Do Irrational Investors Destabilize?

Li, Hao[1]

University of Hong Kong

April, 1998

Author's address: School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong. Email haoli@econ.hku.hk. Phone 852-2859-1048. Fax 852-2548-1152.

Abstract: In a financial market where all investors have valuable private information, full rationality requires that investors have unlimited ability of figuring out the equilibrium model. Instead, I assume that due to lack of knowledge or experience, some investors do not know the equilibrium model and use only their private information in forming their demand. By investigating the investment behavior of these "boundedly rational" investors contrasting it with that of the rational ones, I find that in a market where the two kinds of investors coexist, it is the boundedly rational investors who contribute to price stability. The welfare implication is that, although each investor benefits from conditioning his asset demand on the information carried by the equilibrium price, it can happen that all investors lose by doing so because the equilibrium price becomes too volatile.

JEL Classification Number: D84.

1. Introduction

Since Milton Friedman (1953) made the famous claim that rational speculation should stabilize price while irrational speculation should destabilize, there have been a number of works devoted to offering counterexamples, including two recent contributions by Hart and Kreps (1986), and Stein (1987). But these works address only half of the claim made by Friedman: they show that under certain circumstances rational speculations can destabilize the price. What is lacking in the literature is a comparison between rational and irrational behavior, and an examination of their different effects on price stability.

A comparison between rational and irrational investors is possible only if we have a convincing model of irrational behavior. It is a tautology that one can obtain any "result" by imposing on economic agents an arbitrary pattern of behavior and justifying it by "bounded rationality." Indeed, one apparently unappealing aspect of Friedman's claim on the price-destabilizing effect of irrational investors rests on the stereotypical notion that irrational investors form their expectations of price development in a very irrational way: they somehow believe that if price is high today it is going to go up even further tomorrow. This way of modeling bounded rationality is at best a simplistic view of the behavior of investors whose abilities of forming the right expectations are limited by their knowledge and experiences. In a sense, to model some behavior that is not fully rational, we must "rationalize" bounded rationality, or give more structure to the limit imposed on investors' rationality.

Rationality of investment behavior in a financial market naturally concerns how investors use their available information to form expectations of asset return. This important issue is not addressed by existing models of boundedly rational behavior in the finance literature, such as the "noise traders" in Kyle (1985) who submit random asset demand, and the irrational traders in De Long et al. (1991) who systematically underestimate variance of asset return. Forming expectations of asset return is often a non-trivial task, especially when information about asset return in a financial market is private and dispersed among investors. The standard equilibrium concept in this context is rational expectations equilibrium (see, e.g. Grossman (1976)), where the equilibrium price is a stochastic function of investors' private information and the random asset return, and is therefore informative. A rational investor observes the price and uses both his private information and the information carried by price to determine his demand for the asset. Full rationality thus requires the knowledge of the equilibrium stochastic price function. But to figure out this equilibrium price function, an investor has to solve a fixed-point problem, for rational expectations equilibrium requires each investor's asset demand to be consistent with the equilibrium price function and the equilibrium price in turn to be consistent with individual asset demand through market clearing. Moreover, the investor has to have extensive knowledge about parameters of the financial market such as wealth distribution among investors and their risk preferences, since these parameters in general affect the equilibrium price function.

The level of rationality required in rational expectations equilibrium can be too high to be realistic. I propose a boundedly rational behavior by assuming that boundedly rational investors do not know the equilibrium price function and are thus unable to infer any information from prices. These boundedly rational investors can be interpreted as "outsiders" in financial markets, or "inexperienced" investors. This interpretation captures the notion that what distinguishes an insider from an outsider is often not the former's better access to valuable information, but his ability to decipher valuable information from publicly observed information. An example of insiders is an experienced broker, who has both the trained expertise to solve the fixed-point problem (just as economists do) and the accumulated knowledge about the market (he knows the right values of all the relevant parameters).

With the definition of boundedly rational investors as those who have access to valuable private information but who are incapable of inferring information from equilibrium prices, I consider a financial market populated by both rational and boundedly rational investors, all with valuable private information regarding the return of a risky asset. The equilibrium of the model is similar to the rational expectations equilibrium in that the equilibrium asset price aggregates the diverse information among investors. However, only rational investors are able to exploit the information embodied in the equilibrium price, while boundedly rational investors form their expectations of the asset return based on their private information only. Regarding price stability, defined as dispersion of the realized price around its mean, it is found that equilibrium price becomes more stable as relative size of boundedly rational investors increases. This result can be interpreted as a complete rejection of Friedman's claim: it is the boundedly rational investors that make price stable and rational investors that contribute to its instability.

