposition 1 shows that receiving better signals than type \( L \) in absolute terms is not sufficient for type \( H \) to report truthfully. Rather, type \( H \)'s higher relative improvement in signal quality is crucial. To see this, consider the case when type \( L \)'s signals improve faster. Suppose that \( p_0 \approx p_1 \approx 1 \) and the signals differ, then type \( L \) has more confidence in his second signal than type \( H \) \( (r > \frac{1}{2}) \). Consequently, type \( H \) is more tempted to repeat his first report than type \( L \) even though both his signals are more accurate. In equilibrium, type \( L \) must imitate. This pooling equilibrium is clearly inefficient because the agent’s informative second

21. One off-equilibrium belief supporting this equilibrium is for the principal to believe \( \Pr(H \mid m_0 \neq m_1) = 0 \).

signal is unused, and the principal fails to learn anything new from the second report. The last part of Proposition 1 shows that a similar inefficiency exists when the agent receives multiple signals of the same quality. Together, these results show that strict improvement in signal quality, not just better information, is necessary for the principal to benefit from the agent’s multiple signals.

3.2. Value of consistent reports

One interesting question is whether the sequencing of reports alone carries any information about the agent’s ability. If so, the principal may use such information to improve her first-stage decision before she can observe the true state. This consideration matters in areas such as wars, major joint ventures overseas, or risky medical procedures because the principal (voters, investors, or patients) may only observe the true state after a long time-lag. In the interim period, though, she may benefit from better information of the agent’s ability. Moreover, a major insight of the herding models is that consistent reports are valued by the market as a sign of talent, which gives rise to a lot of empirical work examining whether consultants and forecasters are biased toward appearing consistent (Ehrbeck and Waldmann, 1996; Chevalier and Ellison, 1999).

Formally, the market is considered to value consistency more if
\[ \Pr(H | m_0 = m_1) > \Pr(H | m_0 \neq m_1) \] and to value mind changes more otherwise. Then:

**Proposition 2.** If there exists a unique partial revelation equilibrium under the sequential reports system, before observing the true state in the first stage,

(2.1) the principal values consistency more than mind changes when type L’s equilibrium lying probability \( \pi^* \leq (2p_0 - 1)(2p_1 - 1) \), which occurs when \( w(\hat{\eta}) \) is affine and strictly increasing.

(2.2) the principal values mind changes more than consistency when type L’s equilibrium lying probability \( \pi^* > (2p_0 - 1)(2p_1 - 1) \), which occurs when \( w(\hat{\eta}) \) is sufficiently convex.

Proposition 2 may appear counterintuitive: if the principal does not value consistency in equilibrium, why should a mediocre agent lie against his second, more informative signal to appear consistent? Instead, a mediocre agent should simply tell the truth when his signals disagree. Recall, however, the wage function is very convex when the principal values highly accurate reports disproportionately more than somewhat accurate ones. What are the incentives of a mediocre agent facing such a wage function? When signals differ, he gets either the best future wage \( w(CR) \) with probability \( 1 - r \) or the worst future wage \( w(CW) \) with probability \( r \) by repeating his first report. If he follows his second signal instead, he receives \( w(R) \) or \( w(W) \) with probability \( r \) or \( 1 - r \). Although consistent reports are more likely to be wrong, the cost is relatively small given his lack of confidence in the second signal compared to type \( H \). The riskier consistent reports lead to a higher expected pay-off than changing his mind. Thus, he is more willing to take on this “gamble”. In contrast, the smart agent faces a different probability distribution over the outcomes: his second signal is so much better than his first that repeating it tends to lead to \( CW \), the worst outcome.

What report sequence is a better signal of expert ability, then, depends on the environment and the projects involved. On the one hand, if highly precise information is necessary due to catastrophic financial and human costs of wrong decisions, experts who change their minds may be more valued because they demonstrate confidence in their later and better information. Examples include initiation of political reforms or wars, forecasts of financial crises, plans for

22. Here, both types of agents are lost when their signals differ (\( \Pr(g | g, b; \theta) = \frac{1}{2} \)). Thus, reporting the true second signal does not increase the probability of giving a correct final report or the agent’s perceived ability.

© 2007 The Review of Economic Studies Limited
speculative currency attacks, and preparation for major natural disasters. On the other hand, when the project is more routine and relies less on highly precise information, consistency may be more valued. Examples may include short-term forecast of consumer demand for existing products, fact-finding procedures in law, and cost-benefit analysis of many regular government policies.

Propositions 1 and 2 together yield some interesting predictions in term of expert bias and forecasting errors. In particular, inconsistent reports may be either positively or negatively correlated with forecasting errors. For example, if there exists a full revelation equilibrium and type $H$ is relatively rare, the market is likely to observe many wrong changes of mind. Then, mind changes become positively correlated with predicting errors even though no expert bias is involved. This prediction is consistent with empirical result such as those identified by Ehrbeck and Waldmann (1996). Using experts’ forecasts on the discount rates of new issues of U.S. Treasury bills over time to test forecasting bias, they show that experts changed forecasts too much, and those who make large changes in forecasts made bigger errors. If the pay-off is sufficiently convex, however, a mediocre agent reports too consistently in equilibrium. Then, mind changes may become negatively correlated with forecasting errors.

4. OPTIMAL REPORTING PROTOCOL

Section 3 shows that requiring sequential reports may induce the agent to lie to appear consistent. A frequently observed alternative is for the principal to require report(s) after the agent has received all the signals. This section compares these two reporting protocols. First, it investigates incentives generated when the principal requires a final report. Then, it characterizes which reporting protocol elicits the most information for the principal.

Recall that in the second stage, the agent receives the full expected value of his information. Therefore, the optimal reporting protocol is one that leads to the highest first-stage profit for the principal. She can require one final report ($m_f$) or a final report sequence ($\vec{m}_f = (m_0, m_1)$) after the agent receives both signals. When one final report is required, the agent’s future wage depends solely on its accuracy. Clearly, the agent should report his best estimate. When a sequence of final reports is required, however, the results are more subtle.

Proposition 3.

(3.1) If the principal requires one report $m_f$, the agent reports $m_f = i_1$ in equilibrium regardless of the first signal.