The reasoning behind the positive relation between the relative size of the boundedly rational investors and price stability can be understood as follows. The equilibrium price of the asset enters a boundedly rational investor's asset demand only through the budget constraint, while it enters a rational investor's demand also through its informational content. A boundedly rational investor's demand is therefore more negatively responsive to changes in the price than a rational investor's demand. For example, when the asset price is high a boundedly rational investor reduces his demand for the asset, while a rational investor who knows the equilibrium price function infers that the asset return is likely to be high and so he tends to increase his demand relative to the boundedly rational investor. It follows that the investment behavior of the boundedly rational investor contributes to price stability and that of the rational investors does the opposite.

The above result has direct welfare implications because investors, rational or boundedly rational, are averse to the variations in the equilibrium price. This is especially true when investors are endowed with relatively large quantities of the asset. Clearly, regardless of the relative size of the boundedly rational investors in the market, it pays for each investor to be rational, i.e., to exploit the information conveyed by the price. However, when all investors try to do so, the asset price may become so volatile that all investors become worse off compared with a situation where some investors are boundedly rational.

Financial markets in the emerging economies are known to be much more volatile than in the developed countries.[2] Some commentators in the emerging economies have attributed the price volatility partly to the poor experience of investors with modern financial markets. For example, last December the leading government newspaper in China carried an article titled "Understanding Correctly the Current Stock Market," (*People's Daily*, December 19, 1996), which identifies investors' lack of experience as one of main sources of a 340% increase in stock price index from April 1 to December 9 last year. The article points out that high volatility follows inevitably from unsustainable stock price surges, and goes on to suggest "educating" investors on risks and other aspects of stock market as a way to reduce stock price volatility. The view expressed by the article contradicts the result in this paper about the relation between inexperienced investors and price volatility. High price volatility does not come from inexperienced "speculation." Indeed, the result in this paper also casts doubt on the view expressed in December last year and again February this year by Allan Greenspan, the Chairman of the Federal Reserve of the United States, that "it's not markets that are irrational; it's people who become irrationally exuberant on occasion" (*The New York Times*, February 29, 1997).

The basic model and the usual rational expectations equilibrium are introduced in Section 2. Section 3 defines rational expectations equilibrium with boundedly rational investors, establishes the result that price stability is positively related to the relative size of the boundedly rational investors, and considers the welfare implications of bounded rationality. The last section concludes with some remarks on extending the model to other issues of interest.

2. The Model

2.1. Two-period Economy

The basic model is borrowed from Verrecchia (1982). A description of the model is provided to make this paper self-contained.

Consider a two-period economy with a single consumption good and two assets. One is a risk-free bond with a normalized return of 1. The other is a risky asset with a return X which is distributed normally:[3]

$$X \sim N(\mathrm{E}[X], \sigma_x^2).$$

Let $h_x = 1/\sigma_x^2$ be the precision of X.

There is a continuum of investors in the economy, indexed by $i \in [0,1]$. Each investor i is endowed with b_{0i} units of the risk-free bond and z_{0i} units of the risky asset. His preference is given by a constant risk-aversion utility function in wealth w_i:

$$u(w_i) = -\exp(-w_i/r),$$

where r is the coefficient of absolute risk tolerance. Note that for simplicity, I have assumed that risk tolerance is the same for all investors.

Each investor i has access to a private signal Y_i, which is assumed to be the sum of the random return of the risky asset X and a white noise S_i with variance σ_s^2:

$$Y_i = X + S_i.$$

Thus Y_i is normally distributed with mean $\mathrm{E}[X]$ and variance $\sigma_x^2 + \sigma_s^2$. Let $h_s = 1/\sigma_s^2$ be the precision of S_i. That the variance of S_i is the same for all investors amounts to assuming that the private signals have the same "quality."[4] Furthermore, it is assumed that S_i is independently distributed across i.

Timing of events is as follows. At the beginning of the first period, each investor i learns his endowments b_{0i} and z_{0i}. Let b and z be the per-capita endowment of the risk-free bond and the risky asset respectively. Then a realization of X occurs, unobservable to all investors. Each i is informed of y_i, a realization of his private signal Y_i. Two markets are then opened, one for the risk-free bond and one for the risky asset. Walrasian auctions for the bond and for the asset take place in the two markets subsequently. It is assumed that the price of the risky asset is denominated in the price of the risk-free bond, so that the market-clearing price of the bond can be assumed to be 1. Trading takes place and the markets are closed when the market-clearing price of the risky asset is reached through auctions in the two markets. In the second period, a realization of X is revealed to all investors and each investor consumes his wealth.

2.2. Rational Expectations Equilibrium

The rational expectations equilibrium (see e.g. Grossman (1976), (1977)) in this model is a stochastic equation that relates the price of the risky asset to its unobservable return.

Finding the equilibrium entails solving a fixed-point problem. On one hand, the stochastic price equation is a consequence of market-clearing in the market of the risky asset where individual demands reveal their private information about the return. On the other hand, the stochastic price equation implies that the price reveals useful information about the unobservable return of the risky asset, and therefore the investors in the economy should exploit the information in forming their demands. Stated slightly more succinctly, rational expectations equilibrium is a model of the relation between the price and the return of the risky asset such that if all investors in the economy adopt it in forming their demands for the asset, it will coincide with the actual relation between the price and return.

As is standard now in the literature of rational expectations equilibrium under asymmetric information, a second source of uncertainty besides the return to the risky asset needs to be introduced into the model. Otherwise, with the return as the single source of uncertainty, rational expectations equilibrium would imply that the price perfectly aggregates the private information of the investors so that they do not use their private signals in forming their demands. But then it would be unclear why the price ends up providing all the information about the return in the first place.[5] This second source of uncertainty is provided by the assumption that per-capita endowment of the risky asset, Z, is a random variable unobservable by investors in the economy. Specifically, it is assumed that Z is normally distributed:

$$Z \sim N(0, \sigma_z^2),$$

and Z is independent from X and each S_i. Let $h_z = 1/\sigma_z^2$ be the precision of Z.

The introduction of the new random variable Z has the effect of preventing the price from fully revealing the return of the risky asset. Now rational expectations equilibrium can be defined as stochastic equation that relates the price of the risky asset to its unobservable return and supply.

DEFINITION 2.1. A rational expectations equilibrium is a stochastic price function $P(X, Z)$ such that $\forall x$, z, and y_i,

$$\int_0^1 z_i(b_{0i}, z_{0i}, y_i, P(x, z))di = z,$$

where $z_i(b_{0i}, z_{0i}, w_{0i}, y_i, P(x, z))$ is a solution to the constrained maximization problem

$$\max_{z_i} \mathrm{E}[-\exp(-(b_i + z_i X)/r) | y_i, P(x, z)]$$

such that $b_i + z_i p \leq b_{0i} + z_{0i} p$.

Due to the simplifying assumptions of constant risk-aversion utility function and normal distributions in this model, the rational expectations equilibrium can be calculated explicitly. I will not do so, however, because it can be viewed as a special case of the equilibrium described in the next section.

3. Rationality and Volatility

3.1. Rational expectations equilibrium with boundedly rational investors

The departure from the usual rational expectations equilibrium comes from the assumption that some investors in the economy are not fully "rational," or "boundedly rational." These investors are assumed to form conditional expectations of the return X on the basis of their private signals only. Thus, they ignore the information about the return conveyed by the equilibrium price. One explanation is that these investors are "inexperienced" investors who do not have the knowledge of the equilibrium price function, either due to their inability to solve a complex fixed-point problem, or more plausibly, due to their limited knowledge about the parameters of the economy. The second factor is most reasonable in a heterogeneous environment where complete knowledge about the parameters that permits one to find the equilibrium price function is improbable. If one models limited knowledge about the equilibrium price function formally as uncertainty about the parameters, then what is called boundedly rational investors are in fact fully rational but face incomplete information about the parameters of the model. I will not take this approach here, both because I want to keep the model simple enough to permit explicit calculations, and because such generalization does not add to the insights of the results.

Let $B \subset [0, 1]$ and $R = [0, 1] \setminus B$ be the set of boundedly rational and rational investors in the economy, respectively. Let λ^B be the measure of the set B, or the relative size of the boundedly rational investors in the economy. A rational expectations equilibrium with boundedly rational investors can be defined along the same lines of rational expectations equilibrium given in section 2.2.

DEFINITION 3.1. A rational expectations equilibrium with boundedly rational investors is a stochastic price function $P(X, Z)$ such that $\forall x$, z, and y_i,

$$\int_{i \in R} z_i^R(b_{0i}, z_{0i}, y_i, P(x,z))di + \int_{i \in B} z_i^B(b_{0i}, z_{0i}, y_i, P(x,z))di = z, \tag{3.1}$$

where $z_i^R(b_{0i}, z_{0i}, y_i, P(x,z))$ is as defined in Definition 2.1 and $z_i^B(b_{0i}, z_{0i}, y_i, P(x,z))$ is a solution to the constrained maximization problem

$$\max_{z_i} \mathrm{E}[-\exp(-(b_i + z_i X)/r)|y_i]$$

such that $b_i + z_i p \leq b_{0i} + z_{0i} p$.

Calculating a rational expectations equilibrium with boundedly rational investors is made easy in this model by the assumptions of constant risk-aversion and normal distributions. The crucial consequence of these simplifying assumptions is that demand for the risky asset, by a rational investor as well as by a boundedly rational investor, is independent of his endowed wealth. Furthermore, the demand is linear in his private signal and the price. The market-clearing condition then implies that the equilibrium price is linear in the return and the supply, since the white noises in the signals cancel with each other when aggregated in the price.[6]

PROPOSITION 3.2. *A rational expectations equilibrium with boundedly rational investors is given by the stochastic price equation*

$$P = \pi_0 + \pi_x X - \pi_z Z,$$

where

$$\pi_x = \frac{h_s + h_z h_s^2 r^2 (1 - \lambda^B)}{h_x + h_s + h_z h_s^2 r^2 (1 - \lambda^B)}, \tag{3.2}$$

$$\pi_z = \pi_x / (h_s r), \tag{3.3}$$

and

$$\pi_0 = (1 - \pi_x) E[X]. \tag{3.4}$$

PROOF. The plan of the proof is to postulate some linear form of the equilibrium price equation, find demand functions $z_i^R(b_{0i}, z_{0i}, y_i, P(x,z))$ and $z_i^B(b_{0i}, z_{0i}, y_i, P(x,z))$, and then use the market-clearing condition to verify that the equilibrium price equation takes a linear form and solve for the coefficients in the price equation.