(3.2) If the principal requires a report sequence $\vec{m}_f$ after the agent receives both signals, and if type $H$’s signals improve faster (inequality (1) holds), both types of agents report $m_0 = m_1 = i_1$. Moreover, $m_0$ is not informative in any equilibrium.

Proposition 3 first shows that the agent reports his final signal truthfully if $m_f$ is required: doing so leads to, in expectation, a higher estimate of his ability than reporting $m_f = i_0$. However, the final report $m_f$ is not a sufficient statistic of the agent’s signals: both signal sequences $(b, g)$ and $(g, g)$ lead to the same report, even though the principal forms different opinions of the true state given these sequences. Despite some seeming similarity, Proposition 3 shows that the natural alternative of requiring a sequence of reports at the end differs markedly from the sequential reports model in Section 3.23 It is key to observe that when sequential reports are required, the agent has little incentive to lie in his initial report, which commits him to that report to some extent. When $\vec{m}_f$ is required, however, inducing a truthful initial report becomes more difficult.

---

23. Requiring $\vec{m}_f$ is equivalent to requiring a probabilistic assessment of the state. See Section 5(A) for more details.
because the agent, knowing both signals, can and will modify his entire report sequence in any way to get the highest wage in the next stage.

In particular, Proposition 3 shows that when the smart agent improves faster, full revelation cannot occur even if a unique full revelation equilibrium exists with sequential reports. In the sequential reports system before, type $H$ reports his second signal truthfully because the benefit of a correct final report outweighs any cost of inconsistency. In the final report(s) system, however, there is no need for type $H$ to send inconsistent reports. By reporting $m_0 = m_1 = i_1$, he is likely to be accurate as well as consistent. And type $H$ has more to gain from doing so because he is more confident in his second signal than type $L$. In equilibrium, the first signal is lost.

Having shown the similarity of requiring $m^f$ and $\tilde{m}^f$, the following proposition compares the final report system with the sequential reports system.

**Proposition 4.** If $\eta < \bar{\eta}$,

1. the principal should require sequential reports if a full revelation equilibrium exists. Formally, this occurs if either inequality (1) holds and $p_0 \leq p_0^L$ or inequality (1) does not hold and $p_0 \leq p_0^H$.

2. the principal should require one final report if both types of agents report $m_0 = m_1 = i_0$ in a sequential reports system. Formally, this occurs if $p_0 \geq p_0^H$ and inequality (1) does not hold.

3. if there exists a partial revelation equilibrium in a sequential reports system, the principal should require a final report when the expert market is extremely poor ($\eta \approx 0$). However, if the expert market is not too poor, or if the principal needs highly precise reports (Pr($s = g | m^A_0$) sufficiently high), the principal should require sequential reports.

First, Proposition 4 shows that if the agent reports both signals truthfully when sequential reports are required, the principal should do so. She learns more from two accurate reports than from one final report. But if the mediocre agent’s signals improve faster, the agent’s second report is completely uninformative with sequential reports. Then, requiring a final report is better because it elicits the true, more accurate second signal. In addition, the principal should choose a final report when the inefficiency of the sequential reports system is sufficiently high. As illustrated in Example 2, if type $H$ is exceedingly rare, the principal may receive two uninformative reports from a mediocre agent who repeats himself to protect his reputation. On balance, she makes a better decision with the final report system.

Second, Proposition 4 shows that sequential reports are optimal if the principal’s decision demands highly precise information. For example, suppose that the principal needs to be convinced that $s = g$ with such a high probability that she would not invest with one final report even if $m^f = g$, then one final report is useless for her. But she may invest instead after hearing two good reports because they are more indicative of a good state. This result suggests that sequential reports may be preferable when significant financial or human costs are at stake.

### 5. DISCUSSION AND EXTENSIONS

This section first discusses several assumptions on message space, communication, and expert market used in the model. Then, it turns to two plausible extensions: i) allowing the agent’s ability to be symmetric information, and ii) introducing an additional informative signal.

24. Relatedly, in a single signal model, Prat (2005) shows that the principal may prefer not to observe an agent’s action (similar to a report here) because then he is likely to conform to the ex ante more likely action of a smart agent. Instead, she should observe the action’s outcome (whether the report correctly predicted the state) because the agent’s reputation increases with the good outcome and he will act efficiently.

25. Formally, this occurs if $[p_1 \eta + r(1 - \eta)]g + [(1 - p_1)\eta + (1 - r)(1 - \eta)]b \leq 0$ and $\text{Pr}(g | g, g) > \text{Pr}(g | g)$. 

© 2007 The Review of Economic Studies Limited
A. Assumption on the Message Space. The agent in this model simply reports whether the state is good or bad. One question is whether more information can be communicated in a richer message space, for example, if the agent reports his belief of the state distribution instead. However, binary signals with known signal qualities mean that there are only six possible estimates of the state distribution and that the agent can always report a distribution that is type \( H \)'s. For instance, a mediocre agent will never report that both states are equally likely after receiving \( i_0 \) because doing so would identify him as mediocre.\(^{26}\) Thus, the reports are no more informative than when the messages were limited to \( g \) or \( b \). If the state distribution is more general, however, interval equilibria with a finite number of messages may arise as shown by Crawford and Sobel (1982) and Ottaviani and Sorensen (2006a).

Even in the present setting, a state distribution that should not arise on the equilibrium path can mean that the agent does not want to reveal his signal. In this sense, this question is similar to voluntary disclosure of information: the agent may avoid disclosure by reporting an impossible state distribution. Allowing this gives rise to an additional type of equilibrium in which the agent babbles in his initial report and the principal ignores it. However, this equilibrium depends strongly on the principal’s beliefs, which may not always be reasonable. In some cases, type \( H \) agent may want to signal his ability by reporting his higher quality first signal, whereas type \( L \) does not because his initial signal is useless.\(^{27}\) A practical implication is that if the principal cannot forbid voluntary, informal reports, requiring a final report may not work even if it is optimal.