Suppose that the equilibrium price P is linear in return X and supply Z:

$$P = \pi_0 + \pi_x X - \pi_z Z. \tag{3.5}$$

Since both X and Z are normally distributed, P is also normally distributed. The covariance matrix of the joint distribution (X, Y_i, P) is given by:

$$\begin{bmatrix} \sigma_x^2 & \sigma_x^2 & \sigma_{px} \\ \sigma_x^2 & \sigma_x^2 + \sigma_s^2 & \sigma_{px} \\ \sigma_{px} & \sigma_{px} & \sigma_p^2 \end{bmatrix}. \tag{3.6}$$

Note that $\sigma_{py_i} = \sigma_{px}$. Also, σ_p^2 and σ_{px} can be expressed as functions of the unknown coefficients π_x and π_z:

$$\sigma_p^2 = \pi_x^2 \sigma_x^2 + \pi_z^2 \sigma_z^2,$$

$$\sigma_{px} = \pi_x \sigma_x^2. \tag{3.7}$$

Given a realization y_i of Y_i and a realization p of $P(X,Z)$, $z_i^R(b_{0i}, z_{0i}, y_i, P(x,z))$ can be calculated as follows. By properties of normal distribution,

$$\begin{aligned} \mathrm{E}[-\exp(-(b_i + z_i X)/r)|y_i, p] &= -\exp\left(-\frac{\mathrm{E}[b_i + z_i X|y_i, p]}{r} + \frac{\mathrm{Var}[b_i + z_i X|yi, p]}{2r^2}\right) \\ &= -\exp\left(-\frac{b_i + z_i \mathrm{E}[X|y_i, p]}{r} + \frac{z_i^2 \mathrm{Var}[X|yi, p]}{2r^2}\right). \end{aligned}$$

Straightforward optimization leads to

$$z_i^R(b_{0i}, z_{0i}, y_i, p) = \frac{r(\mathrm{E}[X|y_i, p] - p)}{\mathrm{Var}[X|y_i, p]}. \tag{3.8}$$

Note that $z_i^R(b_{0i}, z_{0i}, y_i, P(x,z))$ is independent of b_{0i} and z_{0i}. The expressions $\mathrm{E}[X|y_i, p]$ and $\mathrm{Var}[X|y_i, p]$ can be obtained from (3.6):

$$\mathrm{Var}^{-1}[X|y_i, p] = h_s + h_x + \frac{h_x \sigma_{px}^2}{\sigma_x^2 \sigma_p^2 - \sigma_{px}^2},$$

$$\mathrm{E}[X|y_i,p] = \gamma_0^R + \gamma_y^R y_i + \gamma_p^R p,$$

where

$$\gamma_0^R = \mathrm{E}[X] - \gamma_y^R \mathrm{E}[X] - \gamma_p^R \mathrm{E}[P],$$

$$\gamma_y^R = h_s \mathrm{Var}[X|y_i,p],$$

$$\gamma_p^R = \sigma_{px} \frac{\mathrm{Var}[X|y_i,p]}{\sigma_x^2 \sigma_p^2 - \sigma_{px}^2}. \tag{3.9}$$

Given a realization y_i of Y_i and a realization p of $P(X,Z)$, $z_i^B(b_{0i}, z_{0i}, y_i, P(x,z))$ can be similarly calculated. The counterpart of (3.8) is

$$z_i^B(b_{0i}, z_{0i}, y_i, p) = \frac{r(\mathrm{E}[X|y_i] - p)}{\mathrm{Var}[X|y_i]}. \tag{3.10}$$

The counterpart of equations (3.9) is

$$\mathrm{Var}^{-1}[X|y_i] = h_s + h_x,$$

$$\mathrm{E}[X|y_i] = \gamma_0^B + \gamma_y^B y_i,$$

where

$$\gamma_0^B = \mathrm{E}[X] - \gamma_y^B \mathrm{E}[X],$$

$$\gamma_y^B = h_s \mathrm{Var}[X|y_i]. \tag{3.11}$$

Substituting (3.9) into (3.8) and (3.11) into (3.10), substituting the resulting expressions into the market-clearing condition (3.1) in Definition 3.1, and using the fact that $\int_{i \in R} y_i = \int_{i \in B} y_i = x$, we have the following equation:

$$\frac{(1-\lambda^B)(\gamma_0^R + \gamma_y^R x + \gamma_p^R p - p)}{\mathrm{Var}[X|y_i,p]} + \frac{\lambda^B(\gamma_0^B + \gamma_y^B x - p)}{\mathrm{Var}[X|y_i]} = \frac{z}{r}. \tag{3.12}$$