B. When the Expert Market is Very Strong. The analysis assumes that \( \eta < \bar{\eta} \), which is sufficient (but not necessary) for the agent’s incentives to be monotonic in \( p_0 \). The value \( \bar{\eta} \in (\frac{1}{3}, 1] \) is derived from the special case of an affine wage function. Because the agent’s incentives depend on the exact shape of his wage function, which in turn depends on the project-specific parameters, a general characterization cannot be given for a very strong expert market. If \( \omega \) is affine, though, the agent reports both signals truthfully if \( \eta \) is sufficiently high. Intuitively, if one is extremely likely to be smart, his reports have little impact on his posterior reputation (Corollary 1, Appendix B).

C. When Ability is Symmetric Information. Whether an agent reports truthfully depends crucially on the relative signal quality improvement between types. In some professions (or stages of one’s career), the agent may not know whether he is smart. Appendix C considers such a model and shows that, when ability is symmetric information, consistent reports always signal high ability in equilibrium (Proposition 5). Intuitively, both the principal and the agent himself believe that a smart agent is more likely to be consistent. Thus, both types of agents face very similar incentives to lie upon receiving conflicting signals. This implies that whether consistency or mind changes is a better signal of talent also depends on how well the agent knows his own ability. In professions where one’s talent is unknown to all parties, consistency is more valued. High-quality information and fast improvement are not enough to ensure that an expert changes his mind and acknowledges a (likely) early mistake: he needs to know himself.

D. Additional Signals. When the agent receives additional signals, the agent’s incentive to report truthfully becomes path dependent. Li (2005) extends this model to a three-signal setting to illustrate a new trade-off of the agent: to appear consistent early to show his confidence in

\(^{26}\) Similarly, after the second signal, he would choose a report that maximizes his expected wage, using the distribution of type \( H \). Reporting \( Pr(s = g \mid i_0 = i_1 = g, H) \) then becomes equivalent to reporting two good signals.

\(^{27}\) One belief supporting this equilibrium is for the principal to believe that anyone who reports a possible state distribution early is mediocre. However, suppose that the principal believes that both types of agents are equally likely to report a possible state distribution, which may be reasonable. Then, for \( p_0 \) sufficiently close to \( \frac{1}{2} \) or \( p_0 \) sufficiently high, type \( H \) can be shown to prefer reporting his true signal \( i_0 \), thus type \( L \) must do so as well. This equilibrium depends crucially on the principal’s out-of-path belief and is not the focus of this paper.
the early signals or to appear consistent late so that his reports are more likely to be correct. The principal may want the third report if improvement in the smart agent’s signal quality levels off. In this case, a mediocre agent may lie against his true third signal if he has lied against his second to appear consistent. This “escalation effect” actually improves type $L$’s incentive to report his second signal truthfully because he knows that if he lies then, he is likely to lie again in the third report and suffer from a big loss in accuracy after three wrong reports. However, if the smart agent’s signal quality improves much more in the final signal than type $L$’s, the principal may not want the third report. In equilibrium, a type $L$ agent may lie more in his early reports because he can still afford to change his mind later. Such counterintuitive effects on the agent’s overall incentives should be considered in designing reporting protocols with many signals.

6. CONCLUSION

When experts are asked to give sequential reports based on private signals of increasing quality, both the sequencing and the accuracy of the reports become a signal of ability. A mediocre agent may repeat his initial report to appear consistent even when the market values mind changes as a prized sign of the fast learners and the talented.

Knowing that the convex implicit incentives may encourage experts to take on too much risk in information revelation opens up many questions. One such question is how these implicit incentives evolve, for example, whether the wage becomes less convex over time as an agent’s ability becomes better known. Another question concerns the optimal reporting protocol if the agent receives many signals over time. This model suggests that requiring very early reports may cause the mediocre agent to commit to a position too early just to appear smart, but requiring late reports alone makes it difficult for the principal to elicit finer information. How the principal optimizes with respect to both the number and the timing of reports is a matter for further research.

APPENDIX A. PROOFS (EXCEPT PROPOSITION 1)

Proof of Lemma 1. (1) First, recall from the text that the principal’s expected profit in stage $B$ is $\Pi^B(m^B, \hat{\eta}) = \sum_{m^B} [Pr(g, m^B)g + Pr(b, m^B)b]u(m^B, \hat{\eta})$. Let $\pi^B(m^B, \hat{\eta}) = Pr(g, m^B)g + Pr(b, m^B)b$ denote her expected profit from investing after $m^B$, then we have

$$
\pi^B(g, g, \hat{\eta}) = \frac{r}{4}g + \frac{1 - r}{4}b + \frac{\hat{\eta}}{2} \left[ (p_0p_1 - \frac{r}{2}) g + \left( (1 - p_0)(1 - p_1) - \frac{1 - r}{2} \right) b \right],
$$

$$
\pi^B(b, g, \hat{\eta}) = \frac{r}{4}g + \frac{1 - r}{4}b + \frac{\hat{\eta}}{2} \left[ (1 - p_0)p_1 - \frac{r}{2} \right] g + \left( p_0(1 - p_1) - \frac{1 - r}{2} \right) b],
$$

$$
\pi^B(g, b, \hat{\eta}) = \frac{1 - r}{4}g + \frac{r}{4}b + \frac{\hat{\eta}}{2} \left[ p_0(1 - p_1) - \frac{1 - r}{2} \right] g + \left( (1 - p_0)p_1 - \frac{r}{2} \right) b],
$$

$$
\pi^B(b, b, \hat{\eta}) = \frac{1 - r}{4}g + \frac{r}{4}b + \frac{\hat{\eta}}{2} \left[ (1 - p_0)(1 - p_1) - \frac{1 - r}{2} \right] g + \left( p_0p_1 - \frac{r}{2} \right) b].
$$

Clearly, each $\pi^B(m^B, \hat{\eta})$ is affine in $\hat{\eta}$, and they can be ranked as $\pi^B(g, g, \hat{\eta}) > \pi^B(b, g, \hat{\eta}) > \pi^B(g, b, \hat{\eta}) > \pi^B(b, b, \hat{\eta})$ for any given $\hat{\eta}$. The principal should choose $\ast = 1$ if $\pi^B(m^B, \hat{\eta}) \geq 0$ and $\ast = 0$ otherwise. Thus, $w(\hat{\eta})$, defined as the sum of $\pi^B(m^B, \hat{\eta})$, is (piecewise) linear in $\hat{\eta}$.

Second, $\pi^B(g, b, \hat{\eta}) < 0$ because $g + b \leq 0$ by assumption. As a result, $\ast = 0$ if the reports are $(g, b)$ or $(b, b)$. What happens if the reports are $(b, g)$ or $(g, g)$? Note that $\pi(g, g, \hat{\eta})$ is strictly increasing in $\hat{\eta}$. Therefore, the slopes of $\Pi^B(m^B, \hat{\eta})_{m^B = a^*}$ can be at most one of three: zero if $b$ is sufficiently negative that the principal never invests, $\pi^B(g, g, \hat{\eta})$ if only $(g, g)$ leads to investment; or the sum of the slopes if both $(b, g)$ and $(g, g)$ lead to investment. In all three cases, the expected profit and thus the wage are nondecreasing in $\hat{\eta}$. The case of $w(\hat{\eta}) = 0$ for all $\hat{\eta}$ is not interesting, thus from now on we assume that the principal invests after some report sequences.

© 2007 The Review of Economic Studies Limited
Next, consider two posterior estimates of the agent’s type \( \eta_1 \) and \( \eta_2 \), and let \( \eta = \gamma \eta_1 + (1-\gamma)\eta_2, \gamma \in (0,1) \). Let \( V(a_1^*(\eta_1)), V(a_2^*(\eta_2)) \) and \( V(a_0^*(\eta)) \) denote the respective wages of the agent in stage \( B \) given these posterior distributions and when the principal takes optimal action. Then,

\[
\gamma V(a_1^*(\eta_1)) + (1-\gamma) V(a_2^*(\eta_2)) \geq \gamma V(a_1^*(\eta_1)) + (1-\gamma) V(a_0^*(\eta_2)) \\
= \gamma \sum_{m^B} \pi^B(m^B, \eta_1) a^*(\eta) + (1-\gamma) \sum_{m^B} \pi^B(m^B, \eta_2) a_0^*(\eta) \\
= \sum_{m^B} \gamma \pi^B(m^B, \eta_1) + (1-\gamma) \pi^B(m^B, \eta_2) a_0^*(\eta) \\
= V(a_0^*(\eta)).
\]

The second equality is true because the action is constrained to be the optimal one given \( \eta \). The third equality is true because each \( \pi^B(m^B, \eta) \) is affine. For example, when \( \hat{w} = \hat{w}(\eta) \) is affine. Because \( \pi(1) \leq \pi(2) \) because the action is constrained to be the optimal one given \( \eta \).

(2) If \( a^*(m^B, \eta) \) depends on the reports only, then the principal takes the same action after \( m^B \) regardless of \( \eta \).

| Page 1189 |

Proof of Proposition 2.

First, in a partial information revelation equilibrium, the principal’s beliefs about the agent’s type before observing the data, but after receiving the reports, are, respectively,

\[
\Pr(H \mid m_0 = m_1) = \frac{p_0 p_1 (1 - p_0) (1 - p_1) \eta}{[p_0 p_1 (1 - p_0) (1 - p_1)] \eta + \frac{1}{\gamma} (1 + \pi^*(1 - \eta))},
\]

\[
\Pr(H \mid m_0 \neq m_1) = \frac{p_0 (1 - p_1) (1 - p_0) p_1 \eta}{[p_0 (1 - p_1) (1 - p_0) p_1] \eta + \frac{1}{\gamma} (1 + \pi^*(1 - \eta))}.
\]

Simple calculation shows that \( \Pr(H \mid m_0 = m_1) > \Pr(H \mid m_0 \neq m_1) \) if \( \pi^* \neq 2(p_0 - 1)(p_1 - 1) \) and \( \Pr(H \mid m_0 = m_1) < \Pr(H \mid m_0 \neq m_1) \) otherwise.

Second, the incentive constraint \( IC_1 \) binds in a partial revelation equilibrium, as shown in the proof of Proposition 1. Let \( f(r, \pi^*) = (w(CR) - w(CW))(1 - r) - (w(R) - w(CW))r \). Since \( \frac{\partial f}{\partial r} < 0 \) and \( \frac{\partial f}{\partial \pi^*} < 0 \), \( a^* \) is non-decreasing. Reporting \( m^* = i_1 \) leads to a higher expected wage.

When \( w(W) = w(CW) = 0 \), \( IC_1 \) simplifies into \( w(CR)(1 - r) < w(R)r \). At \( r = \frac{1}{2} \), \( \pi^* = 2(p_0 - 1) \), which is larger than the cut off value \( (2p_0 - 1)(2p_1 - 1) \), and the principal values mind changes more. 

Proof of Proposition 3.

(3.1) When only one final report is required, the agent chooses \( m^* \) to maximize his expected wage \( \sum w(Pr(H \mid m^*, s))Pr(s \mid i_0, i_1, \theta) \). When \( i_0 = i_1 \), it is clear that the agent reports \( m^* = i_0 = i_1 \). If \( i_0 \neq i_1 \), then \( Pr(i_0 = s) < Pr(i_1 = s) \). Moreover, because \( Pr(H \mid m^* = s) > Pr(H \mid m^* \neq s) \) and \( w(\hat{\eta}) \) is non-decreasing, reporting \( m^* = i_1 \) leads to a higher expected wage.