Two equations in the two unknowns π_x and π_z can be obtained from the above equation:

$$\frac{h_s}{\pi_x} = \frac{(1-\lambda^B)(1-\gamma_p^R)}{\mathrm{Var}[X|y_i,p]} + \frac{\lambda^B}{\mathrm{Var}[X|y_i]},$$

$$\frac{1}{r\pi_z} = \frac{(1-\lambda^B)(1-\gamma_p^R)}{\mathrm{Var}[X|y_i,p]} + \frac{\lambda^B}{\mathrm{Var}[X|y_i]}.$$

Substituting (3.7) and (3.9), and (3.11) into the above two equations and solving them give (3.2) and (3.3). Finally, taking unconditional expectations of the both sides of (3.12) with respect to x and z yields

$$\mathrm{E}[P] = \mathrm{E}[X],$$

which together with (3.5), (3.2) and (3.3) implies (3.4). *Q.E.D.*

Note from the above proof,

$$\mathrm{Var}^{-1}[X|y_i, p] - \mathrm{Var}^{-1}[X|y_i] = \frac{h_x \sigma_{px}^2}{\sigma_x^2 \sigma_p^2 - \sigma_{px}^2}.$$

The above expression can be interpreted as the "informativeness" of the equilibrium price. It shows how much the information about the return X embodied in the equilibrium price improves on the information provided by the private signal y_i. The informativeness of the equilibrium price is negatively related to the correlation between the price and the return X. It is straightforward to verify that the informativeness of the equilibrium price does not depend on the relative proportion λ^B of boundedly rational investors.

3.2. Price stability

Proposition 3.2 holds for any relative proportion λ^B. Simple comparative statics results can be obtained by varying λ^B. In the current static model, price stability may be identified with the variance of the price of risky asset. The question of the relation between rationality of the investors and price stability can be addressed by a comparative statics exercise. The following result is a corollary of Proposition 3.2.

PROPOSITION 3.3. *σ_p^2 is a decreasing function of λ^B.*

PROOF. From (3.2), π_x decreases with λ^B. Using (3.3), (3.7) can be written as:

$$\sigma_p^2 = \pi_x^2(\sigma_x^2 + \sigma_z^2/(h_s r)^2),$$

from which the proposition follows immediately. *Q.E.D.*

The intuition behind the proposition is simple. By (3.10) and (3.11), a boundedly rational investor's expectation of the return X does not depend on p, and p enters his demand for the risky asset through the budget constraint only. This contrasts with the demand for the risky asset by a rational investor. It is easy to see from Proposition 3.2 that in equilibrium p and x are positively correlated. Therefore γ_p^R is positive, and the rational investor's expectation of the return X depends positively on p. Thus, compared with the demand for the risky asset by a boundedly rational investor, the demand for the risky asset by the rational investor is less negatively responsive to increases in p. As the relative size of boundedly rational investors increases in the economy, the total demand for the risky asset becomes more negatively responsive to increases in the price, which makes the equilibrium price more stable. Stated differently, rational investors destabilize the price, while boundedly rational investors stabilize it.

In fact, it is even possible that the demand by the rational investor is an increasing function of p. From (3.8), the necessary and sufficient condition for this to occur is that $\gamma_p^R > 1$. It is straightforward to demonstrate that $\gamma_p^R < 1$ if $\lambda^B = 0$, and that there are parameter values such that $\gamma_p^R > 1$ for some $\lambda^B > 0$. For example, if $\lambda^B = 1$, then $\gamma_p^R > 1$ is equivalent to $h_z r^2 > \sigma_x^2 + \sigma_s^2$.

From Proposition 3.3, π_x decreases as λ^B increases. It might seem unintuitive that the equilibrium price P depends less on the fundamental X as the proportion of boundedly rational investors increases, since boundedly rational investors rely more on their private information about the fundamental than rational investors, who use both their own private information and the information contained in equilibrium price. A closer examination of equations (3.8)-(3.11) reveals that this is not the case: the "elasticity" of demand with respect to private information is the same for the boundedly rational and rational investors.