(3.2) If the principal requires \( m^* = (m_0, m_1) \) after the agent receives both signals, the agent will report \( m_0 = i_0, m_1 = i_1 \) if he receives the highest expected wage from doing so. If his signals agree, it is straightforward to show that both types of agents report truthfully. If his signals differ, for example, \( i_0 = b, i_1 = g \), the following three ICs must hold.
for the agent to report both signals truthfully:
\[
\begin{align*}
\omega(R)\Pr(g \mid b, g, \theta) + \omega(W)\Pr(b \mid b, g, \theta) & \geq \omega(CR)\Pr(g \mid b, g, \theta) + \omega(CW)\Pr(b \mid b, g, \theta), \\
\omega(R)\Pr(g \mid b, g, \theta) + \omega(W)\Pr(b \mid b, g, \theta) & \geq \omega(CR)\Pr(g \mid b, g, \theta) + \omega(R)\Pr(b \mid b, g, \theta), \\
\omega(R)\Pr(g \mid b, g, \theta) + \omega(W)\Pr(b \mid b, g, \theta) & \geq \omega(CW)\Pr(g \mid b, g, \theta) + \omega(CR)\Pr(b \mid b, g, \theta).
\end{align*}
\]
(2)

In particular, the key constraint IC (2) simplifies into the following:
\[
[w(CR) - w(R)](1 - p_0)p_1 \leq [w(W) - w(CW)]p_0(1 - p_1), \quad \text{for type } H;
\]
\[
[w(CR) - w(R)]p_1 \leq [w(W) - w(CW)](1 - r), \quad \text{for type } L.
\]

Recall from the proof of Proposition 1 that if the agent reports truthfully, \( w(CR) - w(W) = w(R) - w(CW) \) at \( p_0 = \frac{1}{2} \), and this gap increases with \( p_0 \) if \( \eta < \eta^* \). Because \( (1 - p_0)p_1 > p_0(1 - p_1) \) and \( r > \frac{1}{2} \), IC (2) fails to hold for any \( p_0 \). Hence, no full revelation equilibrium exists when \( m^f = (m_0, m_1) \) is required.

Next, a comparison of IC (2) for type \( H \) and type \( L \) shows that there are two possible cases if inequality (1) holds. First, if it binds for type \( H \), it holds strictly for type \( L \). That is, if the smart agent reports \( m_0 = i_0 \) with positive probability, type \( L \) reports truthfully. Simple algebra can show, however, that the expected reputational pay-off of consistent reports increases with type \( H \)'s mixing probability, thus type \( H \) will report consistently and type \( L \) needs to imitate. Second, if IC (2) for \( L \) binds, the one for \( H \) fails to hold, which implies that type \( H \) strictly prefers consistent reports if type \( L \) mixes, which is impossible. Thus, no partial revelation equilibrium exists if type \( H \) improves faster.

The only possible equilibrium is a pooling one in which both type \( H \) and type \( L \) report \( m_0 = m_1 = i_1 \). Similar to (3.1), the agent receives higher expected wage by reporting \( m_0 = m_1 = i_1 \) than \( i_0 \). One out-of-path belief supporting this equilibrium is for the principal to believe \( \Pr(m_0 \neq m_1) = \epsilon \) for both types.

**Proof of Proposition 4.**

Lemma 1 shows that the principal will not invest if the final report is \( b \) or if the sequential reports are \((b, b)\) or \((g, b)\). Let \( \Pi_h^1(g), \Pi_h^2(g), \Pi_l^1(g, g), \Pi_l^2(b, g) \), be the expected first-stage profit if \( m^f = g \), the profit if reports are \((g, g)\) in a pooling equilibrium of the sequential reporting game, and the profits if the reports are \((g, g)\) and \((b, g)\) with sequential reports, then

\[ \Pi_h^1(g) = \frac{r}{2}g + \frac{1 - r}{2}b + \frac{(p_1 - r)\eta}{2}(g - b), \quad \Pi_h^2(g) = \frac{1}{4}g + \frac{1}{4}b + \frac{(p_0 - 1)\eta}{4}(g - b), \]

\[ \Pi_l^1(g, g) = \frac{r + (1 - r)\pi^*}{4}g + \frac{1 - r + r \pi^*}{4}b + \frac{\eta}{2}\left[ p_0p_1 - \frac{r + (1 - r)\pi^*}{2}g + \left(1 - p_0\right)(1 - p_1) - \frac{1 - r + r \pi^*}{2}b \right], \]

\[ \Pi_l^2(b, g) = \frac{1 - \pi^*}{4}g + \frac{(1 - \pi^*)(1 - r)}{4}b + \frac{\eta}{2}\left[ (1 - p_0)p_1 - \frac{1 - \pi^*}{2}g + \left(p_0(1 - p_1) - \frac{1 - \pi^*}{2}(1 - r) \right)b \right]. \]

(4.1) In this case, the principal should require sequential reports because there exists a full revelation equilibrium, which provides her with the best information possible.

(4.2) In this case, Proposition 1 shows that both type \( H \) and type \( L \) report \( m_0 = m_1 = i_0 \). When one final report is required, both types report \( m_0 = m_1 = i_1 \). The expected profit is zero if the report in either case is \( b \). When \( m = g \), simple calculation shows the expected profit is higher under the final report system: \( \Pi_h^1(g) - \Pi_l^1(g, g) = \frac{1}{2}[p_1 - p_0]\eta + (r - \frac{1}{2})(1 - \eta)(g - b) > 0 \).

(4.3) When type \( H \) improves faster (inequality (1) holds) and \( p_0 \geq p_0^L \), a unique partial revelation equilibrium exists and there are two possible cases. First, a comparison of the expected profits shows that \( \Pi_h^1(g) > \Pi_h^2(g, g) + \Pi_l^2(b, g) \) for \( \eta \approx 0 \) because the expected profit depends (almost) entirely on the reports provided by the mediocre agent, whose initial signal is useless. Thus, the principal should require one final report to get the true second signal.

Second, consider the case when \( \eta \) is bounded away from 0. Suppose \( b \) is so negative that \( \Pi_h^1(g) = \frac{1}{2}[p_1\eta + r(1 - \eta)]g + \frac{1}{2}[(1 - p_1)\eta + (1 - r)(1 - \eta)b] \leq 0 \), then the final report has no value to the principal because she will not invest even if \( m^f = g \). However, \( \Pi_l^1(g, g) > \Pi_l^1(g) \) when the mixing probability \( \pi^* \) is not too high, in which case type \( L \) lies relatively little and \((g, g)\) is a better signal of state \( g \). Also, \( \Pi_l^1(b, g) > \Pi_l^1(g) \) if the mixing probability \( \pi^* \) is very high and \( p_1 \) is sufficiently high. In this case, type \( L \) repeats his initial report so much that \((b, g)\) is almost surely a sign of type \( H \), whose second signal is very accurate. In either case, requiring sequential reports may lead to higher expected profit.

\( \copyright 2007 \) The Review of Economic Studies Limited
APPENDIX B. ANALYSIS AND PROOF OF PROPOSITION 1

This section first describes the general problem the agent faces after receiving his second signal. Then, it provides some preliminary results before proving Proposition 1.

Assume that the agent reports \( m_0 = i_0 \) in his initial report. After receiving the second signal \( i_1 \), the agent needs to choose \( m_1 \) to maximize
\[
\sum_s w(Pr(H \mid m_0, m_1; s))Pr(s \mid i_0, i_1, \theta),
\]
where \( Pr(s \mid i_0, i_1, \theta) \) is the probability that the true state is \( s \) based on the agent’s information and his type, and \( w(Pr(H \mid m_0, m_1, s)) \) is the agent’s future wage given his reports and the later observed true state. For both types of agents to report \( m_1 = i_1 \) truthfully, four ICs given the history and the agent’s type must hold:

\[
\begin{align*}
(\text{IC}_1^L) & \quad (w(CR) - w(W))Pr(b \mid b, g, L) \leq (w(R) - w(CW))Pr(g \mid b, g, L), \\
(\text{IC}_1^H) & \quad (w(CR) - w(W))Pr(g \mid g, g, L) \geq (w(R) - w(CW))Pr(b \mid g, g, L), \\
(\text{IC}_2^L) & \quad (w(CR) - w(W))Pr(b \mid b, g, H) \leq (w(R) - w(CW))Pr(g \mid b, g, H), \\
(\text{IC}_2^H) & \quad (w(CR) - w(W))Pr(g \mid g, g, H) \geq (w(R) - w(CW))Pr(b \mid g, g, H).
\end{align*}
\]

First, compare probabilities \( Pr(s \mid i_0, i_1, \theta) \) in the above ICs:
\[
\begin{align*}
Pr(g \mid g, g, L) &= \frac{r}{1 - r} > 1 > \frac{Pr(b \mid b, g, L)}{Pr(g \mid b, g, L)} = \frac{1 - r}{r}, \\
Pr(b \mid g, g, H) = \frac{p_0p_1}{(1 - p_0)(1 - p_1)} > 1 > \frac{Pr(b \mid b, g, H)}{Pr(g \mid b, g, H)} = \frac{p_0(1 - p_1)}{p_1(1 - p_0)}.
\end{align*}
\]

Assuming the wage differences on both sides of the IC are non-negative (shown later to be true in equilibrium), then observe that (1) if either \( \text{IC}_2^H \) or \( \text{IC}_1^L \) binds, \( \text{IC}_2^H \) and \( \text{IC}_1^L \) hold strictly. That is, if either type is (weakly) willing to report \( m_1 = i_1 \) after receiving opposite signals, the agent strictly prefers doing so after consistent signals. (2) If inequality (1) holds, then \( Pr(b \mid g, g, H) < Pr(b \mid b, g, L) \) by definition. This means that if \( \text{IC}_1^L \) holds or binds, \( \text{IC}_2^H \) holds strictly. Or when a smart agent improves faster, type \( H \) strictly prefers reporting \( m_1 = i_1 \) if type \( L \) weakly prefers it. (3) If inequality (1) does not hold, and if \( \text{IC}_1^H \) holds or binds, \( \text{IC}_1^L \) holds strictly.

Second, consider the agent’s wage \( w(Pr(H \mid m_0, m_1, s)) \) in the above ICs. As described in the text, suppose that type \( L \) repeats his initial report with probability \( \pi \), the principal’s posterior beliefs of the agent’s ability are
\[
\begin{align*}
\text{(CR)} \quad Pr(H \mid g, g, g) &= \frac{p_0p_1\eta}{p_0p_1\eta + \frac{1}{2}[r + (1 - r)\pi][1 - \eta]}, \\
\text{(CW)} \quad Pr(H \mid b, b, g) &= \frac{(1 - p_0)(1 - p_1)\eta}{(1 - p_0)(1 - p_1)\eta + \frac{1}{2}[1 - r + r\pi][1 - \eta]}, \\
\text{(W)} \quad Pr(H \mid g, b, g) &= \frac{p_0(1 - p_1)\eta}{p_0(1 - p_1)\eta + \frac{1}{2}(1 - r)(1 - \pi)(1 - \eta)}, \\
\text{(R)} \quad Pr(H \mid b, b, g) &= \frac{(1 - p_0)p_1\eta}{(1 - p_0)p_1\eta + \frac{1}{2}(1 - \pi)r(1 - \eta)}.
\end{align*}
\]

Consider the case when the agent reports \( i_1 \) truthfully (\( \pi = 0 \)). Lemma 1 shows that \( \omega' \geq 0, \) thus
\[
\frac{\partial}{\partial p_0} (w(CR) - w(W)) \geq \omega'(W) \frac{\partial}{\partial p_0} (CR - W) \quad \text{and} \quad \frac{\partial}{\partial p_0} (w(R) - w(CW)) \leq \omega'(R) \frac{\partial}{\partial p_0} (R - CW). \tag{3}
\]

Differentiating with respect to \( p_0 \), we have
\[
\begin{align*}
\text{sign} \left( \frac{\partial (CR - W)}{\partial p_0} \right) = \text{sign} \left( - \frac{p_1(1 - p_1)\eta^2 - \frac{r(1 - r)(1 - \eta)^2}{4p_0^2}}{4p_0} \right), \\
\text{sign} \left( \frac{\partial (R - CW)}{\partial p_0} \right) = \text{sign} \left( \frac{p_1(1 - p_1)\eta^2 - \frac{r(1 - r)(1 - \eta)^2}{4(1 - p_0)^2}}{4(1 - p_0)^2} \right). \tag{4}
\end{align*}
\]

(c) 2007 The Review of Economic Studies Limited
Simple calculations show that \( p_1 (1 - p_1) \eta^2 - \frac{(1 - r)(1 - \eta)^2}{4p_0^2} < 0 \) for all \( p_0, p_1, r \) if \( \eta \leq \frac{1}{2} \). Let \( \eta \) solve \( p_1 (1 - p_1) \eta^2 - \frac{(1 - r)(1 - \eta)^2}{4p_0^2} = 0 \), the solution \( \eta \in \left[ \frac{1}{3}, 1 \right] \) (see also the remark on \( \eta \) after the proof). When \( \eta \leq \bar{\eta} \), the L.H.S. of IC\(^1\) increases in \( p_0 \) and the R.H.S. decreases with it. We now prove the main proposition.