Boundedly rational investors in this paper are inexperienced investors who do not know the equilibrium price function and are therefore incapable of inferring information from the price. These investors can be alternatively thought of as "fundamentalists" who are capable of inferring information from the price but do not care to do so because they trust only their private information about the fundamental. The opposite of fundamentalists is "chartists," irrational investors who do not care to use their private information and rely exclusively on information provided by the market price (price charts) in forming their demand. Proposition 3.3 shows that fundamentalists contributes to price stability. Since chartists and fundamentalists behave in opposite ways, intuition suggests that chartists should destabilize. This intuition turns out to be flawed. Following the same analysis as in Proposition 3.2, one can show that when the market is populated by three groups of investors, chartists, fundamentalists (boundedly rational investors), and rational investors, the equilibrium price function is $P = \pi_0 + \pi_x X - \pi_z Z$, where[7]

$$\pi_x = \frac{(1-\lambda^C)h_s + (1-\lambda^B)h_z h_s^2 r^2 (1-\lambda^C)^2}{h_x + (1-\lambda^C)h_s + (1-\lambda^B)h_z h_s^2 r^2 (1-\lambda^C)^2},$$

$$\pi_z = \pi_x / ((1-\lambda^C)h_s r),$$

$$\pi_0 = (1-\pi_x)\mathrm{E}[X],$$

and λ^C is the proportion of chartists in the population. It can be shown from the above equations that π_x decreases in λ^C. Thus, the equilibrium price P depends less on the fundamental X as the proportion of chartists increases.[8] This is clearly because chartists disregard their private information and so there is less information aggregation by the price. Relatedly, the informativeness of equilibrium price, defined as $\mathrm{Var}^{-1}[X|y_i, p] - \mathrm{Var}^{-1}[X|y_i]$, can be shown to decrease as λ^C increases. However, price instability, as measured by σ_p^2, does not necessarily increase as λ^C increases. It can be shown that the necessary and sufficient condition for σ_p^2 to increase with λ^C is

$$\frac{1-\lambda^B}{1/(r^2 h_s h_z) + (1-\lambda^B)(1-\lambda^C)} < \frac{1/(r^2 h_s h_z) - \lambda^B(1-\lambda^C)}{h_x/(r^2 h_s^2 h_z) + (1-\lambda^C)^2}.$$

Thus, changes in the proportion of chartists have ambiguous effect on price stability.

3.3. Welfare implications

The expected utility of a rational investor with b_{0i} and z_{0i}, conditional on y_i and p and given that $z_i^R = z_i^R(b_{0i}, z_{0i}, y_i, p)$, can be written as

$$\begin{aligned}
\mathrm{E}[-\exp(-(b_i + z_i^R X)/r)|y_i, p] &= -\exp\left(-\frac{\mathrm{E}[b_i + z_i^R X|y_i, p]}{r} + \frac{\mathrm{Var}[b_i + z_i^R X|yi, p]}{2r^2}\right) \\
&= -\exp\left(-\frac{b_{0i}}{r}\right)\exp\left(-\frac{(\mathrm{E}[X|y_i, p] - p)^2}{2\mathrm{Var}[X|y_i, p]} - \frac{p z_{0i}}{r}\right).
\end{aligned}$$

Let $U^R(b_{0i}, z_{0i})$ be the unconditional expected utility of a rational investor with b_{0i} and z_{0i}, before y_i and p are realized. Then, by the covariance matrix (3.6), $U^R(b_{0i}, z_{0i})$ is given by

$$-\exp\left(-\frac{b_{0i}}{r}\right)\int_{y_i} dy_i \int_p \frac{f(y_i,p)}{2\pi\sqrt{A}} \exp\left(-\frac{(\mathrm{E}[X|y_i,p]-p)^2}{2\mathrm{Var}[X|y_i,p]} - \frac{pz_{0i}}{r}\right) dp, \tag{3.13}$$

where $2\pi\sqrt{A}f(y_i,p)$ is the probability density of the bivariate normal distribution (Y_i, P), and

$$A = (\sigma_x^2 + \sigma_s^2)\sigma_p^2 - \sigma_{px}^2;$$

$$f(y_i,p) = \exp\left(-\frac{\sigma_p^2(y_i - \mathrm{E}[X])^2}{2A} + \frac{\sigma_{px}(y_i - \mathrm{E}[X])(p-\mathrm{E}[X])}{A} - \frac{(\sigma_x^2+\sigma_s^2)(p-\mathrm{E}[X])^2}{2A}\right).$$

Substituting (3.8) and (3.9) into (3.13), and sufficient simplifications lead to

$$U^R(b_{0i}, z_{0i}) = -\sqrt{\frac{\mathrm{Var}[X|y_i,p]}{\mathrm{Var}[X-P]}} \exp\left(-\frac{b_{0i} + z_{0i}\mathrm{E}[X]}{r} + \frac{z_{0i}^2(\sigma_x^2\sigma_p^2 - \sigma_{px}^2)}{2r^2\mathrm{Var}[X-P]}\right). \tag{3.14}$$