**Proof of Proposition 1.**

(1.1) When the smart agent also improves faster, inequality (1) holds, which is equivalent to \( p_0 \leq p_{\text{ratio}} \equiv \frac{\gamma + \frac{p_1 - 1}{2p_1} - 2\gamma}{\gamma} \). To start with, at \( p_0 = \frac{1}{2} \), it is easy to see that \( w(\text{CR}) - w(W) = w(R) - w(\text{CW}) \) and all the four ICs hold strictly. At \( p_0 = 1 \), the agent always needs to be consistent (\( w(R) = w(W) = w(\text{CW}) = 0 \) and IC\(^L\) is clearly violated if the agent reports truthfully. Since the L.H.S. of IC\(^L\) increases in \( p_0 \) and the R.H.S. decreases in \( p_0 \), there exists a probability \( p_0^L \) such that IC\(^L\) binds when the agent reports truthfully (\( \pi = 0 \)). Similarly, let \( p_0^H \) be the probability that IC\(^H\) binds when the agent reports truthfully. Observe, however, that the three values \( p_0^L, p_0^H, p_{\text{ratio}} \) cannot be ranked in general, because \( p_{\text{ratio}} \equiv \frac{1}{3} \) is a function of \( p_1, r \), but \( p_0^L, p_0^H \) are functions of \( p_1, r, \eta, g, b \). But we can show that there are only two possible cases, as illustrated in Figure 2. First, if \( p_{\text{ratio}} > p_0^L \), then \( p_0^L < p_0^H < p_{\text{ratio}} \). Second, if \( p_{\text{ratio}} \leq p_0^L \), then \( p_{\text{ratio}} \leq p_0^H \leq p_0^L \).

To understand the first case, note that IC\(^L\) binds at \( p_0^L \) by definition. If \( p_0^L < p_{\text{ratio}} \), IC\(^H\) still holds at \( p_0^L \), thus \( p_0^H > p_{\text{ratio}} \). Moreover, it cannot be that \( p_0^H > p_{\text{ratio}} \). Were it the case, IC\(^H\) must bind first at \( p_0^H \) while IC\(^L\) holds strictly, which implies that \( p_0^0 > p_0^H > p_{\text{ratio}} \), a contradiction. Thus, we have \( p_0^L < p_0^H < p_{\text{ratio}} \). The second case can be shown similarly.

For all \( p_0 \leq p_0^L \), from the discussion above, both IC\(^H\) and IC\(^L\) hold. Moreover, since at \( p_0 = \frac{1}{2} \), \( w(\text{CR}) - w(W) = w(R) - w(\text{CW}) \) and the L.H.S. increases while the R.H.S. decreases with \( p_0 \), IC\(^H\) and IC\(^L\) hold strictly. A unique full revelation equilibrium exists.

If \( p_0^L \leq p_0 < p_{\text{ratio}} \), IC\(^L\) is violated and there is no full revelation equilibrium. Consider the mixed strategy that when \( i_1 \neq m_0 \), type \( L \) repeats \( m_0 \) with probability \( \pi \). Note that \( w(\text{CR}) - w(W) \) decreases with \( \pi \) while \( w(R) - w(\text{CW}) \) increases with \( \pi \) because the more likely a type \( L \) agent repeats his initial report, the less likely that the principal thinks that the agent is smart after hearing consistent reports. Thus, the L.H.S. of IC\(^L\) decreases with \( \pi \) while the R.H.S. increases with \( \pi \). At \( \pi = 1 \), that is, when \( L \) always repeats \( m_0 \), L.H.S. \( < \) R.H.S. \( = \) w(i). Because in this case, L.H.S. \( \geq \) R.H.S. at \( \pi = 0 \), there exists a mixing probability \( \pi^* \in (0, 1) \) such that L.H.S. \( = \) R.H.S. at \( \pi^* \). Because IC\(^L\) binds, all other ICs hold strictly and we have a partial revelation equilibrium. In order to see that \( \pi^* \) increases in \( p_0 \), let \( f(\pi, \pi^*) = (w(\text{CR}) - w(W))(1 - r) - (w(R) - w(\text{CW}))r = 0 \). Since \( \frac{\partial f}{\partial \pi} > 0, \frac{\partial f}{\partial \pi^*} < 0, \frac{\partial^2 f}{\partial \pi^2} > 0 \) by the implicit function theorem. Case 1 of Figure 2 illustrates how the continuation equilibria change as \( p_0 \) changes when \( H \) improves faster. If \( p_0 > p_{\text{ratio}} \), the equilibrium behaviour falls into the case when type \( L \) improves faster, which we turn to presently.

(1.2) When type \( L \) improves faster (inequality (1) does not hold), then \( p_0 > p_{\text{ratio}} \). Similar to (1.1), for \( p_0 \) sufficiently close to \( \frac{1}{2} \), there still exists a full revelation continuation equilibrium. However, in this case IC\(^L\) binds first as \( p_0 \) increases. Thus for all \( p_0 \leq p_0^L \), IC\(^L\) holds and the equilibrium is full revelation. For \( p_0 > p_0^L \), IC\(^L\) is violated even though IC\(^L\) still holds strictly. Suppose that type \( H \) repeats his first report with probability \( y \) when \( i_1 \neq m_0 \). Simple
calculation shows that the L.H.S. increases with $\eta$, thus type $H$ will repeat his first report with probability one: the more type $H$ repeats his first report, the more consistent reports become a sign of type $H$. As a result, type $L$ has to repeat his initial report as well. This continuation equilibrium can be supported by the principal’s belief that $\Pr(H \mid m_0 \neq m_1) = 0$. The continuation equilibrium behaviour when type $L$ learns faster is illustrated as Case 2 in Figure 2.