Let $U^B(b_{0i}, z_{0i})$ be the unconditional expected utility of a boundedly rational investor with b_{0i} and z_{0i}, before y_i and p are realized. $U^B(b_{0i}, z_{0i})$ can be calculated in a similar fashion as above:

$$U^B(b_{0i}, z_{0i}) = -\sqrt{\frac{\mathrm{Var}[X|y_i,p]}{D[X-P]}} \exp\left(-\frac{b_{0i} + z_{0i}\mathrm{E}[X]}{r} + \frac{z_{0i}^2(\sigma_x^2\sigma_p^2 - \sigma_{px}^2)}{2r^2 D[X-P]}\right), \tag{3.15}$$

where

$$D[X-P] = \mathrm{Var}[X-P] - \frac{h_x^2(\sigma_x^2 - \sigma_{px})^2\sigma_{px}^2}{h_s \mathrm{Var}[X-P]}.$$

A comparison of (3.14) and (3.15) immediately reveals that $U^R(b_{0i}, z_{0i}) > U^B(b_{0i}, z_{0i})$, for any b_{0i} and z_{0i}. Clearly, this follows from the fact that a boundedly rational investors's demand for the risky asset is not optimal because it does not exploit the information about the return X carried by the equilibrium price. A more interesting question is how the relative size of boundedly rational investors affects the welfare of each rational investor and boundedly rational investor in the economy. It turns out that answer to this question depends on the relative size of boundedly rational investors in the economy and on the endowment of the risky asset of the particular investor in question. For the following proposition, define

$$K = \frac{\sigma_z^2(\sigma_x^2 + \sigma_s^2)}{\sigma_x^2\sigma_s^2 r^2 + \sigma_z^2(\sigma_x^2+\sigma_s^2)}.$$

Note K is a positive constant less than one.

Proposition 3.4. *If $\lambda^B > \sigma_s^2\sigma_z^2/r^2 + 1 - K$, then both $U^R(b_{0i}, z_{0i})$ and $U^B(b_{0i}, z_{0i})$ are increasing in λ^B for any b_{0i} and z_{0i}; for any given λ^B, if the absolute value of z_{0i} is great enough, then both $U^R(b_{0i}, z_{0i})$ and $U^B(b_{0i}, z_{0i})$ are increasing in λ^B.*

Proof. By Proposition 3.2, the equilibrium does not depend on the distribution of endowments b_{0i} and z_{0i}.

The right-hand-side of (3.14) can be viewed as a function of π_x. By Proposition 3.2, π_x is a decreasing function of λ^B. Taking derivatives of (3.14) with respect to π_x then implies that $dU^R(b_{0i}, z_{0i})/d\lambda^B$ is proportional to

$$\left(\frac{z_{0i}}{r}\right)^2 \frac{d((\sigma_x^2\sigma_p^2 - \sigma_{px}^2)/\mathrm{Var}[X-P])}{d\pi_x} - \frac{1}{\mathrm{Var}[X-P]}\frac{d\mathrm{Var}[X-P]}{d\pi_x}. \tag{3.16}$$

It is straightforward to verify that

$$\frac{\sigma_x^2\sigma_p^2 - \sigma_{px}^2}{\mathrm{Var}[X-P]} = \frac{\pi_x^2\sigma_x^2\sigma_z^2}{(1-\pi_x)^2\sigma_x^2(h_s r)^2 + \pi_x^2\sigma_z},$$

so that the first term in (3.16) is always positive. Furthermore, $d\mathrm{Var}[X-P]/d\pi_x$ is proportional $\sigma_s^2\sigma_z^2/r^2 - \lambda^B$, so that the expression given by (3.16) is positive for any z_{0i}, if $\lambda^B > \sigma_s^2\sigma_z^2/r^2$.

Similarly, taking derivatives of (3.15) with respect to π_x implies that $dU^B(b_{0i}, z_{0i})/d\lambda^B$ is proportional to

$$\left(\frac{z_{0i}}{r}\right)^2 \frac{d((\sigma_x^2\sigma_p^2 - \sigma_{px}^2)/D[X-P])}{d\pi_x} - \frac{1}{D[X-P]}\frac{dD[X-P]}{d\pi_x}. \tag{3.17}$$

It can be verified that

$$\frac{\sigma_x^2\sigma_p^2 - \sigma_{px}^2}{D[X-P]} = \frac{\pi_x^2\sigma_x^2\sigma_z^2}{K(1-\pi_x)^2\sigma_x^2(h_s r)^2 + \pi_x^2\sigma_z}.$$

The first term in (3.17) is therefore always positive. Moreover, $dD[X-P]/d\pi_x$ can be shown to be proportional $\sigma_s^2\sigma_z^2/r^2 + 1 - K - \lambda^B$, so that the expression given by (3.17) is positive for any z_{0i}, if $\lambda^B > \sigma_s^2\sigma_z^2/r^2 + 1 - K$.

Therefore, if $\lambda^B > \sigma_s^2\sigma_z^2/r^2 + 1 - K$, both $U^R(b_{0i}, z_{0i})$ and $U^B(b_{0i}, z_{0i})$ are increasing in λ^B for any b_{0i} and z_{0i}. Furthermore, since the equilibrium does not depend on the distribution of endowments b_{0i} and z_{0i}, for any given λ^B, if the absolute value of z_{0i} is great enough, then both $U^R(b_{0i}, z_{0i})$ and $U^B(b_{0i}, z_{0i})$ are increasing in λ^B.