(1.3) When neither type’s signal improves ($p_0 = p_1 = p, r = \frac{1}{2}$), the agent’s belief after receiving conflicting signals becomes $\Pr(g \mid b, g, b) = \frac{1}{2}$. Thus, the key truth-telling constraints $IC_1^L$ and $IC_1^R$ become the same: $w(CR) - w(W) \leq w(R) - w(CW)$. However, this is violated for all $p > \frac{1}{2}$. Suppose that in equilibrium, type $H$ and type $L$ repeat the first report with probability $y$ and $\pi$, respectively. Then clearly, truth telling ($y = \pi = 0$) cannot be part of the equilibrium. Moreover, for any $\pi$, the L.H.S. of the IC increases in $y$, while the R.H.S. decreases in $y$ as in (1.2). Thus, type $H$ will repeat his first report with probability one and type $L$ must do so as well. The equilibrium is one of full pooling in which $m_0 = m_1 = i_0$.

Finally, following the continuation equilibrium above, we need to check whether the agent wants to report $m_0 = i_0$. Type $L$’s first signal is completely uninformative and thus he is indifferent. Type $H$ prefers to report $m_0 = i_0$ if the following IC is true:

$$w(CR)p_0p_1 + w(R)(1 - p_0)p_1 + w(W)p_0(1 - p_1) + w(CW)(1 - p_0)(1 - p_1)$$

$$\geq w(CR)(1 - p_0)p_1 + w(R)p_0p_1 + w(W)(1 - p_0)(1 - p_1) + w(CW)p_0(1 - p_1)$$

$$\Rightarrow (w(CR) - w(R))p_1 \geq (w(CW) - w(W))(1 - p_1).$$

(5)

The L.H.S. of the above is the expected reputational pay-off if type $H$ agent reports truthfully, while the R.H.S. is the expected pay-off if he reports $m_0 \neq i_0$. IC (5) holds if $CR \geq R \Rightarrow W \geq CW$ and IC (5) holds. In a full revelation continuation equilibrium, the discussion in part (1) shows that $CR \geq R \Rightarrow W \geq CW$ and IC (5) holds. In a partial revelation continuation equilibrium, recall that $W \geq CW$ at $\pi = 0$, and $W$ increases in $\pi$ while CW decreases in $\pi$. Thus in a partial information revelation equilibrium, $W \geq CW$. Also, in equilibrium, $CR \geq R$, otherwise type $L$ should deviate by lying less and receiving higher pay-off. Therefore, IC (5) holds in a partial revelation equilibrium. Lastly, in a full pooling equilibrium, IC (5) simplifies to $w(\Pr(i_0 = s)) \geq w(\Pr(i_0 \neq s))$, which is always true. Thus, both types of agent report $m_0 = i_0$ truthfully.

Remark: strong expert market. The model assumes $\eta < \bar{\eta}$. Because the wage function depends on parameter values $g, b$, the cut off $\bar{\eta}$ is a lower bound derived using the limit case of an affine wage function. Inequality (3) and (4) show that Proposition 1 holds for even higher $\bar{\eta}$ in general, thus it is less restrictive. When $\eta$ is very high, however, we can see from (4) that the signs of the L.H.S. and the R.H.S. depend on the specific $w$ and a general characterization cannot be given. But we can show the following:

Corollary 1. When $w(\bar{\eta})$ is affine and strictly increasing in $\bar{\eta}$, a full revelation equilibrium exists for $p_0$ sufficiently close to $\frac{1}{2}$. A pooling equilibrium such that $m_0 = i_0 = m_1$ exists when $p_0$ is sufficiently close to 1, and the second report is uninformative in any equilibrium. For any given $p_0$, a full revelation equilibrium exists if $\eta$ is sufficiently close to 1.

Proof. Similar to that of Proposition 1. ||

APPENDIX C: WHEN ABILITY IS SYMMETRIC INFORMATION

All assumptions remain the same as in the text, except that now both the principal and the agent only know his type distribution. Assume that the agent reports $i_0$ truthfully. Suppose that $m_0 = i_0 = g$, then the following two truth-telling ICs need to hold, depending on whether $i_0 = i_1$ (instead of four in the asymmetric information model):

$$[w(CR) - w(W)]\Pr(g \mid g, g) \geq [w(R) - w(CW)]\Pr(b \mid g, g)$$

($IC_1$),

$$[w(CR) - w(W)]\Pr(g \mid g, b) \leq [w(R) - w(CW)]\Pr(b \mid g, b)$$

($IC_2$).

Analysing these two ICs, we can show that the following is true if the ability is symmetric information.

Proposition 5. If $\eta \leq \bar{\eta} \in \{\frac{1}{2}, 1\}$, there exists a cut-off $\tilde{p}_0$ such that

(5.1) If $p_0 \leq \tilde{p}_0$, there exists a unique full revelation equilibrium in which the agent always reports $m_0 = i_0$ and $m_1 = i_1$. Moreover, $\tilde{p}_0 > p_0^H$ if type $H$ improves faster; and $\tilde{p}_0 > p_0^L$ if type $L$ does.

28. This can be generalized to type $L$ receiving informative signals as long as $r < p$. © 2007 The Review of Economic Studies Limited
If $p_0 > \hat{p}_0$, then there exists a full pooling equilibrium in which the agent always repeats his first report, and the principal believes that $\Pr(H \mid m_0 \neq m_1) = \epsilon$.

The market always values consistent reports more than mind changes in equilibrium: $\Pr(H \mid m_0 = m_1) > \Pr(H \mid m_0 \neq m_1)$.

Proof. See Li (2005).

Acknowledgements. I thank Glenn Ellison and Botond Köszegi for their advice and encouragement. I am also grateful for the insightful comments from George Akerlof, Abhijit Banerjee, Mathias Dewatripont, Edward Schlee, Jean Tirole, the editor, and two anonymous referees. This paper has also benefited from the input of seminar participants at MIT, UCR, UCSD, UCLA, WZB, USC, ASU, UCI, and the 2004 Parallel Meetings of EEA-ESEM. Adapted from Chapter 3 of my MIT dissertation, several earlier drafts of this paper were circulated under the title “Mind Changes and the Design of Reporting Protocols”.

REFERENCES


© 2007 The Review of Economic Studies Limited