Q.E.D.

Although Proposition 3.4 is stated in terms of effect of the relative size of boundedly rational investors on the welfare of individual investors, it can be restated for the welfare of the whole economy. For example, in an economy where all investors are endowed with large (positive or negative) quantities of the risky asset, if some of the investors behave in a boundedly rational fashion, then every investor, including those who are boundedly rational, are better off in terms of greater unconditional expected utility. Thus, although each investor benefits from conditioning his asset demand on the information carried by the equilibrium price, it can happen that all investors lose by doing so. The reason is that risk-averse investors, rational ones and boundedly rational ones alike, dislike volatility of the price, especially when they are endowed with large quantities of the risky asset. By Proposition 3.3, the equilibrium price is less volatile when the relative size of boundedly rational investors increases, and so all investors can benefit such an increase.

4. Concluding Remarks

In some financial markets, it is possible to invest more resources to collect better private information about the return of the asset. For example, investors can spend more time following relevant news reports or examining the trend in the return of the asset. This can be formalized in the present model by allowing them to choose the quality of their private information before the markets open. Higher quality of private information, in the form of greater precision of the signals, can be assumed to be more costly in terms of the single consumption good. As long as the cost of acquiring better private information is the same for all investors, this extension of the model keeps intact the idea that it is the ability to infer information from the price, and not a greater access to superior private information, that distinguishes the experienced investors from the inexperienced investors in financial markets. For a rational investor, the usefulness of his private information depends negatively on how informative the equilibrium price is. Since boundedly rational investors do not know how to infer information from the equilibrium price, they spend more in collecting private information than rational investors do. Therefore, unlike the model presented here, the informativeness of the equilibrium price will be positively related to the relative size of boundedly rational investors. As the price becomes more informative with greater relative size of boundedly rational investors, demands for the risky asset by rational investors become even less negatively responsive to increases in the price, adding to instability of the equilibrium price. It can still be shown that there is a positive relation between the size of boundedly rational investors and price stability, but the relation will be weaker than in the case discussed in the present paper, where rational and boundedly rational investors have the same quality of private signals.[9]

The model of bounded rationality can be useful for discussing other issues in rational expectations equilibrium under heterogeneous information. In particular, because the equilibrium price carries valuable information, boundedly rational investors would be willing to pay for advices from rational ones. Extending the model by allowing for a market for financial consultation is an interesting topic, and it will also be interesting to investigate the effects of a market of financial consultation on the informational content of the equilibrium price and on the stability of the price.

References

Friedman, Milton, *Essays in Positive Economics.* Chicago: University of Chicago Press, 1953.

Grossman, Sanford J., "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information, " *Journal of Finance* 31: 573-585, 1976.

Grossman, Sanford J., "The Existence of Future Markets, Noisy Rational Expectations and Informational Externalities," *Review of Economic Studies* 64:431-449, 1977.

Hart, Oliver D., and Kreps, David M., "Price Destabilizing Speculation," *Journal of Political Economy* 94: 927-952, 1986.

Hellwig, Martin F., "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory* 22:477-498, 1980.

Kyle, Albert S., "Continuous Auctions and Insider Trading," *Econometrica* 53: 1315-1336, 1985.

De Long, J. Bradford, Shleifer, Andrei, Summers, Lawrence H., and Waldmann, Robert J., "The Survival of Noise Traders in Financial Markets," *Journal of Business* 64: 1-19, 1991.

Stein, Jeremy C., "Informational Externalities and Welfare-reducing Speculation," *Journal of Political Economy* 95: 1123-1145, 1987.

Verrecchia, Robert E., "Information Acquisition in a Noisy Rational Expectations Economy," *Econometrica* 50: 1415-1431, 1982.

Footnotes

1. A draft of the paper was completed in 1994. The author would like to thank Michael Chwe and Michael Woodford for helpful discussions. Financial support from the Sloan Foundation Dissertation Fellowship is gratefully acknowledged.

2. See, for example, the report in March 15 issue of *The Economist*.

3. Throughout the paper, random variables are denoted by capitalized letters, and their realizations are denoted by corresponding lower cases.

4. This assumption can be easily modified without changing the results of this paper. See the last section for remarks on a more meaningful way of relaxing this assumption.

5. See Hellwig [1980] for a detailed argument.

6. See Hellwig [1980] for a formal argument based on convergence of a sequence of discrete economies.

7. I use the same notation as the last section to ease the burden.

8. In considering the effect of an increase in λ^C, I implicitly assume that the proportion of rational investors decrease while λ^B remains the same. If the increase in λ^C comes with the decrease in λ^B while the proportion of rational investors remains the same, π_x can either decrease or increase depending on parameters.

9. Proof of this claim is available from the author by request.